Information Acquisition in Auctions

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Abstract

With few exceptions, auction theory takes as given the information bidders have about each other and about the object being sold. We present a general framework in which to discuss the acquisition of information on the part of the bidders. This work will show that

i) different auction forms give different incentives to acquire information. Specifically, a first-price auction gives higher incentives than a second-price auction to acquire information that is positively correlated with the opponent's bid. This is because in a first price auction it is valuable to bid close to the opponents, to minimize the sum paid when winning: in a second price this is immaterial, because the price paid does not depend on the winner's bid. Hence a piece of information that allows to bid closer to the opponents is more useful in a first price than in a second price auction.

ii) the different incentives to acquire information may overturn the well-known Milgrom and Weber result stating that a second-price auction dominates a first-price in terms of revenue to the seller. The force driving this result is that, as we have seen, a first price auction encourages acquisition of information correlated with the opponent's signal. This will result in a highly correlated information structure in a first price auction, relative to a second price. But, in a pure common value setting, bidders with very correlated information will compete away most of the surplus from each other (in this case, an auction closely resembles a Bertrand game), so the difference in the endogenous information structures can result in the first price auction revenue-dominating a second price.

In addition, we trace the connection between revenue-ranking and incentives to acquire information: in a large class of mechanisms, the higher the revenue to the auctioneer, the lower the incentives for the bidders to acquire information.

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1 Introduction

With few exceptions, auction theory takes as given the information bidders have about each other and about the object being sold; much of this literature is concerned with comparing bidding behaviour and revenue to the seller across different auction forms, for a fixed information structure. This work will argue that bidders' incentives to acquire information in the pre-bidding stage vary across auction mechanisms, and presents a general framework suited to compare these incentives across a variety of auction forms.

1.1 Theoretical motivation: Endogeneizing the information structure

The philosophical assumption of most of information economics is that asymmetric information of agents generates many interesting economic phenomena. The agents' information structure has almost always been assumed exogenous, and beyond the control of a 'market maker', or central planner: indeed, mechanism design is the planner's answer to the problems posed by unyielding information structures. But if the planner has no control over the agents' information, from where does the information structure come? We take a step towards realism, recognizing that the information structure is not fixed: instead, it is agents that determine the information structure in a game, before playing it. This has profound implications, because the exercise of comparing two mechanisms with the same information structure – as in mechanism design – begs the question of whether that (same) information structure could possibly result from the agents' own doing: we show that in general this is unlikely. We evaluate the incentives for agents to acquire information, and the way they depend on the mechanism they face: in this paper we do this for auctions, and view this as part of a larger research project on endogenous information structures in economic models.

1.2 Descriptive motivation: modeling R&D in defense procurement

Weapons procurement is an important economic activity: $80 billions of the U.S. Department of Defense's (DoD) budget were devoted to weapons procurement in 1992, roughly half of which was used to subsidize R&D of defense contractors. It is generally acknowledged that one of the most important features of the procurement activity is DoD's objective of inducing R&D on the part of contractors. Bidding in procurement is commonly understood as a way of providing incentives for individual contractors to perform R&D: directly to condition the award of a contract on the quality of the project (or amount of R&D) is often impractical, either because this may be non-verifiable, or because of the political difficulty of awarding a contract to anybody but the lowest bidder. So we ask the question: if a contract can be awarded only on the basis of price, which is the mechanism that gives greater incentives for firms to perform R&D?

The idea is that firms will perform R&D in order better to compete in the bidding phase. There is no difficulty in reinterpreting the acquisition of information about the value of the object as a process of (stochastic) cost reduction in the production of a weapon; at the same time, our model could accomodate the notion of R&D as a process that allows better to judge one's own cost to produce the weapon. The simplifying assumption is that DoD can force the winning bidder to provide the weapon at the agreed price. A first price auction will induce more R&D than a second price, due to correlation in the R&D activity, specifically to the fact that since firms are investigating the cost of the same project, more investigation leads closer to the opponent's bidding. When this element of correlation is absent, and the R&D process is just an independent process of private cost-reduction, the incentives to perform R&D are the same in a first and second price auction (see Tan [13]). But if a common component is present in the R&D process, a first price auction should be preferred to a second price, if the main objective of DoD is to maximize the amount of R&D performed by contractors. This could explain the popularity of the first price format in procurement situations.

1.3 Normative motivation: Reversal of revenue ranking

As we have seen, a first price auction encourages acquisition of information correlated with the opponent's signal. This will result in a highly correlated information structure in a first price auction, relative to a second price. But, in a pure common value setting, bidders with very correlated information will compete away most of the surplus from each other. In this case, an auction resembles closely a Bertrand game, and in the limiting case of perfect correlation between signals, each bidders knows the same thing as his opponents. In this case Bertrand competition will drive bidders' surplus to 0. This is the logic behind the following example.

This example shows how endogeneizing information acquisition may overturn the Milgrom-Weber revenue ranking result. We are thinking of the two-stage game where first players decide simultaneously and independently whether or not to acquire (almost perfect) information on the opponent's signal; then, without having observed the opponent's choice, they engage in competitive bidding.

Example 1 There are two players, 1 and 2, two identically distributed random variables (signals) X_1, X_2, and i.i.d. random variables η_1, η_2 (noise), independent of X_1, X_2 and with small support. Player i is endowed with information X_i. The value of the object to player i is $V_i = X_1 + X_2$.[1] Milgrom and Weber's revenue-ranking result states that the second price auction dominates the first price in terms of expected revenue.

Suppose now that player i can also *covertly* observe $X_j + \eta_j$ (acquire information), at a small cost c.

In a second price auction he will not acquire if the opponent does not acquire (see Theorem 5 below). Thus, the "no acquisition" strategy for both players constitutes an equilibrium.

[1] The role of the η_i component is to guarantee each bidder a component of private information that is unavailable to the opponent. This keeps the bidders' revenue above 0 after information acquisition, which permits existence of an equilibrium with information acquisition.

In a first-price auction, however, the no-acquisition strategy combination is not an equilibrium, because each player will have an incentive to deviate and acquire information at a small cost (acquiring the opponent's signal allows a bidder to minimize the money paid when winning, by bidding just above the opponent's bid). To show that the "both acquire" strategy combination is an equilibrium, let us consider the case in which both players acquire the information on the opponent's signal. Because of the noise component, they will have a strictly positive expected payoff if c is small enough, and acquiring always has a value in a first-price. Hence the strategies of acquiring information and then playing the auction are indeed an equilibrium.

Compare now the two equilibria of the first and second price auction, having taken into account the effect of information acquisition: the payoff to the auctioneer is higher in the first-price auction compared with the second-price. This is because the correlated information structure prevailing in the former game induces a sort of Bertrand competition among players. Indeed, as the noise component η converges to 0, the first price auction will extract all surplus from the players. In the second price auction no such extraction is possible: both players in equilibrium keep a substantial private information element that they exploit to guarantee themselves some surplus. Thus, when information acquisition is taken into account, the first-price revenue-dominates the second-price in a common-value setting, thus reversing the well-known Milgrom and Weber ranking result. $\diamond$

1.4 The structure of this paper

This work is concerned with *covert* information acquisition. That is, players simultaneously choose the accuracy of their signal, and they do not observe the opponents' accuracy choice before bidding. In contrast, some literature has assumed that each bidder's accuracy (though not the realization of their signal) is disclosed before the bidding stage (*overt* information acquisition). Our modeling choice seems appropriate because it is often difficult to gauge other people's information. Besides, overt information acquisition introduces additional strategic considerations (like the possibility of punishing a competitor for acquiring too much information) that make the analysis more complex. The assumption that information gathering is covert allows us to apply results developed in Persico [10]. There, a new concept of "better information", A-order (Accuracy-order), was developed. A-order is a notion tightly suited to affiliated decision problems: it captures the natural idea that "more correlation with the unknown random variable is better". Acquiring information in Blackwell's sense is a special case of this theory.

