Macroeconomic Effects of Devaluation Rate Changes:

Dynamic implications under alternative regimes

of capital mobility

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Abstract: This paper uses a model of endogenous labor supply to study the effects of changes in the devaluation rate in small open economies under different regimes of capital mobility. Under no international capital mobility, a permanent cut in the devaluation rate causes long run consumption and output to rise and is thus non-neutral. This is so even when central bank reserves do not earn interest and hence when a well known channel of non-neutrality is absent. The effects of permanent, temporary and preannounced sequential changes in the devaluation rate are analyzed under capital market imperfections where the interest rate is linked to the net indebtedness of the country. The paper finds that amongst these competing explanations for the current deficits and the booms in real activity that characterized exchange rate based stabilization programs, the "temporary policy" hypothesis seems to be the most robust to alternative specifications. Lastly, the effects of devaluation rate changes are also analyzed under capital controls. The paper documents conditions under which the dynamic implications of controls on capital flows and capital market imperfections are qualitatively similar.

JEL Classification: E2, F3, F4

Key words: Devaluation rates, non-neutralities, imperfect capital mobility

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This paper studies the effect of changes in the devaluation rate in small open economies under regimes of no capital mobility and imperfect capital mobility. The issue regarding the effects of changes in the devaluation rate first attracted attention in the late 1970's when some countries used the rate of devaluation as a nominal anchor in an attempt to reduce chronically high inflation rates.

Among the well known papers on this issue, Calvo (1981) showed in the context of an endowment economy model with no capital mobility that a permanent cut in the devaluation rate would cause a trade surplus in the short run but no long run real effects. Using a similar model, Obstfeld (1986) showed that in settings where central bank foreign exchange reserves earn interest and the government transfers the interest earnings to private agents, a permanent cut in the devaluation rate would not only cause a short run trade surplus but also occasion an increase in consumption in the long run since the increase in foreign exchange reserves induced by the trade surplus would cause a positive wealth effect. Thus, the long run neutrality result obtained by Calvo (1981) would no longer hold.

A parallel and closely connected literature has focused a lot of attention on the stylized facts that emerged about the exchange rate based stabilization programs mentioned above. Three facts about these programs that have attracted the greatest attention are (a) the consumption and output booms that emerged in these countries; (b) the sustained current account deficits that were run by these economies; and (c) the gradual appreciation of the real exchange rate. Details of these programs can be found in Kiguel and Liviatan (1992) and Vegh (1992).

Explanations of these stylized facts have typically centered on four channels: (a) Dornbusch (1982) and Rodriguez (1982) argue that in the presence of price inertia, a cut in the rate of devaluation
lowers the real interest rate which, in turn, sparks the expansion in real activity1; (b) Calvo (1986), Calvo and Vegh (1993) and Calvo and Drazen (1994) argue that if the policy change is not credible and is thus perceived to be a temporary change then it would cause an intertemporal consumption substitution towards present consumption since the transactions costs of consumption would be lower in the current periods relative to the future; (c) Lahiri (1995), Roldos (1994) and Uribe-Echevarria (1993) argue that permanent cuts in the devaluation rate could cause the stylized facts described above due to supply side adjustments even in the absence of either nominal frictions or credibility problems; and (d) Obstfeld (1985) shows that when money and consumption are substitutes, a preannounced sequential reduction in the devaluation rate would cause a consumption boom and current account deficit on impact. However, all the papers mentioned above build and present their arguments in the context of models with perfect capital mobility.

This paper revisits both these issues in the context of alternative regimes of capital mobility. In particular, the paper analyzes the effects of changes in the devaluation rate under regimes of both no capital mobility and imperfect capital mobility using a model where there is an endogenous labor-leisure choice and output is a function of labor. The key feature of the model is that in settings where agents face a cash-in-advance (CIA) on consumption, a cut in the devaluation rate lowers the opportunity cost of holding money and, thus, alters the optimal mix of consumption and leisure. In particular, it induces an increase in both labor supply and consumption.

One should note that this kind of a model has been used before by, among others, Aschauer and Greenwood (1983) to analyze the welfare effects of fixed and flexible exchange rates, by Stockman (1985).

1Calvo and Vegh (1994), however, show in a utility maximizing framework that in order for the fall in the real interest rate to be expansionary the intertemporal elasticity of consumption substitution has to exceed the intratemporal elasticity of substitution between traded and non traded goods. Available econometric estimates of these two elasticities indicate that the opposite is more likely to be true.
to analyze the effects of steady state inflation on the direction and size of international trade, and by Lahiri (1995) to study the effects of exchange rate based stabilization programs under perfect capital mobility.

The paper shows that under no capital mobility, a permanent cut in the rate of devaluation occasions not only a short run trade surplus but also an increase in consumption and output in the long run due to the positive labor supply response. This is so even when central bank reserves earn no interest and hence, the channel for long run non-neutrality in Obstfeld (1986) is absent. A transitory cut in the devaluation rate, on the other hand, causes an initial balance of payments surplus which changes to a deficit before the program collapse. It also causes a boom in output and consumption both of which disappear before the end of the program.

The paper demonstrates that under regimes where international capital markets are imperfect so that international creditors link the interest rate facing the country to a measure of its creditworthiness like its net external indebtedness, a permanent cut in the devaluation rate causes an immediate increase in consumption and output but no current account effects since all adjustments are instantaneous. This result is identical to that under perfect capital mobility. A temporary cut in the devaluation rate, however, causes a consumption and output boom as well as a current account deficit to emerge thus reproducing the stylized facts. The paper also develops conditions under which capital market imperfections and capital controls have similar dynamic effects. In particular, it is shown that the equilibrium dynamics implied by environments where international capital markets are perfect but the government imposes some restrictions on international capital flows through a tax (a "Tobin" tax) are similar to the dynamics under capital market imperfections as long as the government targets the stock of country assets and, thus, varies the Tobin tax inversely with the stock of net country debt.
The paper also investigates the implications of a preannounced sequential lowering of the devaluation rate (as in the Southern Cone countries of Argentina, Chile and Uruguay in the late 1970's). This, as noted above, has been used by Obstfeld (1985) to account for the consumption booms and current account deficits that emerged in these economies. We demonstrate that when money is introduced into the model developed here through the utility function and money and consumption are substitutes, a preannounced disinflation program indeed causes an impact increase in consumption and a current account deficit but also causes an output contraction due to a fall in labor supply. The implied contraction in output contradicts the stylized facts associated with these programs. It is important to note that Obstfeld (1985) did not focus on supply side adjustments since he used a demand driven model with the one good version of the model being an endowment economy model.

