Capital Income Taxation and Risk–Taking in a Small Open Economy

by

PATRICK K. ASEA
University of California, Los Angeles & NBER
and
STEPHEN J. TURNOVSKY
University of Washington, Seattle

Working Paper Number 768
Department of Economics
University of California, Los Angeles
Bunche 2263
Los Angeles, CA 90095–1477
June, 1997

Capital Income Taxation and Risk–Taking in a Small Open Economy*

PATRICK K. ASEA[†]
University of California, Los Angeles & NBER

STEPHEN J. TURNOVSKY
University of Washington, Seattle

to appear, *Journal of Public Economics*

Abstract

How do capital income taxes affect household portfolio choice and growth? We approach this question within the context of a stochastic model of a small open economy in which taxes on income from domestic capital (equity) and foreign bonds affect household portfolio choice, welfare and the growth rate of the economy. The theoretical and numerical analysis demonstrates the important role that risk plays in determining the mean and variability of growth as well as the conditions under which a higher tax rate can be welfare improving. To shed more light on the complex theoretical interaction between taxes and risk–taking we estimate a reduced–form multinomial probit model of household portfolio choice using the method of simulated moments. The empirical evidence is in stark contrast to the conventional wisdom — we find that higher taxes make it less likely that the household will hold risky assets.

Keywords: Capital income taxes, endogenous growth, risk–taking, portfolio choice, multinomial probit, method of simulated moments.
Journal of Economic Literature Classification E6, E62

*We are grateful to John Muellbauer, Steve O'Connell, Keunkwan Ryu, Aaron Yelowitz, the editor, an anonymous referee and seminar participants at UCLA and Oxford for insightful comments. Jessica Tien, Peter Mondo, Julius Lear and Polin Muniko provided excellent and enthusiastic research assistance. Special thanks to Pater Salazar for facilitating collection of the data. This paper was completed when Asea was visiting the Macroeconomics and Growth Division of the World Bank. Asea gratefully acknowledges the hospitality of the World Bank and the financial support of the UCLA Academic Senate Committee on Research. Symbolic computations, graphics and empirical estimates reported in this paper were performed on a Pentium–66 (replacement chip) PC using programs written in GAUSS[TM] and MATHEMATICA (Wolfram (1988)). All programs are available on request from the corresponding author. This paper reflects the authors' views and not those of the World Bank. Any errors and omissions are our joint responsibility.

[†]**Send Correspondence to:** Patrick K. Asea, Department of Economics, UCLA, 405 Hilgard Avenue, Los Angeles, CA 90024. Tel: (310) 206-6869, Fax: (310) 825-9528, e-mail `asea@econ.ucla.edu`

1 Introduction

How do taxes on capital income affect risk–taking and economic growth ? Though this question is hardly new it remains largely unanswered. This gap in the literature is particularly unfortunate because the importance of taxes in influencing entrepreneurial risk–taking, investment and growth has been conjectured since Schumpeter (1934).

There are two competing views on the relationship between taxation and risk–taking. The first view is that the taxation of risky assets *discourages* risk–taking. The second view is that taxation of risky assets *encourages* risk–taking. The intuition behind the second, rather surprising view, is that an increase in the tax rate decreases both the risk and the return to the asset, thus increasing the demand for higher returns (by raising the marginal utility of income) and reducing resistance to bearing more risk (by lowering the marginal disutility of risk). The net result is an increased demand for assets with greater return *and* greater risk.[1]

If the second view is correct, greater risk–taking should lead to an increased demand for risky assets which should lower the cost of risky capital. Therefore, in the long–run, one would expect the lower cost of risky capital to lead to an increase in the economy's capital stock and growth rate. Despite the sharp predictions of this line of reasoning there has been surprisingly little work on the relationship between risk–taking and economic growth.[2]

Understanding the interaction between taxation, risk–taking and growth is important because it is central to one of the most challenging questions facing academics and policymakers — why some poor nations become rich while others remain poor. Numerous empirical studies on the determinants of growth have demonstrated that the share of private investment in GDP is the most important determinant of cross–country differences in growth rates. However, private investment can only take place if entrepreneurs are willing to bear risk. Factors that affect an entrepreneur's willingness to bear risk are therefore important

[1] The second view has been dominant in the academic literature since the seminal work of Domar & Musgrave (1944), Mossin (1968) and Stiglitz (1969) despite the fact that the result holds only under special assumptions, Sandmo (1989).