In section 2, we use this notion to present a model where bidders choose the Accuracy of their signals. Subsection 2.1 introduces the model, and Subsection 2.2 presents the results: Theorem 1, which ensures that Accuracy is the correct notion of "better information", and Theorem 2, showing that a first price auction gives higher incentives to acquire information than a second price. Proposition 1 follows, giving sufficient conditions – given that an equilibrium exists – for equilibrium Accuracy to be higher in a first price than in a second price. Proposition 2 shows a family of cost functions under which an equilibrium exists, and Proposition 1 applies. Finally, an intuition for why a first price auction gives higher incentives to acquire information than a second price is developed in subsection 2.3.

For the two most common auction forms, the one which gives higher revenue to the auctioneer (second price) gives fewer incentives for the bidders to acquire information. This is not a coincidence: in Section 2.4 we trace this connection, and present several other examples of this dycothomy.

In order to validate our "money on the table" interpretation, Section 3 presents a different but related framework, one of *discrete* information acquisition. By this we mean that the choice is whether or not covertly to acquire the observation of an *additional* random variable. Here, the decision variable is not a continuous one (Accuracy), but a yes-no decision. Subsection 3.1 presents the notation and the model. Subsection 3.2 treats acquisition of a signal that is perfectly correlated with the opponent's: Theorem 5 surprisingly shows that there is no incentive to acquire such a signal in a second price auction. Thus, again a first price auction gives more incentive than a second price to acquire information that is very correlated with the opponent's signal. In Subsection 3.3, Theorem 6 shows that the incentive to acquire a signal independent of the opponent's is higher in a second than in a first price auction. This confirms our intuition, since such a signal leads to a bidding behaviour that is less correlated with the opponent's. By resorting to the discrete (two-signal) framework we have thus produced a phenomenon that cannot be observed in the one-signal framework of Section 2, and further validated our interpretation.

Section 4 concludes.

1.5 Related literature

The first cuts at modeling information acquisition in auctions belong to the literature on information aggregation in market mechanisms. In 1981, Milgrom [8] p. 924 observed that in a second-price auction mechanism, "if a bidder were informed of his price and extracted all the information which his price conveys, he could never gain by revising his bid".[2] That paper however did not pursue the question of information acquisition *per se.* In the same period Matthews [7] analytically solved a model of endogenous information acquisition in first-price auction: before engaging in a first price auction, bidders choose the precision of their signal, at a cost. That work concentrated on information gathering with a large number of bidders.[3] An earlier unpublished working paper (Matthews [6]) however, compared a first and second price auction in terms of information acquisition and concluded that, due to a special feature of the statistical structure used, the two auction forms gave the same incentives to acquire information. The results presented in this paper accommodate this very early insight.

In a recent work independent of our, Gaier [2] takes up Matthews' primitives, and finds that a first price auction will give stronger incentives to acquire information than a royalty rate auction. [4] This result is presented here as a part of Theorem 4. It is Matthews' work, Gaier's paper, and Hausch and Li [3] that are closer in spirit to section 2 of the present work. In the

[2] Our Theorem 5 is a rephrasement of this result, in a different context.

[3] This is a special case of the model presented in our next chapter.

[4] In a royalty rate auction the bidding units are not money, but the fraction of the object's value retained by auctioneer.

latter, the authors investigate in a general model the difference in the incentive to acquire information among all independent-private-value auction mechanisms: they find none. We are able to show that the key to this result is the independence assumption, as is made clear in Remark 2.

Hausch and Li then go on to analyze the case of *overt* information acquisition. In this way they are able to generate an example where a first price auction induces *less* information acquisition than a second price (although the effect on the revenue depends on the number of bidders). The difference between overt and covert information acquisition was pointed out in an early paper by Magee (see [5]). He studied a common-value first-price auction in a context where "increasing information" meant choosing a better managerial accounting system. Other ways to generate a difference in information acquisition behaviour have been endogenous number of bidders, and multiplicity of equilibria (see Tan [13] for an example where firms do R&D instead of acquiring information).

Our work suggests that bidders' information acquisition behaviour naturally varies across different auction forms, and that the key to this phenomenon is correlation. Thus, the results we develop here are quite different from those in most of the literature: we do not rely on the number of bidders, on their participation decision, or on the strategic incentive to acquire information in order that the opponents can see that. While these factors may matter in many economic situations, the results and the intuition we present are inherent to the structure of the two mechanisms. As such, they should be shared by a large number of information acquisition models.

2 Continuous information acquisition

In this section we shall be concerned with the question of which mechanism (first or second price auction) induces more information acquisition, when information is acquirable in a continuous fashion. For notational simplicity we will present a two-player environment. However, all the results in this section are valid in the n-player case.

2.1 The model

There are two players, 1 and 2, and an object up for auction, whose value to player i is $u_i = u(V_i, V_j)$. The function $u(\cdot,\cdot)$ is assumed increasing in its arguments. V_1 and V_2 are random variables unobserved by the players. Players share a prior on their distribution, with density $g(v_1, v_2)$ where $g(\cdot,\cdot)$ is a symmetric affiliated density function. Player i observes a signal on the true value of V_i, X_i^θ, at a cost $C(\theta)$. This signal is chosen from a family $\{X_i^\theta\}_{\theta\in[\underline{\theta},\infty)}$, where $\underline{\theta}$ is a real number, and X_i^θ is distributed according to the density $f_{X_i}^\theta(x_i \mid v_i)$. It is assumed that the family of signals is A-ordered. Furthermore we require that $f_X^\theta(x \mid v)$ exhibits the monotone likelihood ratio property. In our setting, this requirement is equivalent to affiliation of X_i^θ and V_i; given this statistical structure, all random variables in this model are affiliated (see Theorem 1 (ii) in Milgrom and Weber [9]). Denote with $\tilde{u}(v_i, x_j)$ the expected value of the object to player i conditional on $V_i = v_i$ and $X_j = x_j$. Formally, $\tilde{u}(v_i, x_j) = E(u(V_i, V_j) \mid V_i = v_i, X_j = x_j)$.

The information acquisition game consists of two stages:

1) each player i chooses a θ_i, independently from and simultaneously with his opponent.

2) after having observed the realization of $X_i^{\theta_i}$, but not the opponent's choice of θ_j, each player casts a bid for the object.

We will assume that the density of $X_i^{\theta_i}$ conditional on v_i is $f_{X_i}^{\theta_i}(x_i \mid v_i)$, and that for each θ_1, θ_2, the joint distribution of signals and value is

$$f^{\theta_1\theta_2}(x_1, x_2, v_1, v_2) = f_{X_1}^{\theta_1}(x_1 \mid v_1) f_{X_2}^{\theta_2}(x_2 \mid v_2) g(v_1, v_2).$$

This model contains the case of independent signals (and hence independent private values), when $g(v_1, v_2) = g(v_1)g(v_2)$. Moreover, although formally not a special case of the above model, all the results in this section hold in the familiar mineral rights case, where $V_1 \equiv V_2 \equiv V$. These can be seen as the two limiting cases, when the correlation between signals goes from 0 (independent signals) to a maximum (mineral rights).

Remark 1 In order more realistically to model R&D, consider the following more general specification for the value to player i: $u_i = u(V_i, V_j, X_i, \theta_i)$, where $u(\cdot,\cdot,\cdot,\cdot)$ is strictly increasing in its first argument, and nondecreasing in its last three arguments.Thus, increasing Accuracy has a value-enhancing effect, and receiving a high signal is good for the payoff. All the results of Section 2 are unchanged when this additional element is introduced. $\diamond$

Let us now introduce some notation, about the marginal benefit from increasing Accuracy. The first piece of notation is for the marginal benefit from increasing Accuracy starting from θ, when the opponent has instead Accuracy η and moreover (wrongly) thinks the situation is symmetric at η. Let

$AMR_m(\eta,\theta) :=$ the marginal revenue from increasing θ in mechanism m (First or Second price) to a player 1 that has accuracy θ and best responds to a player 2 who has accuracy η and plays the symmetric equilibrium strategy for a mechanism m where both players have accuracy level η.