One should note that the particular model of variable labor supply (endogenous labor-leisure choice) adopted here is intended to be a stylized representation of some basic realities in chronic high inflation economies. It is meant to capture the fact that in these economies agents devote a disproportionate amount of time and resources towards efficient money management - coordinating transactions, optimally reducing average daily cash holdings, etc. Reductions in the rate and variability of inflation typically reduce the amount of time and resources devoted to these activities and, hence, are likely to induce an increase in effective labor supply.\textsuperscript{2}

The next section presents the model and the results under the assumption of no capital mobility. Section III studies the model under imperfect capital mobility while the last section concludes. An appendix to the paper studies sequential cuts in the devaluation rate when money and consumption are

\textsuperscript{2}An alternative but analytically more complicated modelling strategy would be to introduce a transactions cost technology involving money and leisure time where both inputs reduce the costs of effecting transactions. All the results reported here can be qualitatively reproduced as long as money and leisure time are substitutes in the transaction cost technology.
substitutes by relaxing the cash-in-advance constraint and introducing money into the model through the utility function.

II. The model under no capital mobility

Consider a small open economy which consumes and produces a single traded good. The world price of the good is exogenous and constant and, for convenience, is normalized to unity. With no impediments to trade purchasing power parity ensures that the domestic currency price of the good is given by the nominal exchange rate, E. The economy is inhabited by identical, infinitely lived agents who derive utility from consumption and leisure. Thus, the representative agent maximizes the present discounted value of lifetime utility given by

\[ V = \int_{t=0}^{\infty} e^{-\rho t} [u(c_t) + v(l_t)] dt \]  

(1)

where \( u \) and \( v \) are concave and twice differentiable in their respective arguments and where \( c \) denotes consumption while \( l \) denotes leisure. \( \rho \) is the subjective rate of time preference which is exogenous and constant. Each agent has an endowment of one unit of time which can be allocated to labor or leisure. Thus, at each point in time we must have

\[ L + l = 1 \]  

(2)

where \( L \) denotes labor. The traded good is produced using labor as the only input and an increasing, twice differentiable, strictly concave production technology. Thus,

\[ y = f(L), \quad f' > 0, f'' < 0 \]  

(3)

There is no international capital mobility and the only available asset is domestic money. Agents face
a cash-in-advance constraint on consumption:

\[ m_t \geq \alpha c_t \]  

(4)

where \( m_t \) are real balances at time \( t \). As long as the nominal interest is positive (4) will hold with equality. Throughout the paper I shall confine attention to paths along which the nominal interest rate is positive and hence, (4) holds as an equality. Since we shall essentially engage in a local analysis of the model around the steady state, the assumption is reasonable as long as the steady state interest rate is positive.

The government in this economy is assumed to print money and make lump sum transfers to private agents, \( g \). The government's budget constraint is given by

\[ g = \pi m \]  

(5)

where \( \pi \) is the devaluation rate. Note that since the domestic currency price of the good is the nominal exchange rate, the domestic inflation rate, \( \pi \), is just the devaluation rate. Hence, the government just compensates private agents for the depreciation of their money balances due to inflation. Note that I assume that central bank foreign exchange reserves do not earn interest. Since the attempt here is to investigate the consequences of changes in the devaluation rate and since it is well known from Obstfeld (1986) that government transfers of its interest earnings on foreign exchange reserves would cause permanent changes in the devaluation rate to have real effects, this is a natural assumption to make. It eliminates a potential channel of non-neutrality.

Since the focus of the paper is on the effects of changes in the exchange rate policy, it will be assumed throughout the paper that the government fixes the time path of the nominal exchange rate. Thus, monetary policy is endogenous with the government accommodating changes in domestic money demand through changes in money supply.

With the no international capital mobility assumption, the flow budget constraint for the
representative agent is given by

\[ \dot{m} = f(1-L) + g - c - \pi m \]  

(6)

where time subscripts on the variables have been dropped for notational convenience and where the time endowment constraint for the agent, equation (2), has been substituted into the production function\(^3\). We can now combine (6) with the cash-in-advance (CIA) constraint, equation (4), to get

\[ \dot{m} = f(1-L) + g - \left( \frac{1+\alpha \pi}{\alpha} \right) m \]  

(7)

The representative agent maximizes (1) subject to (4) and (7) by choosing leisure, \( l \), at every instant. Note that given \( m \) and \( g \), choosing \( l \) implies choosing the accumulation of domestic real money balances.

Among the first order conditions for the agent’s problem are

\[ v'(l) = f'(1-L) \lambda \]  

(8)

\[ \lambda = \frac{1}{\alpha} \left[ 1+\alpha(\rho+\pi) \right] - u\left( \frac{m}{\alpha} \right) \]  

(9)

where \( \lambda \) is the costate variable associated with \( m \). It is easy to see from (8) and (9) that in steady state we must have

\[ \rho + \pi = \frac{u'(c)f'(L) - v'(l)}{\alpha v'(l)} \]  

(10)

Note that \( \rho + \pi \) would equal the nominal interest rate in steady state if we introduced a domestically traded nominal bond into the model. A necessary condition for the optimality of steady state consumption would then be the equality of the domestic real interest rate with the rate of time preference, \( \rho \). Hence, in

\[^3\text{For convenience, the household has been modelled as both a consuming and producing unit. Separating the two decision making units leaves the results unchanged.}\]
steady state $\rho + \pi$ would equal the nominal interest rate. Since we are restricting attention to paths along which the nominal interest rate is positive, the right hand side of (10) has to be positive as well.

We can use (8) to implicitly solve for $l$ as a function of $\lambda$:

$$l = H(\lambda), \quad H' > 0$$  \hspace{1cm} (11)

Equation (11) shows that as wealth rises leisure rises as well. We are now in a position to fully characterize the equilibrium path for this economy. From (5), (7), (9) and (11) one can see that paths satisfying optimality and general equilibrium must simultaneously satisfy equation (9) and

$$\dot{m} = f(l - H(\lambda)) - \frac{m}{\alpha}$$  \hspace{1cm} (12)

The right hand side of (12) gives the trade balance for this economy.