[2] See Corsetti (1992) and Levine (1991) for some closely related work.

in understanding the relationship between private investment and growth. In this paper we focus on one of these factors — capital income taxes. Many countries have attempted to encourage private investment through tax incentives.[3] However, such tax incentives will be of uncertain effectiveness in achieving the alleged (growth) goals because we know so little about the relationship between the household's asset choices that are influenced by tax policy and the real factors underlying growth performance.

In this paper we use theory, new data and a powerful econometric method to examine the relationship between capital income taxes, household portfolio choices (risk–taking) and economic growth. We start our analysis at the macroeconomic level by developing a stochastic general equilibrium model of a small open economy in which taxes on domestic capital (equity) income and foreign bonds affect household portfolio choice, welfare and the growth rate of the economy. Production takes place by means of a linear technology in which capital is the only factor of production. With the production function thus exhibiting constant returns to scale in the input being accumulated, the equilibrium is one of ongoing, endogenously–determined growth. The economy is subject to various stochastic disturbances of both domestic and foreign origin, all of which are taken to be Brownian motion processes. Accordingly, the model can be characterized as a "stochastic" endogenous growth model.[4]

An important feature of our analysis is a focus on open–economy aspects of taxation. With recent reductions in barriers to capital mobility, economists have begun to pay more attention to the international ramifications of tax policy. Most of this literature adopts the neoclassical framework and analyzes the macrodynamic effects of various forms of taxation on both the domestic economy and the world economy. For example, Sen and Turnovsky (1990) analyze the effects of an investment tax credit on the accumulation of capital and

[3]See Lent (1967), Shah and Toye (1978) on tax incentives. See for instance Asea, Mendoza and Milesi (1995) who demonstrate the adverse effect of factor income taxes on private investment in a cross–country study of OECD countries using aggregate data on marginal effective taxes.

[4]In this respect, the model can be viewed as an extension of the deterministic A–K models developed by Barro (1990), Jones and Manuelli (1990) and Rebelo (1991) to a small open economy. As a stochastic model, it builds on previous contributions by Eaton (1981), Turnovsky (1993) for a closed economy and by Stulz (1981), Grinols and Turnovsky (1994) for an open economy.

the dynamics of the current account of a small open economy. Nielsen and Sorensen (1991) analyze the implications of various forms of capital taxation (dividend tax, capital gains tax, and investment tax credit) in a similar type of economy, though using a somewhat different model. Bovenberg (1986), Sibert (1990), Frenkel, Razin and Sadka (1991) and Turnovsky and Bianconi (1992) address these types of issues within a two country world economy. Much of this literature focuses on the distinction between residence–based and source–based principles of taxation and the implication this has for the viability of the international tax system, Giovannini (1990), Sinn (1990), Frenkel and Razin and Sadka (1991). However as Slemrod (1988) has suggested, the stringent arbitrage conditions characterizing the viability of alternative exchange rate regimes is essentially an artifact of the absence of risk and the corner solutions this yields with respect to portfolio decisions. The viability issue ceases to be important in the presence of risk and risk averse agents.

The main insight from the theoretical and numerical analysis is the important role risk plays in determining growth rates, welfare and household portfolio choices. For instance, among other results, we demonstrate how the share of domestic capital in the optimal portfolio can raise or lower the mean growth rate of the economy depending on the predominant sources of risk. We also demonstrate conditions under which a higher tax rate can be welfare improving.

Having explored the theoretical macroeconomic interrelationships, we turn to the empirical micro–level relationship between taxation and portfolio choices. In our empirical analysis we utilize previously unused household–level data from South Africa. We specify the household's portfolio choice problem as multinomial probit (MNP). Estimation is implemented using McFadden's (1989) method of simulated moments (MSM).[5] We use a MNP model because risk–taking is best modeled as a *discrete* choice between a finite set of portfolios in which certain portfolios may be closer substitutes than others. Studies that use (indepen-

[5]See Uhler & Cragg (1971) for an early application of the multinomial probit model to analyze the structure of asset portfolios of households.

dent) binomial probit models treat the decision to hold an asset as if it was independent of the existence of alternatives other than holding a zero amount of the asset. However, multinomial probit (MNP) models are difficult to estimate because they require the computation of high dimensional numerical integrals. The method of simulated moments utilizes the orthogonality principle to construct unbiased estimates of complicated choice probabilities without direct numerical integration. The MSM thus provides a computationally inexpensive means of estimating choice probabilities when the choice set is large (more than 4 alternatives).[6]

The empirical results are striking — we demonstrate that a higher tax rate *decreases* the probability of holding risky assets in our sample. This empirical result runs counter to the conventional view (discussed above) that increasing the tax on a risky asset *increases* holdings of the risky asset.