Let now

$$MR_m(\theta) := AMR_m(\theta,\theta).$$

This is the marginal revenue from increasing Accuracy when both bidders have Accuracy θ. We denote with $MC(\theta)$ the marginal cost of increasing Accuracy.

2.2 The results

The following theorem proves that A-order is the right concept of "better information" for a first or second price auction: It says that increasing one's Accuracy is beneficial, irrespective of the opponent's strategy and Accuracy level. To prove this result, in view of the theory presented in Appendix A, it suffices to show that one is facing an affiliated decision problem, when playing against an opponent in a first or second price auction.

Theorem 1 *Suppose that, in either a first or a second price auction, player 2 has accuracy level η and plays an increasing strategy $b_2^{\eta}(\cdot)$, and player 1* *has accuracy level θ and plays his best response to $b_2^\eta(\cdot)$. Then increasing θ is beneficial to player 1 if the family of signals is A-ordered. In other words, $AMR_F(\eta,\theta), AMR_S(\eta,\theta) \geq 0$ for all η, θ.*

Proof: See Appendix C. ∎

The next theorem proves that – when at a symmetric equilibrium – the marginal revenue of information is always higher in a first price auction than in a second price, i.e. $MR_F \geq MR_S$.

Theorem 2 *Suppose we are at a symmetric equilibrium of a first or second price auction with a given information level $\theta_1 = \theta_2 = \theta$, and suppose the family of signals is A-ordered. Then $MR_F(\theta) \geq MR_S(\theta)$ for all θ.*

Proof: In view of theorem 8, it is enough to show that $\frac{\partial}{\partial x_1} u_F^\theta(v_1, x_1) \succeq \frac{\partial}{\partial x_1} u_S^\theta(v_1, x_1)$, where the index $m = F, S$ refers to first and second price auction, respectively. For a second price auction we have, at a symmetric equilibrium,

$$u_S^\theta(v_1, x_1) = \int_{-\infty}^{x_1} \left[\tilde{u}(v_1, y) - b_2^\theta(y)\right] f_{X_2}^\theta(y \mid v_1) dy$$

A first price has

$$u_F^\theta(v_1, x_1) = \int_{-\infty}^{x_1} \left[\tilde{u}(v_1, y) - b_1^\theta(x_1)\right] f_{X_2}^\theta(y \mid v_1) dy,$$

whence

$$u_F^\theta(v_1, x_1) - u_S^\theta(v_1, x_1) = \int_{-\infty}^{x_1} \left[b_2^\theta(y) - b_1^\theta(x_1)\right] f_{X_2}^\theta(y \mid v_1) dy$$

We need to show that $\frac{\partial}{\partial x_1} u_F^\theta(v_1, x_1) - \frac{\partial}{\partial x_1} u_F^\theta(v_1, x_1)$ is **(QM)**. But this expression is just

$$\left[-b_1^\theta(x_1) - b_1^{\theta\prime}(x_1) \frac{F_{X_2}^\theta(x_1 \mid v_1)}{f_{X_2}^\theta(x_1 \mid v_1)} + b_2^\theta(x_1)\right] f_{X_2}^\theta(x_1 \mid v_1) \tag{1}$$

which is readily verified to be **(QM)** in v_1, in view of affiliation. ∎

Remark 2 If V_1 is independent of V_2, then expression (1) is independent of v_1, hence $MR_F(\theta) = MR_S(\theta)$. Thus, independent signals imply that both auction forms will give the same incentive to acquire information. $\diamond$

A necessary condition for a symmetric equilibrium (θ, θ) is that $MC(\cdot)$ intersects $MR_m(\cdot)$ from below in θ. Since Theorem 2 ensures that $MR_F \geq MR_S$, we have

Proposition 1 *Suppose a symmetric pure-strategy equilibrium exists for a first and second price information-acquisition game. If $MC(\cdot)$ only intersects $MR_S(\cdot)$ once, then the equilibrium accuracy in the first price auction will be higher than or equal to that in a second price auction.*

Proof: Since an equilibrium for the second price game exists, it will be the unique $\tilde{\theta}$ where $MC(\tilde{\theta}) = MR_S(\tilde{\theta})$. Moreover, since $MC(\cdot)$ only intersects $MR_S(\cdot)$ once, it must be that $MC(\theta) < MR_S(\theta)$ for all $\theta < \tilde{\theta}$. Theorem 2 gives that $MR_S(\cdot) \leq MR_F(\cdot)$, so we can write

$$MC(\theta) < MR_S(\theta) \leq MR_F(\theta) \text{ for } \theta < \tilde{\theta}.$$

Since $MC(\cdot) = MR_F(\cdot)$ is a necessary condition for equilibrium, this inequality implies that the equilibrium accuracy for a first price auction is not lower than $\tilde{\theta}$. ∎

The question now is which cost functions will fulfill the requirements of the above Proposition. A cost function of the form

$$C(\theta) = K\theta^{\alpha}$$

will certainly meet all the requirements for an α high enough. Indeed, as α grows, this cost function converges to one that is 0 for $\theta \in [0, 1)$ and is infinite for $\theta > 1$. We are thus guaranteed a unique upcrossing for both MR_m functions at $\tilde{\theta}_m$ near 1, and that $MC(\theta) < AMR_m(\tilde{\theta}_m, \theta)$ if and only if $\theta < \tilde{\theta}_m$. That an equilibrium exists for both games is implied by the specific functional form of $C(\cdot)$, which allows us to concentrate on a neighbourhood of $\theta = 1$ instead of checking global optimality. But then local first- and second-order conditions are enough to yield an equilibrium, and those are satisfied for α high enough. We have thus argued the following

Proposition 2 *For every set of primitives there exists a family of cost functions $C^{\alpha}(\theta) = K\theta^{\alpha}$ and an $\underline{\alpha}$ such that, for every $\alpha > \underline{\alpha}$ there exists a unique pure-strategy symmetric equilibrium for a the first and second price game, and the equilibrium accuracy is higher in the first than in a second price auction.*

The consequence of this for revenue comparison are, however, ambiguous. Consider for example the pure common value model: there, the auctioneer's revenue has two global maxima,[5] one at $\theta = 0$, and the other at $\theta = \infty$. [6]

Thus, when Accuracy is very low, the revenue to the auctioneer must be decreasing in θ. On the other hand, when Accuracy is very high, the revenue must be increasing in θ. Hence, an increase in the equilibrium precision will benefit the auctioneer only if the extant value of θ is high. This may be assumed at the outset, choosing $\underline{\theta}$ very large. Furthermore, when $\theta \to \infty$ we know that $MR_S(\theta) \to 0$. This is because in this case the bidding strategies converge to v in probability, and thus very little can be gained from becoming more informed. Thus, whenever we are in this region, it is reasonable to believe that any cost function such that $MC(\cdot)$ is increasing will intersect the $MR_S(\cdot)$ function only once. In this case, endogeneizing information acquisition should lead to a higher equilibrium Accuracy in a first price auction. This reduces the revenue difference between first and second price, and the ranking may be reverted, compared to the case of a given information precision.

2.3 Interpretation: Money on the table

It is apparent that the proof of Theorem 2 hinges on the properties of the quantity $\frac{F(x|v)}{f(x|v)}$, which is decreasing in v because of affiliation; we can interpret this quantity as an index of the "money left on the table" by the winner in a first-price auction. The quantity $\frac{F(x|v)}{f(x|v)}$ can be very close to 0, expressing that $f(x \mid v)$ is very large relative to $F(x \mid v)$, viz. very little money (in a

[5]This is pointed out in Matthews [7].

[6]We represent with $\theta = 0$ a completely uninformative signal, and with $\theta = \infty$ the perfectly informative signal.

probabilistic sense) is left on the table. Alternatively, this quantity can be very large, witnessing that one could safely reduce his bid and still win with almost the same probability – lots of money are left on the table.

The proof of Theorem 2 then makes clear that an additional tiny bit of correlation with v is more useful in a first price auction because it allows to reduce the impact of the "money-on-the-table" term.