The system defined by (9) and (12) when linearized around a steady state has two roots of opposite signs and, hence, exhibits saddle path dynamics. With the negative root driving the system all adjustments are monotonic along the stable path. Figure 1 shows the dynamics of the system through a phase plane. The $\dot{\lambda} = 0$ locus is downward sloping while the $\dot{m} = 0$ locus is upward sloping. The only path which converges to a steady state is the saddle path (labelled SS). All other paths diverge from the steady state over time. If the initial value of $m$ is different from its steady state value then the co-state variable, $\lambda$, adjusts to put the system on the saddle path which ensures convergence to the steady state.

II.1 Permanent cut in the devaluation rate:

A permanent cut in the devaluation rate causes $\pi$ to fall which induces the $\dot{\lambda} = 0$ locus to shift out to the right as shown in figure 2(a). The new saddle path is SS$_1$ which lies above SS$_0$. On impact, $\lambda$ jumps up to point a on SS$_1$ and thereafter the system travels along SS$_1$ to asymptotically regain steady
state at point b which is characterized by a higher $\lambda$ and $m$.

The implied time paths of output and consumption are shown in figure 2(b). From (11) we know that leisure is inversely related to $\lambda$. This implies that on impact output jumps up since the upward jump in $\lambda$ causes leisure to fall and labor supply to rise. Over time $\lambda$ falls as wealth grows. With the rise in money balances leisure rises as well which causes output to fall over time. Hence, on impact, output overshoots its long run equilibrium level and then approaches it from above. The new steady state output level, $y_1$, is greater than the old steady state level $y_0$. This is obvious from the fact that in the new steady state $\lambda$ is greater relative to its old equilibrium level.

Since $m$ is given at any instant, the CIA constraint on consumption implies that consumption cannot jump on impact. Over time $m$ rises which implies that consumption rises as well. During the transition the economy runs a balance of payments surplus which coincides with the trade balance surplus. The surplus narrows over time as output falls and consumption rises and is eliminated asymptotically. The balance of payments implications of the policy change are identical to the implications in Calvo (1981). However, the long run consumption and output dynamics are quite different.

The key feature to note is that in this model a change in the devaluation rate is not neutral in the long run. This is so despite the fact that central bank reserves are assumed not to earn interest and, hence, the central channel for the non-neutrality of devaluation rate changes in Obstfeld (1986) is absent. The intuition for the result is that the change in the devaluation rate alters the optimal mix of consumption and leisure and changes the equilibrium consumption and output levels.

It is worth noting that studying a model similar to the one developed over here, Lahiri (1995) has shown that under perfect capital mobility a permanent cut in the devaluation rate would cause an immediate increase in both consumption and output but no transitional dynamics or current account
movements since all adjustments are instantaneous. As seen above, without international capital mobility there is a similar response of output on impact of the cut. But now there are important transitional dynamics as the economy is forced to run a trade surplus in order to build up money balances to purchase the increased desired consumption.

II.2 Temporary cut in the devaluation rate:

It is by now well known that in a number of countries attempts to reduce chronically high inflation through reductions in the devaluation rate ended in failures with the program collapses coinciding with maxi-devaluations and reversion to higher devaluation rates. As mentioned above, a popular explanation of the consumption and output booms along with the current account deficits that emerged in these economies has in fact centered on this temporariness of policies. Since most of the work on this issue has been done in the context of perfect capital mobility, it is of interest to understand the effects of temporary policies in the absence of perfect capital mobility. In this section I focus on the case of no capital mobility while the case of imperfect capital mobility will be dealt with in the next section.

I consider the following experiment: at time $t=0$ the devaluation rate is lowered from $\pi_0$ to $\pi_1 < \pi_0$ for $0 \leq t < T$ and from time $T$ $\pi_t = \pi_0$ for $t \geq T$. Figure 3(a) illustrates the dynamic consequences of this policy. At time $t=0$ the $\lambda = 0$ locus shifts out to the new $\lambda_1 = 0$ locus with the new associated saddle path given by $SS_1$. However, at time $T$ the system shifts back to the original $\lambda = 0$ locus. The dynamic adjustment of the economy is dictated by the requirement that at time $T$ the economy has to be on the original saddle path $SS_0$ in order to ensure convergence to the steady state. Hence, at $t=0$ the system jumps up to a point such as $c$ to get on a dynamic path which hits $SS_0$ at exactly time $T$. Thereafter, the system travels along $SS_0$ to asymptotically regain steady state at the original equilibrium point.
The time paths of consumption and output are shown in figure 3(b). On impact output jumps up due to increased labor supply. But over time output falls monotonically till time $T$, undershooting its long run equilibrium level, $y_0$, in the process. From time $T$ onwards output starts rising monotonically towards its long run equilibrium level. Consumption, on the other hand, rises for a while but then starts declining before the program collapse date, $T$. This happens since the economy starts running down its excess money balances in anticipation of the program collapse and the associated higher devaluation rate from time $T$.

The balance of payments jumps into an initial surplus as money balances grow. However, the surplus falls over time as output declines while consumption grows. The surplus disappears completely when consumption reaches its maximum value. Thereafter, a balance of payments deficit emerges since the decline in output is greater than the fall in consumption. The deficit reaches a maximum at time $T$ and starts shrinking thereafter as output starts growing again and is eliminated asymptotically.

To summarize the results above, we have seen that appending a very simple supply side specification through an endogenous labor-leisure choice to an otherwise standard model causes long run real effects of permanent changes in the devaluation rate. We have also seen that the implications of exchange rate based stabilizations under no capital mobility are very different from those under perfect capital mobility. Whereas with perfect capital mobility the model implies immediate adjustments to permanent changes and hence no current account effects, with no capital mobility there are important transitional dynamics as the economy runs trade account imbalances in order to alter its money balances.

It is worth noting that the qualitative nature of the results remain unchanged when money is introduced into the model through the utility function rather than a cash-in-advance constraint as long as money and consumption are complements. This is to be expected since the cash-in-advance constraint
imposes complementarity between money balances and consumption.

The primary lesson to be drawn from the results is that a cut in the devaluation rate increases desired money balances. The change in desired money balances will alter the desired level of any variable which is correlated with money. This, in turn, will cause long run real effects of a permanent cut in the devaluation rate. Hence, as long as money is positively (negatively) related to consumption or negatively (positively) related to leisure, a permanent cut in the devaluation rate will increase (decrease) long run consumption and output. Note that a change in either consumption or leisure will change the steady state level of the other since in steady state consumption has to equal output.