The rest of the paper proceeds as follows. In Section 2 we describe the basic structure of the stochastic small open economy and derive the macroeconomic equilibrium. In Section 3 we demonstrate both analytically and numerically the effect of tax changes on equilibrium portfolio shares, growth rates, the variance of the growth rate and welfare. Section 4 describes the data, discusses the method of simulated moments and the empirical evidence. The concluding section discusses the significance of our results. Detailed derivations are provided in an Appendix.

2 A Stochastic Small Open–Economy

In this section we describe the basic structure of a small open economy that specializes in the production of a single good. The economy is assumed to be sufficiently small in the world production of this good, so as to have no impact on its market price.

[6]This is because it is possible to reduce a 4 alternative problem to one which requires integration of only a bivariate normal density.

2.1 *Prices and Asset Returns*

The economy is inhabited by a representative household who consumes both the domestically produced good, and a second good that is imported from abroad. Being small in the markets for both goods, the relative price E of the imported good, in terms of the domestically produced good taken as numeraire, is given exogenously and is assumed to be generated by the geometric Brownian motion process

$$\frac{dE}{E} = \epsilon dt + de, \tag{1}$$

where ϵ is the instantaneous expected rate of change in the relative price and de is a temporally independent, normally distributed, random variable with mean zero and variance σ_ϵ^2.[7]

In addition, the representative household holds three assets in her portfolio: traded bonds $B^\star$; domestic bonds B which are nontraded[8]; and equity claims (which may also be traded) on physical capital K. There is no money, so the model is real. Traded bonds are assumed to be denominated in terms of foreign output, so that their price in terms of the numeraire also follows (1). Capital is measured in terms of the numeraire. The domestic bonds are assumed to be consols paying one unit of output over the instant dt. The relative price of these domestic bonds (in terms of the numeraire) is assumed to evolve according to the stochastic process

$$\frac{dQ}{Q} = \eta dt + dq, \tag{2}$$

where η is the instantaneous expected rate of change and dq is a temporally independent,

[7] The assumption of geometric Brownian motion is introduced, in part, to exclude the chance of negative prices and quantities. This assumption is not entirely harmless. For example, (1) implies that there is a tendency for continuous real depreciation in this economy.

[8] If domestic bonds were internationally traded, the risk parity conditions between the foreign and domestic bond would become exogenously determined by risk conditions and preferences in the rest of the world, since the domestic economy is small. Rather than impose such a condition arbitrarily, or attempt to model the entire economy of the rest of the world, we assume that domestic bonds are nontraded, thereby determining the risk parity condition between the two assets endogenously in the market of the small open economy We should also add that the model does not belong to the conventional CAPM family because the menu of assets is more limited by the absence of a riskless domestic asset.

normally distributed random variable with mean zero and variance σ_q^2. While the representative household perceives the stochastic process (2) as being parametrically given, it will ultimately be determined as part of the macroeconomic equilibrium. In contrast to ϵ which by assumption is constant, η is time varying as will be shown below.

With domestic and foreign bonds paying deterministic coupons of $(1/Q)dt$, $i^\star dt$, respectively, and with their respective prices following (2) and (1) , the real rates of return on domestic and foreign bonds, expressed in terms of the domestic good as numeraire, are:

$$dR_b = r_b dt + du_b; \qquad r_b = \frac{1}{Q} + \eta; \quad du_b \equiv dq, \tag{3}$$

$$dR_f = r_f dt + du_f; \qquad r_f = i^\star + \epsilon; \quad du_f \equiv de. \tag{4}$$

where the foreign interest rate. $i^\star$ is exogenously given.

Production of domestic output Y using domestic capital K is given by the following simple stochastic constant returns technology

$$dY = \alpha K dt + \alpha K dy, \tag{5}$$

where α is the (constant) marginal physical product of capital and dy is a temporally independent, normally distributed random variable with zero mean and variance σ_y^2. Equity investment is the real investment opportunity represented by this technology. Hence, in the absence of adjustment costs to investment, the before–tax real rate of return on equity (capital) is

$$dR_k = r_k dt + du_k; \qquad r_k = \alpha; \quad du_k \equiv \alpha dy. \tag{6}$$

2.2 *Household Optimization*

The representative household's asset holdings are subject to the wealth constraint

$$W = K + QB + EB^\star, \tag{7}$$

where W denotes real wealth, expressed in terms of the domestic good as numeraire. In addition, over the instant dt she is assumed to purchase output of the two commodities at the nonstochastic rates $C_d(t)dt$, $C_m(t)dt$ respectively.