2.4 Information acquisition and Revenue Ranking

It is well known that – when the information structure is exogenously fixed – the second price auction gives the auctioneer a higher expected profit than the first price auction (see Milgrom and Weber [9]). So Theorem 2 establishes that the auction form which – for a fixed information level – maximizes the auctioneer's revenue is the one where bidders will acquire more information. This relationship is not coincidental: in this subsection we will present other pairs of auction forms where it holds, and theoretically explain the connection.

The intuition behind the result is as follows: given an information structure, the revenue to the auctioneer is the mirror image of the revenue to the seller. So, when the revenue to the auctioneer is low, agents have a high expected revenue. For agents to be able to keep a high expected revenue, it has to be that bidder with a high signal have a much higher expected revenue than types with a low signal. So the expected revenue conditional on a signal must be very variable in the signal: but when this is the case, there will be a high value to having better information. To formalize this intuition we will compare the (sufficient) condition that allows to rank auction forms in terms of revenue, with the (sufficient) condition that gives the marginal revenue of information.

Consider two decision problems, constituted by two different payoff functions $u_I(\cdot,\cdot)$ and $u_{II}(\cdot,\cdot)$, associated to the same statistical structure, as described in Appendix A. Following the interpretation of a constant sum game (like an auction), we can call these decision problems *mechanisms*, and agree that the revenue to the auctioneer is inversely related to the revenue of the decision maker (who impersonates a bidder). In Appendix B we establish that a *sufficient* condition for mechanism II to give higher revenue to the auctioneer than mechanism I is that (for the notation refer to Appendix A)

$$u_I^\theta(v_1, x_1) - u_{II}^\theta(v_1, x_1) \text{ be quasi-monotone in } v_1 \tag{2}$$

This condition allows to recover all existing revenue-ranking results for sealed-bid auctions. Compare condition (2) with the *sufficient* condition for problem I to induce more information acquisition than problem II, which is that

$$\frac{\partial}{\partial x_1}\left[u_I^\theta(v_1, x_1) - u_{II}^\theta(v_1, x_1)\right] \text{ be quasi-monotone in } v_1$$

So, to compare two auction forms in terms of the revenue to the seller it suffices to answer the question " is $u_I^\theta(v_1, x_1) - u_{II}^\theta(v_1, x_1)$ quasi-monotone in v_1?": to compare them in terms of incentives to acquire information it suffices to ask "is $\frac{\partial}{\partial x_1}\left[u_I^\theta(v_1, x_1) - u_{II}^\theta(v_1, x_1)\right]$ quasi-monotone in v_1?". The answer to these two questions coincides for several pairs of mechanisms. One such pair are the first and second price auctions. Another pair are the war of attrition and the all-pay auction, as we shall see in the following theorem:

Theorem 3 *Suppose we are at a symmetric equilibrium (in increasing strategies) of an all-pay auction and a war of attrition with a given information level $\theta_1 = \theta_2 = \theta$, and suppose the family of signals is A-ordered. Then the revenue to the auctioneer is higher in the war of attrition than in the all-pay auction, and $MR_{APA}(\theta) \geq MR_{WOA}(\theta)$ for all θ.*

Proof: The war of attrition has been shown to yield a higher expected revenue to the auctioneer than the all-pay auction (see Krishna and Morgan [4]): an easy way to see this is to observe that

$$u_{WOA}^\theta(v_1, x_1) = \int_{-\infty}^{x_1}\left[\tilde{u}(v_1, y) - b_2^\theta(y)\right] f_{X_2}^\theta(y \mid v_1)dy + \int_{x_1}^{+\infty} b_1^\theta(x) f_{X_2}^\theta(y \mid v_1)dy$$

and

$$u_{APA}^\theta(v_1, x_1) = \int_{-\infty}^{x_1}\left[\tilde{u}(v_1, y) - b_1^\theta(x)\right] f_{X_2}^\theta(y \mid v_1)dy + \int_{x_1}^{+\infty} b_1^\theta(x) f_{X_2}^\theta(y \mid v_1)dy,$$

so we have $u^{\theta}_{WOA}(v_1, x_1) - u^{\theta}_{APA}(v_1, x_1) = u^{\theta}_{S}(v_1, x_1) - u^{\theta}_{F}(v_1, x_1)$. This shows that the relationship between war of attrition and all-pay auction – both in terms of revenue to the auctioneer and in terms of information acquisition – is the same as that between the second and the first price auction. ∎

Yet another case is the relationship between the first price and the "mineral rights auction", in the (pure common values) case where $V_1 \equiv V_2 \equiv V$ and $F^{\theta}_{X}(x|v) = \left[\frac{x}{v}\right]^{\theta}$ on $[0, v]$.[7] The contents of the following theorem are not new (except for the proof of existence of equilibrium in increasing strategies): we believe the method of proof is. The revenue-ranking result is due to Riley [11], while the result about marginal revenue of information is due to Gaier [2].

Theorem 4 *In the pure common values case a symmetric equilibrium in strictly increasing strategies exists for the mineral rights auction. When* $F^{\theta}(x|v) = \left[\frac{x}{v}\right]^{\theta}$ *on* $[0, v]$, *then the mineral rights auction gives higher revenue to the auctioneer than a first or second price, and* $MR_{MRA}(\theta) \leq MR_F(\theta) = MR_S(\theta)$.

Proof: For the proof that the mineral rights auction has an equilibrium in increasing strategies, see Appendix C. To see that the mineral rights auction yields higher expected revenue to the auctioneer than a first price auction, consider that

$$u^{\theta}_{MRA}(v, x_1) = \int_{-\infty}^{x_1} v\left[1 - r(x_1)\right] f^{\theta}_{X_2}(y \mid v)dy$$

and

$$u^{\theta}_{F}(v, x_1) = \int_{-\infty}^{x_1} \left[v - b^{\theta}_{1}(x_1)\right] f^{\theta}_{X_2}(y \mid v)dy,$$

so

$$u^{\theta}_{F}(v, x_1) - u^{\theta}_{MRA}(v, x_1) = \int_{-\infty}^{x_1} \left[vr(x_1) - b^{\theta}_{1}(x_1)\right] f^{\theta}_{X_2}(y \mid v)dy.$$

[7] A mineral rights auction is an auction where bidders bid a fraction r of the value of the object. The bidder who bids the highest fraction wins, and pays that fraction to the auctioneer. For literature on the mineral rights auction, see Riley [11] and Gaier [2].

This expression is quasi-monotone in v, and so we are done. In addition, because of the specific form of the signal distribution the first price auction gives the same revenue to the auctioneer than a second price auction, [8] which proves that a mineral rights auction also revenue-dominates a second price.

To prove our claim about the marginal revenue of information, let us compute

$$\frac{\partial}{\partial x_1}\left[u_F^\theta(v,x_1) - u_{MRA}^\theta(v,x_1)\right] = \left[vr(x_1) + \left(r'(x_1) - b_1^{\theta\prime}(x_1)\right)\frac{F_{X_2}^\theta(x_1 \mid v)}{f_{X_2}^\theta(x_1 \mid v)}\right] f_{X_2}^\theta(x_1 \mid v). \tag{3}$$

Since the expression $\frac{F_{X_2}^\theta(x_1|v)}{f_{X_2}^\theta(x_1|v)}$ is independent of v in our case, we can conclude that expression (3) is quasi-monotone in v and therefore a first price auction gives higher incentives to acquire information than a mineral rights auction, so $MR_{MRA}(\theta) \leq MR_F(\theta)$. The fact that $MR_F(\theta) = MR_S(\theta)$ is again a consequence of our choice of information structure, and has been shown by Matthews [6]. ∎

3 Discrete Information Acquisition

In this section we consider a model that is different from the one we called "continuous information acquisition". Here, additional information is acquired observing an *additional* random variable, which is informative either about the true value of the object or the opponent's signal, or about both. The purpose of this section is to validate our "money on the table" interpretation for the difference between the two auction forms with respect to information acquisition. We will consider the decision of acquiring an additional signal, and confirm that the incentives to do so are consistent with the previous section's results.