Lastly, this model also permits some clear welfare statements regarding permanent cuts in the inflation rate. Rewriting equation (10) we get the steady state marginal rate of substitution between consumption and leisure to be \[ \frac{u'}{v'} = \frac{1 + ai}{f'} \] where \( i \) denotes the nominal interest rate. It is easy to check that the corresponding equation for an economy without a CIA constraint would not have the term involving \( i \) in the numerator on the right hand side of the preceding equation. As pointed out by Aschauer and Greenwood (1983), the CIA constraint on consumption introduces a wedge in the marginal product of labor and hence distorts the allocation between consumption and leisure. Hence, a permanent cut in the inflation rate increases welfare by reducing this distortion. Further, to attain the allocation in an undistorted economy, the monetary authority has to set \( \pi = -\rho \) which is Friedman's Optimum Quantity of Money rule.

\[ \text{We should also note that even if the utility function is separable in consumption and money there would still be real effects due to permanent changes in the devaluation rate as long as the utility function is non separable in money and leisure. In particular, all the results obtained here can be qualitatively reproduced as long as money and leisure are substitutes.} \]
III. Imperfect capital mobility

We now extend the model to the case where agents have access to world capital markets in which they can trade in bonds denominated in terms of the traded good. Given that the world price of the good is constant and normalized to unity, the face value of a bond is also unity. The bonds pay as interest \( r \) units of the good per unit of time. The interest rate, \( r \), is taken as given by atomistic agents who are individually too small to affect market conditions.

The intertemporal budget constraint facing the representative agent is now given by

\[
\int_{t=0}^{\infty} e^{-\gamma t(1-l)} dt = b_0 + \int_{t=0}^{\infty} e^{-\gamma t} (c_t - g_t + \mu_mp_t) dt
\]

where \( b_0 \) denotes the stock of outstanding private foreign debt at time \( t=0 \) while \( \mu_m \) denotes the seigniorage revenue raised by the government. Time differentiation of (13) yields the flow budget constraint for the agent:

\[
\dot{b} = rb + c + \mu m - f(1-l) - g
\]

where time subscripts on the variables have been dropped for notational convenience. Noting that

\( \dot{m} = \mu m - \pi m \)

and letting \( a \) (\( = b-m \)) denote net private debt, we can now use (4) and (14) to get:

\[
\dot{a} = ra + (1 + \alpha i)c - g - f(1-l)
\]

where \( i \) is the nominal interest rate which is given by \( r + \pi \). This follows from the interest rate parity condition due to capital mobility and the purchasing power parity condition \( P = E \) where \( P \) is the domestic currency price of the good. (15) shows that the effective price of consumption includes the cost of holding domestic money balances which are needed for purchasing consumption. Note that it has been assumed in deriving (15) that \( i \) is always positive so that the cash-in-advance constraint given by (4) is binding.
It is well known that in the presence of capital mobility, if central bank reserves do not earn interest then a change in the devaluation rate causes wealth effects when agents change the composition of their asset portfolio between domestic money holdings and foreign bond holdings (see Obstfeld 1986 for a discussion of this issue). In order to remove any real effects due to these wealth effects, I shall assume that central bank foreign exchange reserves earn interest at the world interest rate $r$ and are included in the lump sum transfers made to private agents. This is a natural assumption to make since the focus of this paper is on the potential non-neutralities of devaluation rate changes due to the endogenous labor-leisure choice. Thus, the lump sum transfers, $g$, made by the government to private agents is given by

$$r d_t + \pi m_t = g,$$

where $d_t$ are government foreign exchange reserves while $\pi m$ is the inflation tax ($\pi$ denotes the rate of inflation). The domestic credit rule given by (2) implies that the government increases domestic credit at the rate $\pi$ and reserves and real balances grow at the same rate. Obstfeld (1985) has a detailed discussion of this issue.

Among the first order conditions for the agent are

$$u'(c) = (1 + \alpha t)\lambda$$

$$v'(l) = f'(1-l)\lambda$$

$$\lambda = (\rho - r)\lambda$$

where $\lambda$ is the costate variable associated with $a$. Equations (17) and (18) can be used to solve for consumption and leisure as functions of $\lambda$ and the nominal interest rate, $i$. In particular, we have
\[ c = M(\lambda, 1 + \alpha t), \quad M_1 < 0, M_2 < 0 \] (20)

\[ l = H(\lambda), \quad H' < 0 \] (21)

where \( M_i \) denotes the partial derivative of \( M \) with respect to its \( i \)th argument. Equation (20) shows that consumption is declining in both \( \lambda \) and \( i \) while (21) shows that leisure, \( l \), is decreasing in \( \lambda \).

Differentiating (20) with respect to time we get

\[ \dot{c} = M_1 \dot{\lambda} + M_2 \alpha t \dot{\alpha} \] (22)

It is easy to see from equations (4) and (22) that determining the time paths of \( \lambda \) and \( n \) also determines the time paths of \( c, m \) and \( l \) and hence, the time path of output.

We define net foreign indebtedness as \( n = b - d \). Given this definition and since, as noted before, the specified domestic credit rule implies \( \dot{m} = \dot{d} \), we must have \( \dot{n} = \dot{d} \). Noting this and substituting (16) into (15) we get

\[ \dot{n} = mn + c - f(1 - l) = \dot{h} \] (23)

The right hand side of (23) denotes the current account deficit, \( h \).

III.1 Capital Market Imperfections

The first case we analyze is one of a world capital market imperfection. It is assumed that for the economy as a whole, international creditors link the interest rate to the net indebtedness of the country. In other words, the net indebtedness of the country is used by creditors as a proxy for the creditworthiness of the economy. In particular, I assume that
\[ r = r^* + t(n), \quad t' > 0 \] (24)

where \( n \) is net country debt (private debt minus public assets) and \( r^* \) is the risk free international interest rate. Hence, the interest rate rises with net country debt. Importantly, it is assumed that private agents take the interest rate as given.