The household's objective is to select these rates of consumption, together with her portfolio of assets, to maximize the expected value of discounted utility

$$\mathbb{E}\int_0^{+\infty} \frac{1}{\gamma}(C_d^{\theta}C_m^{1-\theta})^{\gamma} e^{-\rho t} dt \qquad -\infty < \gamma < 1; \quad 0 \leq \theta \leq 1; \tag{8}$$

subject to the wealth constraint (7) and the stochastic wealth accumulation equation, expressed in real terms as

$$dW = W[n_k dR_k + n_b dR_b + n_f dR_f] - (C_d + EC_m)dt - dT, \tag{9}$$

where $n_k = \frac{K}{W}$ is the share of portfolio held in the form of capital; $n_b = \frac{QB}{W}$ is the share of portfolio held in the form of domestic bonds; $n_f = \frac{EB^{\star}}{W}$ is the share of portfolio held in the form of traded bonds; dT is taxes paid (described below). With utility being represented by the constant elasticity function, $r \equiv 1 - \gamma$ measures the constant coefficient of relative risk aversion. The value $\gamma = 0$ corresponds to the logarithmic utility function.[9]

The government is assumed to tax the various sources of income in accordance with

$$dT = \tau_k K(r_k dt + du_k) + \tau_b QB(r_b dt + du_b) + \tau_f EB^{\star}(r_f dt + du_f), \tag{10}$$

where τ_k, τ_b, τ_f denote the rates at which the three sources of income are taxed.

Substituting for the portfolio shares n_i into (7), and for (3), (4), (6) and (10) into (9), the stochastic optimization problem can be expressed as choosing the consumption–wealth ratios $C_d/W, C_m/W$ and portfolio shares n_i to maximize (8) subject to

$$\frac{dW}{W} = \psi dt + dw, \tag{11}$$

[9]Strictly speaking, the logarithmic utility function emerges as

$$\lim_{\gamma \to 0}\left[\frac{(C_d^{\theta}C_m^{1-\theta})^{\gamma} - 1}{\gamma}\right].$$

This function differs from (8) by the subtraction of the term -1 in the numerator and the two forms of utility function have identical implications.

$$n_k + n_b + n_f = 1, \tag{12}$$

together with (1), where for convenience, we denote the deterministic and stochastic components of the rate of asset accumulation dW/W by

$$\psi \equiv n_k(1-\tau_k)r_k + n_b(1-\tau_b)r_b + n_f(1-\tau_f)r_f - \frac{C_d}{W} - \frac{EC_m}{W}, \tag{13}$$

$$dw \equiv n_k(1-\tau_k)du_k + n_b(1-\tau_b)du_b + n_f(1-\tau_f)du_f. \tag{14}$$

In performing the optimization, the representative household takes the rates of return on the assets and the relevant variances and covariances as given. However, these will all ultimately be determined in the equilibrium to be derived below.

The maximization of (8) subject to (11) – (14) and (1) is straightforward and derivations of the solution are provided in the Appendix.[10] Defining aggregate consumption expressed in terms of domestic output by

$$C \equiv C_d + EC_m,$$

the first order optimality conditions are:

$$C_d = \theta C, \tag{15}$$

$$EC_m = (1-\theta)C, \tag{16}$$

$$\begin{aligned} \frac{C}{W} &= \frac{1}{1-\gamma}\Big\{\rho + \epsilon\gamma(1-\theta) - \gamma\beta - \frac{1}{2}\gamma(\gamma-1)\sigma_w^2 - \frac{1}{2}\gamma(1-\theta) \\ &\quad [\gamma(1-\theta)+1]\sigma_e^2 + \gamma^2(1-\theta)\sigma_{ew}\Big\}, \end{aligned} \tag{17}$$

$$\begin{aligned} [(1-\tau_b)r_b - (1-\tau_k)r_k]\,dt &= (1-\gamma)\mathrm{cov}\left[dw, (1-\tau_b)du_b - (1-\tau_k)du_k\right] \\ &\quad + \gamma(1-\theta)\mathrm{cov}\left[de, (1-\tau_b)du_b - (1-\tau_k)du_k\right], \end{aligned} \tag{18}$$