In Section 2 we have seen that the first- and second-price auction formats give different incentives to acquire information. Furthermore, we have

[8] This is because in our case the quantity $\frac{F(x|v)}{f(x|v)}$ is independent of v: see Milgrom and Weber [9], the proof of Theorem 15 on page 1109.

ascribed this systematic difference to the peculiar format of the second-price auction, that renders a bidder indifferent to his own bid (conditional on winning); that is, in a second price auction there is no use to reducing the amount of money left on the table (say, the difference between one's bid and the highest opponent's). In a first-price instead, this can be the source of great profit, which leads to a first-price auction valuing more information that is correlated with the opponent's bidding.

In the past section however, players observed only one random variable, and a more informative random variable had to be more correlated at once with the true value of the object and with the opponent's bidding: whence, the result that in this setting *always* a first-price gives more incentives to acquire information than a second price. If we allow a more generous structure for information acquisition, namely one where more than one signal is observable, we can explore the effect that the correlation of a signal has on the incentive to acquire it. In such a setting, we are not constrained any more by a theorem imposing a certain correlation of an additional piece of information in order for it to be "more information". Indeed, any additional signal will yield a more precise information structure. We are thus free to consider acquiring a signal that has various degree of correlation with the opponent's signal, or with the true value of the object, without one necessarily implying the other; and we can look for validation of our "money on the table" explanation for the difference between the two auction formats.

3.1 The model

Again, we assume two players: player 1 observes signal X, player 2 observes signal Y and X, Y are affiliated. Player i has valuation V_i for the object, where V_i is a random variable, and all the hypotheses are those of Milgrom–Weber's "generalized symmetric model" (see Milgrom and Weber [9]). In our model, it will be possible to covertly acquire the knowledge of an additional random variable Z. We will proceed in subsections, according to the various correlations of Z with the opponent's signal. We will calculate the incentives to acquire that signal when the *status quo* is the symmetric equilibrium with only X and Y known.

As a piece of notation, let $v_1(x,y) := E(V_1 \mid X = x, Y = y)$, and $v_1(x,z,y) := E(V_1 \mid X = x, Z = z, Y = y)$. Moreover, let $\mathcal{V}_1(x,y) := E(V_1 \mid X = x, Y < y)$ and $\mathcal{V}_1(x,z,y) := E(V_1 \mid X = x, Z = z, Y < y)$.

3.2 Acquiring Totally Correlated Information

This section is concerned with the case in which it is possible to acquire $Z \equiv Y$, i.e. the very opponent's signal (or a garbling of it). It is clear that such information, if available, allows one to reduce to 0 the amount of money left on the table; our intuition suggests that this sort of information should be more valuable in a first- than in a second-price auction. Indeed, we present the striking result that in a second-price auction such information is useless. This result appears in Milgrom [8], in a different context.

Some intuition for this result may be gathered observing that, when player 1 receives a signal in a second price auction, there are two cases: 1's signal is higher than 2's, or *vice versa*. In the first case, suppose that he gets to know y: 1 was winning before (remember his signal is higher than the opponent's); now 1's dominant strategy is to bid his expected value for the object, which is $v(x,y)$ (there is no more winner's curse here, because 1 knows whatever he could discover by winning). But this is higher than $v(y,y)$, which is 2's bidding, and thus 1 will continue to win. The second case is symmetrically treated.

Theorem 5 *Consider the symmetric equilibrium of a second-price auction with affiliated values. Then covertly learning the realization of the opponent's signal has no value.*

Proof: The problem of a player 1 with signal x, when he does not know y, is to choose an opponent's type k that maximizes

$$\int_{-\infty}^{k} [v(x,y) - v(y,y)] f(y \mid x) dy.$$

This is true because player 2's bidding strategy is strictly increasing. Since affiliation implies that the quantity $v(y,y) - v(x,y)$ is **QM** in y, the optimal

k solves

$$v(x,k) = v(k,k).$$

Thus, the optimized utility for player 1 is

$$\int_{y:v(x,y)\geq v(y,y)} [v(x,y) - v(y,y)] f(y \mid x) dy.$$

Suppose instead that player 1 knows y. In this case, his expected valuation for the object is $v(x,y)$. Moreover, player 1 knows that player 2 will bid $v(y,y)$, and he will therefore want to win the object when $v(x,y) \geq v(y,y)$. Hence, player 1's expected profit from the auction is

$$\int_{y:v(x,y)\geq v(y,y)} [v(x,y) - v(y,y)] f(y \mid x) dy,$$

which is the same as that when he did not know y. ∎

Remark 3 In a first-price auction, the knowledge of the opponent's signal is clearly useful. ◇

This result asseveres our interpretation, and is quite striking for the reason that, in a common value model, information about the opponent's signal should give a clearer idea of the object's worth, and hence should be valuable. In a private value model, this result is not at all new, and is a consequence of the equilibrium strategy in a second price auction being a dominant strategy.

3.3 Acquiring Independent Information

In this section we will examine the incentives to acquire the knowledge of an additional random variable, Z, which is taken to be independent of Y, the opponent's signal. We will see that in this case a second price auction gives more incentive to acquire independent information than a first price. This again fits well with our interpretation: here the bidding behaviour after information acquisition becomes less correlated with the opponent's bidding, since it has to reflect information that is independent of it. Thus, new information is not helpful in reducing the money on the table and actually, when used, leads one to reduce his bidding's correlation with the opponent's. It is then not surprising that this form of information acquisition yelds a higher payoff in a second-price auction. If we wanted to mimick this result in Section 2's parlance, we would have to take a $T_{\eta,\theta,v}(\cdot)$ such that $\frac{\partial}{\partial v}T$ is negative, i.e. the deviation from the old bidding behaviour is negatively correlated with the opponent's bidding: however, we know that such a perturbation actually represents a decrease in the informational content of the signal, and thus will never be acquired.

In the following, we will need to impose the following assumption:

A 1 $\frac{\partial^2}{\partial y \partial x} E(v(Y,Y) \mid Y < y, X = x) \geq 0.$

This plays the role of a stochastic single-crossing assumption, and is needed in the proof of Lemma 2 below. Let us present two cases in which assumption A 1 is satisfied.

Example 2 Let $v(Y,Y) = Y^\alpha$ for $\alpha > 0$, and $f(x,y) = \frac{4}{5}(1+xy)$ on $[0,1]^2$. Then $f(y \mid x) = \frac{2(1+xy)}{2+x}$, and

$$\frac{f(s \mid x)}{F(y \mid x)} = \frac{2(1+xs)}{2y + xy^2}.$$

Therefore,

$$E(v(Y,Y) \mid Y < y, X = x) = \int_0^y s^\alpha \frac{f(s \mid x)}{F(y \mid x)} ds = \frac{2}{2y + xy^2}\left[\frac{y^{\alpha+1}}{\alpha+1} + x\frac{y^{\alpha+2}}{\alpha+2}\right].$$

We can compute

$$\frac{\partial}{\partial x} E(v(Y,Y) \mid Y < y, X = x) = \frac{2y^{\alpha+3}}{[2y + xy^2]^2}\left[\frac{\alpha}{(\alpha+1)(\alpha+2)}\right]$$

and

$$\frac{\partial^2}{\partial x \partial y} E(v(Y,Y) \mid Y < y, X = x) = (\text{ pos. const. })[2(\alpha+1) + (\alpha-1)xy].$$

which is positive for $\alpha > 0$. ◇

Example 3 Suppose that X and Y are identically distributed according to

$$f(x,y) = \int_{-\infty}^{+\infty} f(x \mid v) f(y \mid v) g(v) dv,$$

where $f(\cdot \mid v)$ is the density of a random variable distributed as a Uniform on $[0, v]$. Then it is easy to check that $E(v(Y,Y) \mid Y < y, X = x)$ is independent of x for $y \in [0, v]$, that is for all y that can be realized, whereby A 1 holds with equality. ◇

Theorem 6 *Consider the symmetric equilibrium in strictly increasing strategy for a first- and second-price auction with affiliated information structure, and assume A 1. Then the incentive to acquire Z (independent of Y) in a second price auction is higher or equal than in a first price.*

Proof. See Appendix C. ∎

4 Conclusions

This paper has focused on the incentives that different auction mechanisms (first and second price) give to the acquisition of information. In our models, first bidders simultaneously acquire information; then, without observing the opponent's information structure, they engage in competitive bidding.