Substituting equations (20), (21) and (24) in equations (19) and (23) we get the two fundamental differential equations that describe this economy:

\[ \dot{\lambda} = (\rho - r^* - t(n)) \lambda \] (25)

\[ \dot{n} = (r^* + t(n))n + M(\lambda, 1 + \alpha i) - f(1 - H(\lambda)) \] (26)

The system defined by (25) and (26) exhibits saddle path dynamics in the neighborhood around a steady state. Figure 4 shows the dynamics of the system. The \( \dot{\lambda} = 0 \) locus is vertical at the unique level of net foreign debt, \( n^* \), that is consistent with steady state. The slope of the \( \dot{n} = 0 \) locus is ambiguous as it can be upward sloping or downward sloping. Figure 4 has been drawn under the assumption that the \( \dot{n} = 0 \) locus is upward sloping\(^5\). One should note that none of the qualitative characteristics of the dynamic behaviour of \( \lambda \) and \( n \) due to the policy experiments carried out below depend on the slope of the \( \dot{n} = 0 \) locus. The only convergent path is the saddle path, SS, which is, however, unambiguously upward sloping. Divergence from \( n^* \) implies that \( \lambda \) jumps on to the saddle path to ensure convergence to steady state. We are now in a position to study the impact of changes in the exchange rate policy in this model.

1. **Permanent cut in the devaluation rate:**

Starting from a steady state, a permanent reduction in the devaluation rate causes the nominal

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\(^5\)The \( \dot{n} = 0 \) locus would be upward sloping if \( \rho + n t' > |M \alpha t'| \), i.e., if the revaluation effect of the interest rate increase on the existing stock of debt outweighs the reduction in consumption that the implied increase in the nominal interest rate causes.
interest rate to fall. This shifts the \( n = 0 \) locus to shift up to the \( n_1 = 0 \) locus as shown in figure 4. \( \lambda \) immediately jumps to its new higher steady state level and the system locks into its new steady state at point b instantaneously. There is no effect on the current account as consumption jumps up one-for-one with output. Note that the new steady state is characterized by higher consumption and output levels relative to the old steady state. Thus, as before, a cut in the devaluation rate is no longer neutral and has long run real effects.

Intuitively, agents increase their money holdings by borrowing from the world capital market and turning the increased borrowings over to the central bank. Thus, central bank reserves increase by the same amount as the increase in private foreign borrowings which leaves the economy’s net foreign debt, \( n \), unchanged. The instantaneous adjustment to the new steady state is identical to the perfect capital mobility case analyzed by Lahiri (1995). There too the instantaneous adjustment of output to its new steady state implied that there were no transitional dynamics and hence no current account effects.

As in the no capital mobility case, there are some clear welfare implications. Combining equations (17) and (18) we can see that, as before, the CIA constraint on consumption distorts the choice between consumption and leisure. Thus a reduction in the nominal interest rate is clearly welfare enhancing since it reduces the distortion.

2. **Temporary cut in the devaluation rate:**

As in the case of no capital mobility, a transitory cut in the devaluation rate is modelled as an experiment where the rate is lowered from \( \pi_0 \) to \( \pi_1 < \pi_0 \) for \( 0 \leq t < T \) and \( \pi_t = \pi_0 \) for \( t \geq T \). Figure 5(a) shows the implications for our dynamic system. As before, the \( n = 0 \) locus shifts up to the new \( n_1 = 0 \) locus. However, at time \( T \) the system reverts to the original configuration. Thus, the dynamics from \( t = 0 \) are dictated by the stability requirement that the system has to be on the original saddle path, \( SS_0 \).
at precisely time $T$. Thus, on impact, the economy jumps to a point such as $b$ which puts it on an unstable path that hits $SS_b$ at exactly time $T$.

The implied time paths of output and consumption are shown in figure 5(b). Upon impact consumption jumps up but thereafter it could increase or decrease depending on whether the dominant effect is the positive effect of a falling $\lambda$ (due to the increase in the interest rate induced by rising indebtedness) or the negative effect of the rising effective cost of consumption due to the rising nominal interest rate. Figure 5(b) has been drawn under the assumption that the positive effect dominates and hence, consumption rises between times 0 and $T$. From time $T$ onwards consumption rises unambiguously since both $\lambda$ and $n$ are falling. Thus, at time $T$ consumption jumps down to undershoot its long run steady state level and then grows over time to asymptotically regain its old steady state level $c_0$.

One should note that the jump down in consumption at time $T$ is greater than the jump up in consumption at time $t=0$. This can be seen from the fact that consumption at any time is a function of both $\lambda$ and the nominal interest rate. At time $t=0$ $\lambda$ jumps up which damps the upward jump in consumption due to the cut in the nominal interest rate. At time $T$ $\lambda$ cannot jump since time consistency requires that there can be no perfectly anticipated jumps in $\lambda$. This combined with the fact that by construction the rise in the nominal interest rate at time $T$ is exactly equal in magnitude to the fall in $i$ at $t=0$ implies that the jump down in consumption at time $T$ exceeds the upward jump in consumption at $t=0$. This is exactly what figure 5(b) shows.

The time path for output can be derived from the time path for $\lambda$. Equation (22) implies that along paths where $\lambda$ is falling labor supply rises and, hence, output grows. As can be seen from figure 5(a), $\lambda$ jumps up on impact and then declines monotonically. Hence, on impact, output jumps up but
then starts declining monotonically to asymptotically regain its original steady state value, $y$.

The consumption and output dynamics described above imply that at $t=0$ a current account deficit emerges due to the initial increase in consumption exceeding the corresponding increase in output. The current deficit widens till time $T$ due to declining output and increasing consumption. One should note that even if consumption falls during this period the current deficit still widens as output falls faster than the fall in consumption. At time $T$ the current deficit gives way to a current surplus as the economy retires its excess debt holdings on its way to regaining the steady state level of debt, $n$.

The above implies that a temporary cut in the devaluation rate would cause a consumption and output boom accompanied by a current account deficit. This seems to fit well with the stylized facts associated with exchange rate based stabilization programs described above. Furthermore, model predicts that the boom in real activity would disappear towards the end of the program which also seems to conform with the observed facts regarding this class of stabilization programs.

3. A sequential reduction in the devaluation rate:

As noted in the introduction, the Southern Cone countries of Argentina, Chile and Uruguay in the late 1970’s and Argentina, Brazil, Mexico and Israel in the mid-1980’s used the exchange rate as a nominal anchor in their attempt to lower chronically high three digit inflation rates. The Southern Cone countries, in particular, attempted to achieve their goal by announcing a table for the nominal exchange rate (called "tablitas" for little tables). These tables, in effect, gave the schedule for the gradual reduction in the devaluation rate.