[10]See Merton (1971), Malliaris and Brock (1982) for a detailed discussion of the methods involved.

$$\begin{aligned}[(1-\tau_f)r_f-(1-\tau_k)r_k]\,dt &= (1-\gamma)\mathrm{cov}\left[dw,(1-\tau_b)du_f-(1-\tau_k)du_k\right] \\ &\quad +\gamma(1-\theta)\mathrm{cov}\left[de,(1-\tau_f)du_f-(1-\tau_k)du_k\right], \qquad (19)\end{aligned}$$

where

$$\beta \equiv n_k(1-\tau_k)r_k + n_b(1-\tau_b)r_b + n_f(1-\tau_f)r_f.$$

and the expressions for σ_w^2, σ_e^2, σ_{ew} which appear in (17) are calculated from equations (1) and (13) above.

Equations (15) and (16) describe the consumption of the two goods as fixed fractions of overall consumption expenditure, expressed in terms of domestic output. Equation (17) is the solution for the aggregate consumption–wealth ratio. In the case of the logarithmic utility function $\gamma = 0$, (17) reduces to the familiar relationship $C/W = \rho$. Equation (18) expresses the differential after–tax real rates of return on domestic bonds and capital in terms of their relative risk differentials, as measured by the covariance of their returns with the return on the overall portfolio. Equation (19) is an analogous relationship between the differential after–tax real rates of return on traded bonds and capital. Solving (18) and (19) in conjunction with the normalized wealth constraint (12), one can determine the household's portfolio demands n_i.

2.3 *Government Policy*

Government policy is described by the choice of government expenditure, the issuing of debt, and the collection of taxes, all of which must be specified subject to its budget constraint. This may be expressed in real flow terms as

$$d(QB) = dG + (QB)dR_b - dT, \qquad (20)$$

where dG denotes the stochastic rate of real government expenditure.

Government expenditure policy is specified by

$$dG = g\alpha K dt + \alpha K dz, \tag{21}$$

where dz is an intertemporally independent, normally distributed, random variable with zero mean and variance σ_z^2. According to this specification, the mean level of public expenditure is assumed to be a fraction g of the mean level of output, with a proportional stochastic disturbance.[11]

As is evident from the discussion of the household described in Section 2.2 above, government expenditure has no direct impact on the behavior of the private sector. It can be interpreted as being either a real drain on the economy, or alternatively as some public good that does not affect the marginal utility of private consumption or the productivity of private capital. This is a standard assumption and enables us to isolate the effects of distortionary taxation from government expenditure; see Rebelo (1991).

With expenditures specified in (20) and tax collection in (10) the government finances any deficit by issuing long bonds (consols) paying a unit return over the instant dt. Such bonds have a market price Q, which is endogenously determined in the market. Thus in this economy, the government sets expenditure levels and tax rates independently, floating as many bonds as needed to finance the budget. Substituting (21) and (10) into (20) and dividing by W, the rate of accumulation of government bonds is given by

$$\begin{aligned} \frac{dQB}{QB} &= \frac{1}{n_b}\{[(g - \tau_k \alpha n_k + (1-\tau_b) r_b n_b - \tau_f r_f n_f]\, dt \\ &\quad + \alpha n_k dz - \tau_k n_k du_k + (1-\tau_b) n_b du_b - \tau_f n_f du_f\}. \end{aligned} \tag{22}$$

2.4 *The Goods Market and the Current Account*

Net exports are determined by the excess of production over domestic uses, $dY - dC -$

[11]For the equilibrium to be one of ongoing endogenous growth, government expenditure must grow with the economy. If that were not so, such an equilibrium would eventually become unsustainable. The notion that the government seeks to claim a share of domestic output is a natural one, but other rules determining G, including that it be determined by tax revenues are also possible.

$dK - dG$. Balance of payments equilibrium, in turn, requires the transfer of new foreign bonds (in excess of interest on earlier issues) to finance net exports of the domestic economy. This is expressed in real terms by the relationship

$$d(EB^{\star}) = [dY - dC - dK - dG] + (EB^{\star})dR_f. \tag{23}$$

Dividing by W, this can be expressed as

$$n_f \frac{d(EB^{\star})}{EB^{\star}} + n_k \frac{dK}{K} = \left[\alpha(1-g)n_k + n_f r_f - \frac{C}{W}\right] dt + \alpha n_k(dy - dz) + n_f du_f. \tag{24}$$

We should note, however, that the accumulation equations (11) (22) and (24) are not independent; any two imply the third.