Two different but related technologies of information acquisition have been examined. The first dealt with choosing – in a continuous fashion – the informational content of one signal. A very general new concept of "better information", Accuracy, has been used; as a special case, it contains increasing information in Blackwell's sense. For this model, we have found that incentives to acquire information are always higher in a first than in a second price auction. We have given an intuition for this result, based on the different value of bidding close to the opponent when winning: in a second price auction this is immaterial, since the price paid is the opponent's bid. In a first price this has importance, since bidding close to the opponent minimizes the sum paid. Hence, a first price format values information that is correlated with the opponent's bidding more than a second price. Sufficient conditions have been given for the equilibrium Accuracy to be higher in a first than in a second price information acquisition game. The connection between incentives to acquire information and revenue to the seller has been explained, and verified in a number of auction forms.

To verify the robustness of our intuition, a different (but related) model of information acquisition has been developed, one where acquiring information means observing an additional random variable, besides one's signal. In accord with our intuition, it has been shown that acquiring a signal that is very correlated with the opponent's is more useful in a first than in a second price auction. Conversely, acquiring a signal independent of the opponent's is more useful in a second price: this is because it leads to bidding farther from the opponent, which is penalized in a first price format.

Finally, an example has been presented where endogeneizing information acquisition produces a reversal of Milgrom and Weber's revenue-ranking result.

Appendix A

Background Theory

This subsection reports two results from Persico [10]. Subsection 2.2 draws on this theory, and can indeed be seen as an application of these concepts.

Let us define a **payoff function** as a function

$$u(v,a): \mathcal{V} \times \mathcal{A} \rightarrow \Re.$$

Here, a represents an action, and v is an unknown parameter, seen as the realization of a random variable V. Let $g(v)$ be a prior for V, with c.d.f. $G(v)$.

The decision maker cannot observe V, but can observe a **signal**, a random variable X^{η} with conditional density $f^{\eta}(x \mid v)$. The associate c.d.f. is denoted by $F^{\eta}(x \mid v)$. This signal will be chosen – prior to observing its realization – from a **family of signals** $\{X^{\eta}\}_{\eta \in E}$, where E is an interval of the real line.

A payoff function together with a signal X^{η} and a prior for V constitute a **decision problem**, the problem being

$$\max_{a \in \mathcal{A}} \int_{\mathcal{V}} u(v,a) dG^{\eta}(v \mid x).$$

Define $a^{\eta}(x)$ as

$$a^{\eta}(x) \in \ \text{argmax}_{a \in \mathcal{A}} \int_{\mathcal{V}} u(v,a) dG^{\eta}(v \mid x)$$

and let

$$u^\eta(v,x) := u(v, a^\eta(x)).$$

and

$$R(\eta) := \int_{\mathcal{V}} \int_{\mathcal{X}} u(v, a^\eta(x)) dF^\eta(x \mid v) dG(v)$$

be the expected revenue to the decision maker with signal X^η.

Let

$$MR(\eta) := \frac{d}{d\theta} \int_{\mathcal{V}} \int_{\mathcal{X}} u(v, a^\theta(x)) dF^\theta(x \mid v) dG(v) \Big|_{\theta=\eta},$$

denote the marginal revenue to the decision maker from increasing η, i.e. choosing a slightly higher signal.

Definition 1 *We say that a real function $H(v)$ is* **quasi-monotone** *if*

(QM) $\qquad H(v) > 0 \Rightarrow H(v') \geq 0 \quad$ *for all $v' > v$.*

Definition 2 *A real function $u(v,a)$ has the* **weak single crossing property in $(a;v)$ on $\mathcal{I}$** *if, for any fixed pair $a' > a$, we have*

(WSCP)$\mathcal{I}$ $\qquad D_{a,a'}(v) := u(v,a') - u(v,a)$ *satisfies* **(QM)$\mathcal{I}$**.

Definition 3 *Given two real functions $u_I(v,a)$ and $u_{II}(v,a)$, we say that* $u_I \succeq u_{II}$ **on $\mathcal{I}$** *if $u_I - u_{II}$ satisfies* **(WSCP)$\mathcal{I}$**.

Definition 4 *Given a family $\{X^\eta\}_{\eta \in E}$, the transformation $T_{\eta,\theta,v}(\cdot)$ is defined by*

$$T_{\eta,\theta,v} : \Re \rightarrow \Re : \text{is increasing} \quad \textit{and} \quad T_{\eta,\theta,v}(X^\eta \mid v) \sim X^\theta \mid v.$$

Definition 5 *Consider a family of signals $\{X^\eta\}_{\eta \in E}$. We say that this family of signals is* **A-ordered** *by η if, given any η in E, we have $\frac{\partial}{\partial \theta} T_{\eta,\theta,v}(x)\Big|_{\theta=\eta}$ is nondecreasing in v, for all v and x in the support of $X^\eta \mid v$. If this quantity is strictly increasing in v, we say that the family is* **strictly A-ordered**.

Theorem 7 *Consider an A-ordered family of signals and assume that, for all x, $\frac{\partial}{\partial x} u^\eta(v,x)$ satisfies* **(QM)**. *Then $MR(\eta) \geq 0$.*

Theorem 8 *Suppose two payoff functions, say $u_I(v,a)$ and $u_{II}(v,a)$ are associated to the same A-ordered statistical structure, giving rise to two information acquisition problems. If for all x and η we have $u_I^\eta(v,x) \succeq u_{II}^\eta(v,x)$, then for all η, $MR_I(\eta) \geq MR_{II}(\eta)$.*

Appendix B

Revenue Ranking

In the notation of Appendix A, let for $m = I, II$

$$R_m(x,z) := \int_{-\infty}^{+\infty} u_m(v, a_m^\theta(x)) dG^\theta(v \mid z)$$

denote the (expected) revenue in decision problem m to an agent observing signal z who plays according to the optimal strategy as if he had observed x. Denote by

$$\Re_m(x) := R_m(x,x),$$

the expected revenue to type x who plays his optimal action. Consider two mechanisms, I and II, and assume $\Re_I(\underline{x}) = \Re_{II}(\underline{x})$ where $\underline{x}$ is the lowest type (signal). Then a *sufficient* condition for $\Re_I(x) \geq \Re_{II}(x)$ for all x is that

$$\Re_I(x) = \Re_{II}(x) \Rightarrow \Re'_I(x) \geq \Re'_{II}(x) \tag{B.1}$$

Now observe that, for $m = I, II$ we have

$$\Re'_m(x) := \frac{d}{dx}\Re_m(x) = \frac{\partial}{\partial x}\Re_m(x,z)\bigg|_{z=x} + \frac{\partial}{\partial z}\Re_m(x,z)\bigg|_{z=x} =$$

$$\frac{\partial}{\partial z}\Re_m(x,z)\bigg|_{z=x}.$$

where the last equality follows from the first order conditions with respect to x. Hence, we can rewrite B.1 as

$$\Re_I(x) = \Re_{II}(x) \Rightarrow \frac{\partial}{\partial z}\Re_I(x,z)\bigg|_{z=x} \geq \frac{\partial}{\partial z}\Re_{II}(x,z)\bigg|_{z=x} \tag{B.2}$$

Using the specific structure of the payoff function, we can rewrite the above as

$$\int_{-\infty}^{+\infty} \left[u_I(v, a_I^\theta(x)) - u_{II}(v, a_{II}^\theta(x))\right] dG^\theta(v \mid z) = 0 \Rightarrow \tag{B.3}$$

$$\frac{\partial}{\partial z}\int_{-\infty}^{+\infty} \left[u_I(v, a_I^\theta(x)) - u_{II}(v, a_{II}^\theta(x))\right] dG^\theta(v \mid z)\bigg|_{z=x} \geq 0$$

Since $G^\theta(v \mid z)$ has the monotone likelihood ratio property, a *sufficient* condition for (B.3) is that

$$u_I(v, a_I^\theta(x)) - u_{II}(v, a_{II}^\theta(x)) \text{ be quasi-monotone in } v \tag{B.4}$$

(for this last fact see Athey [1], or Persico [10] Lemma 2).