I model this kind of a policy experiment as a sequential, two part reduction in the rate of devaluation of the domestic currency. In particular, suppose at time $t=0$ the devaluation rate is lowered from $\pi_0$ to $\pi_1 < \pi_0$ for $0 \leq t < T$ and it is announced that time $T$ the rate will be further reduced to $\pi_2 <
π₁ for t ≥ 0.

Figure 6(a) depicts the effect of this policy on our dynamic system. At t = 0 the n₁ = 0 locus shifts up to the n₁ = 0 locus. At time T there is a further shift of the n₁ = 0 locus to the n₂ = 0 locus. Thus, from time t = 0 till time T the dynamics are given by the system associated with the saddle path SS₁ while the convergent path from time T is given by the saddle path SS₂. In order to get to SS₂ at precisely time T, at time t = 0 the economy jumps to a point such as b which puts it on an unstable path that hits SS₂ at time T. Thereafter, the system travels along SS₂ to asymptotically regain steady state. Note that between times 0 and T net country indebtedness, n, is falling while from time T onwards, n monotonically rises towards its unique steady state level n*_T.

The dynamic implications of this policy for consumption and output are shown in figure 6(b). On impact, output jumps up due to the upward jump in λ. Thereafter, λ increases throughout and hence labor supply and output rise monotonically.

Consumption, on the other hand, jumps up on impact but the jump is smaller in magnitude relative to the jump in output since at time t = 0 a current surplus emerges. Between times 0 and T the surplus widens as output rises while consumption either falls or rises at a rate slower than the rise in output. Figure 6(b) depicts the consumption path on the assumption that the negative effect of the rise in λ dominates the positive effect of the declining effective price of consumption due to the falling nominal interest rate. Recall that the nominal interest rate declines as n falls.

At time T, the second cut in the nominal interest rate causes another upward jump in consumption which induces consumption to overshoot its long run steady state value c*... From time T consumption declines monotonically. This decline from time T is unambiguous since both n and λ are rising from time T onwards. The jump up in consumption at time T causes a current deficit to emerge at that time which
is only eliminated asymptotically.

Thus, the sequential reduction in the devaluation rate does indeed cause an expansion in real activity with output increasing over time and consumption jumping up at the two transition dates with its long run equilibrium value exceeding the original steady state level. However, it fails to account for the current deficits that emerged during the these programs.

As noted in the introduction the preannounced sequential disinflation mechanism has been used by Obstfeld (1985) to explain the boom in consumption and the current account deficits that emerged in the Southern Cone countries. The results derived here are consistent with his results since he showed that when money and consumption are complements, a sequential disinflation program would cause a current surplus to emerge. Note that the cash-in-advance constraint used here imposes a complementary relationship between consumption and money balances.

In the appendix to the paper it is shown that when money enters the model through the utility function and is a substitute for consumption, the model reproduces Obstfeld’s result on the current account deficit. However, under these conditions a disinflation program also causes output to jump down and decline monotonically over time as labor supply declines monotonically over time. The implied output contraction, however, contradicts the experience of these countries. These economies exhibited not only consumption booms and current deficits but also output booms.

The intuition for the output contraction result described above is that when consumption and money are substitutes, the disinflation program reduces the shadow value of wealth. As long as leisure is a normal good, this induces a decrease in labor supply which causes the output contraction. Note that Obstfeld (1985) analyzed an endowment economy model and thus ignored supply side adjustments. Thus, this channel of explanation of the stylized facts proposed by Obstfeld (1985) does not seem to be robust

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III.2 Capital Controls

Proposals for restricting the flow of private capital across countries have been around for a long time. Even during the Bretton Woods era of fixed exchange rates, the IMF recommended full convertibility only on current account transactions leaving countries free to impose restrictions on capital account transactions. This reflected a belief that such restrictions would dampen speculative capital flows which could be extremely destabilizing. Since the December 1994 Mexican crisis, proposals for some restrictions on capital flows gained strength once again as nervous international investors started pulling their capital out of other Latin American countries as well in what is now popularly referred to as the "tequila" effect.

Restrictions on capital flows can, potentially, be imposed in a variety of ways such as outright bans, taxes on capital flows, dual exchange rates - one for current account transactions and another for capital account transactions etc. In this subsection I investigate the impact of capital controls which are imposed through a tax on international capital flows. In particular, I focus on a "Tobin" tax where the government imposes a tax, \( t \), on interest income from foreign bond holdings. The tax rate is taken as given by private agents who can borrow and lend freely at the interest rate \( r \) in perfectly competitive international capital markets. This implies that the effective interest rate facing private agents is \( r - t \).

Noting this, we can rewrite the flow budget constraint for the representative agent as

\[
\dot{a} = (r - t) a + (1 + \alpha) c - g - f(1 - l)
\]  

(27)

where \( a (= b - m) \) denotes net private debt while the interest parity condition implies that the nominal interest rate, \( i \), is now given by \( r + \pi - t \). Note that net private debt, \( a \), would be negative if private
foreign borrowing, \( b \), is negative. In that event, equation (27) would look like the standard equation incorporating a tax on capital income from abroad. The first order conditions for individual optimality given by equations (17) and (18) remain unchanged while the evolution equation for the costate variable, \( \lambda \), gets modified as

\[
\dot{\lambda} = [r - (r - \tau)] \lambda
\]  

(28)

Optimal paths for consumption and leisure are still given by equations (20) and (21). In order to abstract away from any income effects emanating from the capital taxation, it is assumed that the government rebates all its tax revenues (both seignorage as well as the capital tax) through lump sum transfers to the private agent. Thus, the government's consolidated budget constraint is now given by

\[
r d + \pi m + tb = g
\]  

(29)

where, as before, \( d \) are foreign reserves of the central bank while \( tb \) denotes revenues from the Tobin tax. Substituting (29) into (27) yields the current account equation for this economy:

\[
n = r n + c - f(1-l)
\]  

(30)

where we have once again used the relationship \( d = n \) which is implied by the domestic credit rule.