2.5 *Macroeconomic Equilibrium*

We now combine the elements introduced above to determine the overall stochastic equilibrium of the small open economy. The equilibrium determines the rates of consumption and savings; the value and rates of return on all assets; the portfolio allocation; the economy's investment and growth rate together with its current account; and the risk characteristics of each asset. The exogenous factors include: the initial stocks of assets K_o B_o $B_o^{\star}$, and the initial real exchange rate E_o; the stochastic process generating the real exchange rate (ϵ, de); the stochastic process describing government expenditure (g, dz); the tax rates (τ_k, τ_b, τ_f) the stochastic process characterizing technology (α, dy); and the preference parameters (γ, ρ). The three exogenous stochastic processes (de, dy, dz) are assumed to be mutually uncorrelated.[12] The remaining stochastic disturbances for prices and wealth, namely dq, dw, are determined endogenously and can be expressed as functions of the exogenous shocks. The endogenous variances and covariances can then be determined and an overall mean–variance equilibrium derived.

[12] This assumption is made for convenience only and nonzero patterns of correlation could be introduced if desired.

Our objective is to reduce the model to a simple set of core relationships, which jointly determine the deterministic and stochastic components of the macroeconomic equilibrium. From the relationships in (15)–(19) it is reasonable to posit that if assets have the same stochastic characteristics through time, they will generate the same allocation of portfolio holdings, as well as the same consumption–wealth ratio.[13] Our strategy, therefore, is to look for an equilibrium in which portfolio shares, n_i, and the consumption–wealth ratio C/W are nonstochastic functions of the underlying parameters of the model and observe that the restrictions thus implied are in fact consistent with this solution.[14]

We derive the equilibrium in two interrelated steps. In step one we first solve for the stochastic relations implied by the equilibrium. Using these, we then solve for the deterministic conditions that describe the economy. These relationships themselves imply relations among the stochastic components. Thus, for the equilibrium to be consistent we must check that the constraints implied by the deterministic dynamics are satisfied by the stochastic relations and vice versa.

The intertemporal constancy of portfolio shares implies

$$\frac{dK}{K} = \frac{dQB}{QB} = \frac{d(EB^{\star})}{EB^{\star}} = \frac{dW}{W} = \psi dt + dw. \tag{25}$$

Focusing on the stochastic components of equations (11), (22) and (24) and equating in accordance with (25) (while recalling the definitions of du_k, du_b, du_k appearing in (3), (4) and (6), enables us to solve for dw, and dq as follows

$$dw = \alpha\omega(dy - dz) + (1-\omega)de, \tag{26}$$

$$dq = \frac{\alpha\left[\omega - n_k(1-\tau_k)\right] dy - \alpha\omega dz + \left[(1-\omega) - n_f(1-\tau_f)\right] de}{n_b(1-\tau_b)}. \tag{27}$$

[13] For a general utility function, the optimal portfolio shares and consumption–wealth ratio will be functions of time. The constancy is a consequence of the constant elasticity utility function, the linear production function, and the assumption that the means and variances of the underlying stochastic processes are constant through time.

[14] As in all rational expectations equilibria, this procedure does not rule out other equilibria in which these constancy properties do not prevail. But if such an equilibrium can be found it is certainly a legitimate one, and one of genuine economic significance.

where we define

$$\omega \equiv \frac{n_k}{n_k + n_f},$$

to be the share of capital in the traded portion of the household's portfolio.

The solutions (26), (27) enable us to compute all the necessary variances and covariances. In particular, we note the following expressions, which appear in the consumer optimality conditions (17)-(19)

$$\sigma_w^2 = \alpha^2\omega^2(\sigma_y^2 + \sigma_z^2) + (1-\omega)^2\sigma_e^2; \qquad \sigma_{ew} = (1-\omega)\sigma_e^2 \tag{28}$$

$$\begin{aligned}\mathrm{cov}[dw, (1-\tau_b)du_b - (1-\tau_k)du_k] &= \left\{\alpha^2\omega\left[-(1-\omega) + \left(\frac{n_b + n_k}{n_b}\right)\tau_k\right]\sigma_y^2 \right. \\ &\quad \left. + \frac{\alpha^2\omega^2}{n_b}\sigma_z^2 + (1-\omega)\left[(1-\omega) + \frac{n_f}{n_b}\tau_f\right]\sigma_e^2\right\} dt\end{aligned} \tag{29}$$