Appendix C

Miscellaneous Results

Proof of Theorem 1.

Proof. Because increasing Accuracy is not observed by the opponent, when calculating the revenue from doing so we take his behaviour as fixed. Thus, we are dealing with a decision problem. In view of Theorem 7, it suffices to show that, for a first or second price, $\frac{\partial}{\partial x} u^\theta(v_1, x_1)$ satisfies **(QM)**. Let us start with a second price: there, denoting by $b_1^\theta(\cdot))$ the best response of player 1 with Accuracy θ to $b_2^\eta(\cdot))$,

$$u_S^\theta(v_1, x_1) = \int_{-\infty}^{b_2^{\eta^{-1}}(b_1^\theta(x_1))} \left[\tilde{u}(v_1, y) - b_2^\eta(y)\right] f_{X_2}^\eta(y \mid v_1) dy$$

whence

$$\frac{\partial}{\partial x_1} u_S^\theta(v_1, x_1) = b_2^{\eta^{-1}\prime}(b_1^\theta(x_1)) b_1^{\theta\prime}(x_1) \left[\tilde{u}(v_1, b_2^{\eta^{-1}}(b_1^\theta(x_1))) - b_1^\theta(x_1)\right] f_{X_2}^\eta(b_2^{\eta^{-1}}(b_1^\theta(x_1)) \mid \tag{C.1}$$

This is the product of a **QM** function (in brackets) with nonnegative functions of v_1, which is therefore **QM**.

For a first price, we have

$$u_F^\theta(v_1, x_1) = \int_{-\infty}^{b_2^{\eta^{-1}}(b_1^\theta(x_1))} \left[\tilde{u}(v_1, y) - b_1^\theta(x_1)\right] f_{X_2}^\eta(y \mid v_1) dy$$

whence

$$\frac{\partial}{\partial x_1}u_F^{\theta}(v_1,x_1) = b_1^{\theta\prime}(x_1)\left\{b_2^{\eta-1\prime}(b_1^{\theta}(x_1))\left[\tilde{u}(v_1,b_2^{\eta-1}(b_1^{\theta}(x_1)))-b_1^{\theta}(x_1)\right]\times \right.$$
$$\left. \times f_{X_2}^{\eta}(b_2^{\eta-1}(b_1^{\theta}(x_1))\mid v_1)-F_{X_2}^{\eta}(b_2^{\eta-1}(b_1^{\theta}(x_1))\mid v_1)\right\} \quad \text{(C.2)}$$

The same reasoning as before applies, since

$$b_2^{\eta-1\prime}(b_1^{\theta}(x_1))\left[\tilde{u}(v_1,b_2^{\eta-1}(b_1^{\theta}(x_1)))-b_1^{\theta}(x_1)\right]-\frac{F_{X_2}^{\eta}(b_2^{\eta-1}(b_1^{\theta}(x_1))\mid v_1)}{f_{X_2}^{\eta}(b_2^{\eta-1}(b_1^{\theta}(x_1))\mid v_1)}$$

is monotonic in v_1 (recall $\frac{F(x_2|v_1)}{f(x_2|v_1)}$ is decreasing in v_1 because X_2 and V_1 are affiliated), and hence **QM**. Thus, again $\frac{\partial}{\partial x_1}u^{\theta}(v_1,x_1)$ satisfies **(QM)** and we are done. ▮

Proposition 3 *In a symmetric mineral rights auction with pure common values and affiliated signals an equilibrium in strictly increasing strategies exists.*

Proof: To prove the result it suffices to show that

$$\frac{\partial}{\partial x_1}u_{MRA}^{\theta}(v,x_1)=\left[v\,(1-r(x_1))-r'(x_1)\frac{F_{X_2}^{\theta}(x_1\mid v)}{f_{X_2}^{\theta}(x_1\mid v)}\right]f_{X_2}^{\theta}(x_1\mid v)$$

is quasi-monotone in v, which is clearly the case since)$\frac{F_{X_2}^{\theta}(x_1|v)}{f_{X_2}^{\theta}(x_1|v)}$ is decreasing in v by affiliation. ▮

Proof of Theorem 6.

In order to prove Theorem 6, we need to present a number of Lemmas. From now on we will refer to "equilibrium" quantities as to those concerning the equilibrium with no information acquisition, the one in which Z in not known by anybody; and to "optimal" quantities as to those regarding the optimal decision of player 1 that knows X, Z and is responding to a player 2 that plays "equilibrium" strategy. First, recall the following result from Milgrom and Weber [9].

Lemma 1 *Consider the symmetric equilibrium in strictly increasing strategies of any sealed bid auction mechanism m. Let $C^m(y;x)$ denote player 1's expected payment conditional on winning with signal $X = x$ againts all opponents' types lower than y. Then*

a) if X and Y are independent, $C^m(x;x)$ is independent of m.

b) if X and Y are affiliated, $C^S(x;x) \geq C^F(x;x)$, where F and S stand for first and second price auction, respectively.

Proof: See Milgrom and Weber. ∎

The following Lemma can be interpreted in the following way: if the additional piece of information leads one to modify one's bidding so as to reduce its correlation with the opponent's bidding, then a second-price auction will make a fuller use of it; this is because there one is not hampered by the need to "stay near" the opponent's bid. So, the behaviour in a second price auction will be more responsive to the additional piece of information. Of course, we must define the appropriate unit of measure for "staying near" and "more responsive". Indeed, one cannot use for this purpose the bidding units (money), since the second price auction has naturally more variance in its equilibrium bidding behaviour than a first price. The comparison has to be done, instead, in "opponent's types over which to win", so the unit if measure must be "types".

Lemma 2 *Consider the symmetric equilibrium in strictly increasing strategies of a first- and second-price auction with affiliated information, and assume A 1. The optimal opponent type over which to win is more reactive to additional independent information in a second price than in a first price.*

Proof: Suppose 1 knows $X = x$, 2 knows $Y = y$, and 1 also covertly observes Z, independent of Y. Let $y^m(x,z)$ denote the optimal opponent's type over which to win in mechanism m, i.e.

$$y^m(x,z) \in \text{argmax}_y \quad [\mathcal{V}(x,z,y) - C^m(y;x)]\, P(Y < y \mid X = x),$$

where $C^M(y;x)$ denotes the expected cost – conditional on observing X – of winning against types y or lower, in mechanism m. This quantity is independent of Z since Y is. Let $\Pi^m(x,z,y)$ denote the expected profit with observation z, i.e.

$$\Pi^m(x,z,y) = [\mathcal{V}(x,z,y) - C^m(y;x)]\, P(Y < y \mid X = x).$$

The first order conditions with respect to y read

$$\frac{\partial}{\partial y}\Pi^m(x,z,y)\bigg|_{y=y^m(x,z)} = \frac{\partial}{\partial y}\mathcal{V}_3(x,z,y)F(y \mid x) + \mathcal{V}(x,z,y)f(y \mid x) -$$

$$C_1^m(y;x)F(y \mid x) - C^m(y;x)f(y \mid x)\big|_{y=y^m(x,z)} = 0. \tag{C.3}$$

Part i): if there is a $z_0(x)$ such that $y^F(x, z_0(x)) = x$ (i.e. upon learning $Z = z_0(x)$ the bidding remains unchanged in a first price auction), then $y^S(x, z_0(x)) = x$ (the bidding is unchanged in a second price auction too).