The tax rate, \( t \), is a policy variable which can be varied by the government at its discretion. In this paper, I investigate a particular kind of policy whereby the government uses the tax to target the stock of net country debt, \( n (= d - b) \). In particular, I assume that

\[
t = t(n), \quad t' < 0
\]  

(31)

Thus, the tax rate rises when country debt falls or, equivalently, country assets rise and it falls when country debt (assets) rises (fall). In other words, the government makes it more attractive for agents to save when debt is rising and makes it less attractive to save by reducing the effective interest rate when
country assets are rising. Using equations (28), (30) and (31) we get the system of equations which drives the system:

\[ \dot{\lambda} = [\rho - (r - \tau(n))] \lambda \]  

\[ \dot{n} = r n + M(\lambda, 1 + \alpha i) - f(1 - H(\lambda)) \]  

This system looks very similar to the system given by equations (25) and (26) which described the capital market imperfection case. The differences are essentially quantitative but not qualitative. Note that previously the market imposed risk premium on foreign borrowing was increasing in the level of debt, \( n \), thereby increasing the effective interest rate. In equation (32), the tax rate, \( \tau \), falls as \( n \) rises thereby increasing the effective interest rate, \( r - \tau \). Not surprisingly, the system, as before, is saddle path stable and looks, qualitatively, identical to the case of a capital market imperfection which was depicted in figure 4. Adjustments to changes in the devaluation rate are also qualitatively identical. Thus, as long as the capital controls are structured to target the stock of country assets/debt, there is no difference between the type of capital market imperfection studied here and restrictions on capital flows.

IV. Conclusions

This paper has studied the impact of changes in the devaluation rate in a small open economy when there are restrictions on capital flows. In particular, we have seen that when agents supply labor elastically, changes in the devaluation rate of the domestic currency always have real effects. In environments where there is no international capital mobility and agents need to hold money balances for consumption purposes, a permanent cut in the devaluation rate causes an increase in long run output and consumption along with a transitional trade surplus. This is the case even in situations where central bank foreign exchange reserves do not earn interest and, hence, the channel of non-neutrality in Obstfeld
(1986) is absent. A temporary cut in the rate of devaluation also causes a consumption and output boom along with a trade surplus. But these dynamics reverse direction before the program ends. These results remain qualitatively unchanged when money enters the model through the utility function rather than a cash-in-advance constraint as long as money and consumption are complementary goods.

The primary lesson to be drawn from the results is that a cut in the devaluation rate increases desired money balances. The change in desired money balances will alter the desired level of any variable which is correlated with money. This, in turn, will cause long run real effects of a permanent cut in the devaluation rate. Hence, as long as money is positively (negatively) related to consumption or negatively (positively) related to leisure, a permanent cut in the devaluation rate will increase (decrease) long run consumption and output.

The extension of the model to the imperfect capital mobility case where agents do have access to world capital markets but where international creditors link the interest rate facing the economy to a measure of creditworthiness of the country such as its net indebtedness, yielded interesting insights. In this situation a permanent reduction in the devaluation rate causes an immediate increase in output and consumption but no current account dynamics since all adjustments take place instantaneously. A temporary cut, on the other hand, causes an expansion in real activity and a current account deficit during the program both of which disappear once the program is abandoned. We also saw that as long as governments try to restrict the international flow of private capital through a Tobin tax but use the tax to target the stock of country assets/debt, the dynamics under capital controls and capital market imperfections look very similar.

Lastly, we also saw that a sequential reduction in the devaluation rate as in the "tablitas" would indeed cause a boom in real activity but would induce a current surplus rather than the deficits that
actually emerged. Further, even when money is introduced through the utility function and, as in Obstfeld (1985), money and consumption are substitutes, a preannounced disinflation program does induce a current deficit but also causes a contraction in output which is a counterfactual. The results indicate that of the different channels of explanation that have been offered as rationalizations of the stylized facts associated with exchange rate based stabilization programs, the imperfect credibility or "temporary policy" hypothesis seems to be the most robust to different model specifications. Other channels of explanation seem to work in some environments but not in others.

A useful extension of the model developed here would be to enrich the production technology to include both capital and labor as inputs and to make capital accumulation subject to an adjustment cost. Using this specification, Lahiri (1995) has shown that with perfect capital mobility a permanent cut in the devaluation rate would cause the model to qualitatively reproduce the stylized facts. Roldos (1994) and Uribe-Echevarria (1993) get similar results by using models where capital accumulation is subject to a cash-in-advance constraint. As originally demonstrated by Stockman (1981), in this environment permanent reductions in the inflation rate cause an expansion in output due to capital accumulation. Reworking these models under the assumption of imperfect capital mobility would be a test of the robustness of the results obtained in those works. Further, it would also give us a better sense of the relative strengths of the "supply-side adjustments under permanent cuts" hypothesis vis-a-vis the "temporariness" hypothesis as plausible explanations of the stylized facts associated with exchange rate based stabilization programs.
REFERENCES


____ and Allan Drazen, 1994, "Uncertain Duration of Reform: Dynamic Implications", mimeo, University of Maryland.


Dornbusch, Rudiger, 1982, "Stabilization Policies in Developing Countries: What Have We Learned?", World Development 10, pp. 701-708.


____, 1986, "Capital Controls, The Dual Exchange Rate, and Devaluation", Journal of International


Roldos, Jorge E., 1994, "Supply-Side Effects of Disinflation Programs", mimeo, IMF.


APPENDIX

In this appendix to the paper we investigate the implications of relaxing the cash-in-advance constraint and introducing money through the utility function for the imperfect capital mobility case.

Consider a representative agent who maximizes the present discounted value of lifetime utility given by

\[ V = \int_{t=0}^{\infty} e^{-pt}[u(c,m) + v(l)] dt \]  

(A1)

where \( u \) and \( v \) are concave and twice differentiable in their respective arguments. Agents are allowed to trade in assets in world capital markets. However, as in equation (13) in section III of the main text, international creditors link the going interest rate to the net debt of the country. As before, individual agents take the interest rate as given due to their small measure relative to the market. The equation of motion describing the evolution of net private debt, \( a = b - m \), is given by

\[ \dot{a} = ra + c + im - g - f(1-l) \]  

(A2)

The representative agent maximizes (A1) subject to (A2). Among the first order conditions for this problem are

\[ u_c(c,m) = \lambda \]  

(A3)

\[ u_m(c,m) = \lambda i \]  

(A4)

\[ v'(l) = \lambda f'(1-l) \]  

(A5)

\[ \dot{\lambda} = (\rho - \gamma) \lambda \]  

(A6)

Totally differentiating (A3), (A4) and (A5) and applying Cramer's rule yields the implicit solutions for
c, m and I in terms of \( \lambda \) and i:

\[
c = \psi(\lambda, i), \quad \psi_{\lambda < 0}, \psi_i > 0 \text{ as } u_{cm} < (>) 0
\]

(A7)

\[
m = M(\lambda, i), \quad M_\lambda < 0, M_i < 0
\]

(A8)

\[
l = H(\lambda), \quad H' < 0
\]

(A9)

One should note that partial derivatives of \( c \) and \( m \) with respect to \( \lambda \) assume normality of both goods.