$$\mathrm{cov}[de, (1-\tau_b)du_b - (1-\tau_k)du_k] = \left[(1-\omega) + \frac{n_f}{n_b}\tau_f\right]\sigma_e^2 dt, \tag{30}$$

$$\mathrm{cov}[dw, (1-\tau_f)du_f - (1-\tau_k)du_k] = \left\{-\alpha^2\omega(1-\tau_k)\sigma_y^2 + (1-\omega)(1-\tau_f)\sigma_e^2\right\} dt, \tag{31}$$

$$\mathrm{cov}[de, (1-\tau_f)du_f - (1-\tau_k)du_k] = (1-\tau_f)\sigma_e^2 dt. \tag{32}$$

Note that these expressions are not complete solutions, as they are expressed in terms of the portfolio shares which are yet to be determined.

We may now collect the equations of the complete system. First, substituting (31) and (32) into the optimality condition (19) (recalling the definitions of r_k, r_f given in (3), (4) and (6)) leads to the following solution for ω the share of capital in the traded portion of the investor's portfolio

$$\omega = \frac{\alpha(1-\tau_k) - (i^\star + \epsilon)(1-\tau_f) + (1-\gamma\theta)(1-\tau_f)\sigma_e^2}{(1-\gamma)\left[\alpha^2(1-\tau_k)\sigma_y^2 + (1-\tau_f)\sigma_e^2\right]}. \tag{33}$$

Choosing ω in accordance with (33) ensures the equality of the risk–adjusted after–tax rates of return on the two traded assets.

The above expression highlights two factors determining the optimal portfolio ω. The first is the speculative component which depends upon the differential after–tax real rate of return, while the second reflects the hedging behavior on the part of the investor. This depends upon the relative tax–adjusted variances associated with the returns on the two traded assets.

Next, substituting (29) and (30) into the household's optimality condition (18) implies the following expression for the after–tax rate of return on domestic bonds

$$\begin{aligned} r_b(1-\tau_b) &= \alpha(1-\tau_k) + (1-\gamma)\omega\alpha^2\left(\left[-(1-\omega) + \left(\frac{n_k+n_b}{n_b}\right)\tau_k\right]\sigma_y^2\right. \\ &\quad \left. + \frac{\omega}{n_b}\sigma_z^2\right) + [(1-\gamma)(1-\omega)+\gamma(1-\theta)]\left[(1-\omega)+\frac{n_f}{n_b}\tau_f\right]\sigma_e^2. \end{aligned} \tag{34}$$

Third, combining (25) with the deterministic component of (24), implies the following expression for the equilibrium growth rate ψ

$$\psi = \omega\left(\alpha(1-g) - \frac{C/W}{n_k}\right) + (1-\omega)(i^\star + \epsilon), \tag{35}$$

Equation (35) expresses the equilibrium growth rate of the domestic economy as a weighted average of the domestic and foreign sources of income.

Equations (33)–(35) together with the households equilibrium conditions (15)–(17) expressions for σ_w^2, σ_{ew}^2 in (28), the portfolio shares "adding up" condition (12), and consumer accumulation equation (13) form a complete system determining the equilibrium values of

$$\frac{C_d}{W}; \quad \frac{C_m}{W}; \quad \frac{C}{W};$$

the portfolio shares n_k n_b, n_f; the real rate of return on domestic bonds r_b; and the equilibrium growth rate ψ. In addition, the instantaneous attainment of portfolio equilibrium involves an initial price of domestic bonds Q_o, and corresponding initial wealth W_o determined by

$$Q_o = \left(\frac{n_b}{n_k + n_f}\right)\left(\frac{K_o + E_o B_o^\star}{B_o}\right); \qquad W_o = \frac{K_o + E_o B_o^\star}{n_k + n_f}. \tag{36}$$

Finally, combining (34) with (3) enables one to determine η. With r_b as given in (34) being stationary through time, it follows that η is time varying as the price Q evolves through time; see (2).