We can rewrite the first order conditions in (C.3) as

$$\mathcal{V}_3(x,z_0(x),x)F(x \mid x) + \mathcal{V}(x,z_0(x),x)f(x \mid x) - C_1^F(x;x)F(x \mid x) - C^F(x;x)f(x \mid x) = 0. \tag{C.4}$$

However, *equilibrium* first order conditions tell us that

$$C_1^F(x;x)F(x \mid x) + C^F(x;x)f(x \mid x) = \mathcal{V}_2(x,x)F(x \mid x) + \mathcal{V}(x,x)f(x \mid x)$$
$$= C_1^S(x;x)F(x \mid x) + C^S(x;x)f(x \mid x)$$

Substituting the last term for the first one in (C.4), we see that $z_0(x)$ also solves $y^S(x, z_0(x)) = x$. Therefore, $z_0(x)$ leaves the bidding unchanged in a second price auction, too.

Part ii): we want to show that for $z > z_0(x)$, we have $x \leq y^F(x,z) \leq y^S(x,z)$.

Take any $y > x$: *equilibrium* first-order conditions tell us that, for any y

$$\left[C_1^F(y;y) + C^F(y;y)\frac{f(y \mid y)}{F(y \mid y)}\right] = \left[C_1^S(y;y) + C^S(y;y)\frac{f(y \mid y)}{F(y \mid y)}\right]$$

Now, $C^S(y;y) > C^F(y;y)$ (Lemma 1), and $\frac{f(y|s)}{F(y|s)}$ is nondecreasing in s, by affiliation. Therefore, since $x < y$, we obtain

$$\left[C_1^F(y;y) + C^F(y;y)\frac{f(y \mid x)}{F(y \mid x)}\right]\bigg|_{y=y^I(x,z)} \geq \left[C_1^S(y;y) + C^S(y;y)\frac{f(y \mid x)}{F(y \mid x)}\right]\bigg|_{y=y^I(x,z)}.$$

Since $C^F(\cdot;\cdot)$ (and hence $C_1^F(\cdot;\cdot)$) do not depend on their second argument, using A 1 we get

$$\left[C_1^F(y;x) + C^F(y;x)\frac{f(y \mid x)}{F(y \mid x)}\right]\Bigg|_{y=y^I(x,z)} \geq \left[C_1^S(y;x) + C^S(y;x)\frac{f(y \mid x)}{F(y \mid x)}\right]\Bigg|_{y=y^I(x,z)}.$$

Plugging this inequality into the following expression

$$\frac{\partial}{\partial y}\Pi^m(x,y,z) = \left\{\left[\mathcal{V}_3(x,z,y) + \mathcal{V}(x,z,y)\frac{f(y \mid x)}{F(y \mid x)}\right] - \left[C_1^m(y;x) + C^m(y;x)\frac{f(y \mid x)}{F(y \mid x)}\right]\right\} F(y \mid x), \tag{C.5}$$

we see that equilibrium conditions imply

$$\frac{\partial}{\partial y}\Pi^F(x,y,z) \leq \frac{\partial}{\partial y}\Pi^S(x,y,z) \text{ for all } x, z, y > x.$$

Since $y^m(x,z)$ is found equating expression (C.4) to 0, this shows that

$$y^F(x,z) > x \Rightarrow y^S(x,z) \geq y^F(x,z). \tag{C.6}$$

Furthermore, since $\mathcal{V}(x,z,y)$ is nondecreasing in z, it is clear that $z > z_0(x) \Rightarrow y^F(x,z) \geq x$, whereby taking into account equation (C.6), we conclude

$$z > z_0(x) \Rightarrow y^S(x,z) \geq y^F(x,z).$$

Part iii) Reasoning symmetrically for the case where $y^F(x,z) < x$ we obtain $y^S(x,z) < y^F(x,z) < x$. ∎

Remark 4 Of course, when X and Y are independent, A 1 holds with equality, and Lemma 2 reads "the optimal opponent types over which to win is equally reactive to independent information in a first as in a second price auction". ◇

Lemma 3 *Suppose Z is independent of Y, and $v(x,z,y)$ is increasing in z. Then $\frac{\partial}{\partial z}\mathcal{V}(x,z,y)F(y \mid x)$ is nondecreasing in y.*

Proof:

$$\frac{\partial^2}{\partial y \partial z}\mathcal{V}(x,z,y)F(y \mid x) = \frac{\partial^2}{\partial y \partial z}\int_{-\infty}^{y} v(x,z,s)f_Y(s \mid x)ds =$$

$$= f_Y(y \mid x)\frac{\partial}{\partial z}v(x,z,y)$$

which is greater than 0 by assumption. ∎

We are finally ready to present the main result of this section.

Proof of Theorem 6:

Proof: Let

$$G^m(x,z) := \max_y \left[\mathcal{V}(x,z,y) - C^m(y;x)\right] P(Y < y \mid X = x) - Q^m(x,z)$$

denote the gain in mechanism m from covertly learning $Z = z$. $Q^m(x,z)$ is the expected profit in mechanism m to player 1 when $Z = z$, he does not know Z and therefore plays his symmetric equilibrium strategy. As usual, let $y^m(x,z)$ denote the optimal opponent's type over which to win in mechanism m when observing $X = x$ and $Z = z$ (the argmax in the above problem), and let

$$G^m(x,z,z') := \left[\mathcal{V}(x,z,y^m(x,z')) - C^m(y^m(x,z');x)\right] P(Y < y^m(x,z') \mid X = x) - Q^m(x,z).$$

$G^m(x,z,z')$ denotes the expected gain in mechanism m when observing x, z but playing (subotptimally) $y^m(x,z')$. It is clear that $G^m(x,z) = G^m(x,z,z)$.

By definition of $y^m(\cdot,\cdot)$ we know that

$$G_3^m(x,z,z) := \frac{\partial}{\partial z'}G^m(x,z,z')\bigg|_{z'=z} = 0. \qquad \text{(C.7)}$$

We can also compute that

$$\frac{\partial}{\partial z}Q^m(x,z) = \frac{\partial}{\partial z}\mathcal{V}(x,z,x)F_Y(x \mid x) \text{independent of } M. \qquad \text{(C.8)}$$

Now, observe that

$$G_2^m(x,z,z) := \frac{\partial}{\partial z}G^m(x,z,z')\bigg|_{z'=z} = \frac{\partial}{\partial z}\mathcal{V}(x,z,y^m(x,z'))F(y^m(x,z') \mid x)\bigg|_{z'=z}.$$

By Lemma 3 this quantity is increasing in y^m. Furthermore, Lemma 2 says that if $z > z_0(x)$, then $x \leq y^I(x,z) \leq y^{II}(x,z)$, and the opposite inequalities hold if $z < z_0(x)$. Therefore,

$$\begin{array}{lll} G_2^F(x,z,z) \leq G_2^S(x,z,z) & \text{if} & z > z_0(x) \\ G_2^F(x,z,z) \geq G_2^S(x,z,z) & \text{if} & z < z_0(x) \end{array} \tag{C.9}$$

We are interested in the quantity $G^m(x,z) = G^m(x,z,z)$, the the gain from acquiring the observation of Z when $Z = z$. So, observe that

$$G^F(x, z_0(x)) = G^S(x, z_0(x)) = 0$$

and we can compute

$$\frac{d}{dz}G^F(x,z) = G_2^F(x,z,z) + G_3^F(x,z,z) - \frac{\partial}{\partial z}Q^m(x,z).$$

In view of equations (C.7), (C.8) and (C.9), we can conclude that

$$\begin{array}{lll} \frac{d}{dz}G^F(x,z) \leq \frac{d}{dz}G^S(x,z) & \text{if} & z > z_0(x) \\ \frac{d}{dz}G^F(x,z) \geq \frac{d}{dz}G^S(x,z) & \text{if} & z < z_0(x) \end{array} \tag{C.10}$$

This implies, integrating forward and backward from $z_0(x)$,

$$G^F(x,z) \leq G^S(x,z).$$

Taking expected value over all possible values of z, we find that the incentives to acquire Z are higher in a second than in a first price auction. ∎

Remark 5 If X and Y are independent, then the incentive to acquire Z independent of Y is the same in a first and second price auction. This follows from the proof of Theorem 6, taking into account Remark 4. ◇

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