Noting that the domestic credit rule given by equation (17) of the text implies that \( \dot{m} = \dot{d} \) (recall that \( d \) denotes foreign exchange reserves with the central bank), we can substitute (17), (A7) and (A9) into (A2), and (13) into (A6) to derive the two fundamental differential equations that characterize the economy:

\[
\dot{\lambda} = (\rho - r^* - r(n)) \lambda
\]

(A10)

\[
\dot{n} = (r^* - r(n)) n + \psi(\lambda, i) - f(1 - H(\lambda))
\]

(A11)

where \( n ( = b - d) \) is the net debt of this economy. The system given by (A10) and (A11) when linearized around a steady state is saddle path stable. The \( \dot{\lambda} = 0 \) locus is vertical at the unique level of debt which is consistent with steady state while the \( \dot{n} = 0 \) locus is upward sloping. As figure 7(a) shows, the associated saddle path is upward sloping.

A permanent cut in the devaluation rate, \( \pi \), leaves the \( \dot{\lambda} = 0 \) locus unchanged but shifts the \( \dot{n} = 0 \) locus. The direction of the shift, however, is ambiguous and depends on the sign of \( u_{cm} \). Once again there are three cases to be analyzed:

(i) \( u_{cm} > 0 \): As figure 7(a) shows, in this case of complementarity between consumption and money, the cut in \( \pi \) causes the \( \dot{n} = 0 \) locus to shift upwards to the new \( \dot{n}_1 = 0 \) locus. Starting from steady state at point e, the system immediately jumps to its new steady state point a with an unchanged n and a higher
λ. Consumption and output instantaneously jump up to their new, higher steady state levels. Thus, the qualitative implications are identical to the CIA case analyzed in section III of the paper. It is easy to see that the effects of a temporary cut in the devaluation rate with \( u_m \) are also qualitatively very similar to the results of section III. As noted above, this is to be expected due to the complementarity between consumption and money that is imposed by the cash-in-advance (CIA) constraint.

(ii) \( u_m < 0 \): This is the case where \( c \) and \( m \) are substitutes. In this event, a cut in \( \pi \) shifts the \( \bar{n} = 0 \) locus down to the new \( \bar{n}_1 = 0 \) locus as shown in figure 7(b). If the cut is permanent the system immediately jumps down from point \( e \) to its new steady state point \( a \). The new steady state is characterized by lower consumption and output. The dynamics associated with a temporary cut in \( \pi \) can be derived in an analogous manner.

This case is especially interesting in the context of this paper since Obstfeld (1985) used an endowment economy model to argue that when money and consumption are substitutes, a preannounced sequential disinflation program would cause a current deficit and an increase in consumption on impact. As in the text, we model this experiment as a two-part reduction in the devaluation rate - a cut at time \( t = 0 \) followed by a second reduction at time \( T > 0 \). Figure 7(b) depicts the effects of this policy on our dynamic system.

On impact, the \( \bar{n} = 0 \) locus shifts down to the new \( \bar{n}_1 = 0 \) locus. At time \( T \) the second cut in \( \pi \) shifts the \( \bar{n}_1 = 0 \) locus further down to the \( \bar{n}_2 = 0 \) locus. In terms of figure 7(b), on impact the system jumps down to point \( b \) and then travels along an unstable path which reaches \( SS_2 \) at precisely time \( T \). Thereafter the economy moves along \( SS_2 \) to asymptotically regain steady state at point \( c \) which is characterized by the same \( n \) but a lower \( \lambda \) relative to the initial steady state.

As in Obstfeld (1985), upon impact a current deficit emerges which lasts till time \( T \). Thereafter,
the economy starts running down its excess debt holdings. During the transition consumption rises since λ falls monotonically, assuming that the intertemporal effect dominates. At time T, however, consumption jumps down due to the discrete jump up in m induced by the second cut in π. The behaviour of consumption at time t=0 is ambiguous. The opposing effects induced by the cut in π and the associated fall in λ imply that consumption could jump up or down.

The output dynamics associated with this experiment are particularly interesting. Since (A9) implies that the qualitative behaviour of output mimics the behaviour of λ, output jumps down on impact and declines monotonically thereafter. In the new long run equilibrium both output and consumption are lower relative to the initial steady state.

The implied contraction in output that is occasioned by a sequential disinflation program contradicts a key stylized fact associated with the Southern Cone experiments. Those economies exhibited booms in output as well as the well documented consumption booms and current deficits. It is also worth recalling that the demand driven environment investigated in Obstfeld (1985) precluded any independent supply side effects.

(iii) \( \nu_{cm} = 0 \): In this event c and m are independent and consumption is independent of the nominal interest rate, i. Hence, a cut in π has no effect on the dynamic system. The only adjustment that takes place is that agents increase their money holdings by increasing b thus leaving n unchanged. Thus, when the utility function is completely separable across consumption, money and leisure, devaluation rate changes are neutral and have no real effects.
Figure 1: Dynamic system under no capital mobility
Figure 2: Permanent cut in $\pi$ under no K mobility

2(a): Dynamic system

2(b): Consumption, output
Figure 3: Temporary cut in $\pi$ under no $K$ mobility

3(a): Dynamic system

3(b): Consumption, output
Figure 4: Permanent cut under imperfect K mobility
Figure 5: Temporary cut in $\pi$ under imperfect mobility

5(a): Dynamic system

5(b): Consumption, output
Figure 6: Sequential cut in $\pi$

6(a): Dynamic system

6(b): Consumption, output
Figure 7: Money in the utility function and imperfect K flows

$7(a): u_{cm} > 0$

$7(b): u_{cm} < 0$