Eliminating the real rate of return on domestic bonds, r_b , and substituting, yields the following closed form solution for the key economic variables:

$$\omega = \frac{\alpha(1-\tau_k) - (i^\star + \epsilon)(1-\tau_f) + (1-\gamma\theta)(1-\tau_f)\sigma_e^2}{(1-\gamma)[\alpha^2(1-\tau_k)\sigma_y^2 + (1-\tau_f)\sigma_e^2]}, \tag{37}$$

$$\begin{aligned} \psi \;=\; & \frac{1}{1-\gamma}\Big\{\alpha(1-\tau_k) - \rho - \gamma\epsilon(1-\theta) + \frac{1}{2}(\gamma-1)\alpha^2\omega[2(1-\tau_k) + \omega(\gamma-2)] \\ & \sigma_y^2 + \frac{1}{2}\alpha^2\omega^2(\gamma-1)(\gamma-2)\sigma_z^2 + \Big([1+\gamma(\omega-\theta)]\left[1-\omega+\frac{1}{2}\gamma(\omega-\theta)\right] \\ & + \frac{1}{2}\omega(1-\omega)(\gamma-2)\Big)\sigma_e^2\Big\}, \end{aligned} \tag{38}$$

$$\begin{aligned} \frac{C}{W} \;=\; & \rho + \gamma[\epsilon(1-\theta) - \psi] - \frac{1}{2}\gamma(\gamma-1)\alpha^2\omega^2(\sigma_y^2 + \sigma_z^2) \\ & - \frac{1}{2}\gamma[(\omega-\theta)[(1+\gamma(\omega-\theta)] + \omega(1-\omega)]\sigma_e^2, \end{aligned} \tag{39}$$

$$\frac{n_k}{\omega} = n_k + n_f = \frac{C/W}{\omega(1-g) + (1-\omega)(i^\star + \epsilon) - \psi}. \tag{40}$$

Equations (37)–(40) determine the equilibrium in a simple recursive manner. First, (33), which simply repeats (20), expresses ω in terms of the relevant tax rates, the relative risks of the two traded assets, enabling us to write $\omega = (\tau_k, \tau_f, \sigma_y^2, \sigma_e^2)$. Given ω, (20) then determines the equilibrium growth rate ψ in the form $\psi = \psi(\tau_k, \omega, \sigma_y^2, \sigma_z^2, \sigma_e^2)$. Having determined both ω and ψ, (39) determines the consumption–wealth ratio C/W as $C/W = C/W(\omega, \psi, \sigma_y^2, \sigma_z^2, \sigma_e^2)$. Finally, (40) implies the total share of traded assets, $n_k/\omega \equiv n_k + n_f$, in the form $n_k/\omega = n_k/\omega(\omega, \psi, \sigma_y^2, \sigma_z^2, \sigma_e^2)$. From these "core" solutions, other equilibrium solutions can be obtained.[15]

A number of observations can be made about the equilibrium. First, the equilibrium is completely independent of the tax rates on domestic interest income τ_b. Though the government offers a unit return on its bonds, the after–tax real return to bond holders depends upon the market equilibrium and adjusts as needed to produce the required after–tax return regardless of the magnitude of the before tax return taxed by the government.

Second, the tax rate on capital (τ_k) and on foreign bonds (τ_f) influences the consumption–wealth C/W ratio and the total portfolio share of traded assets (n_k/ω), only through their effects on ω and ψ. The effect of tax rates on growth can be obtained from (38) and will be discussed further below.

Third, the equilibrium growth rate ψ is independent of mean government expenditure g. While an increase in g reduces the real growth rate directly (35), this is exactly offset by the fact that it also increases n_k, thereby increasing the consumption–capital ratio and maintaining the overall rate of growth unchanged; see also Eaton (1981). By contrast, the growth rate does depend (positively) upon the variance of government expenditure.

Finally, the equilibrium must satisfy certain feasibility conditions. First is the transversality condition, which for the constant elasticity utility function, is given by

$$\lim_{t\to\infty} \mathbb{E}\left[W^{\gamma} e^{-\rho t}\right] \tag{41}$$

Using (25), the condition (41) can be shown to reduce to the condition $C/W > 0$, as originally shown by Merton (1969). With the equilibrium being one of balanced real growth, in which all real assets grow at the same rate, (41) also implies that the intertemporal government budget constraint is met, so that the equilibrium is intertemporally viable. Using (37) – (39), the condition $C/W > 0$ can be shown to imply a constraint on the tax rates, and other parameters, though this constraint is automatically met in the case of the logarithmic

[15]From (37)- (40) we observe that the equilibrium is indeed one having a constant consumption–wealth ratio and constant portfolio shares, thereby validating the initial assumption.

utility function ($\gamma = 0$).

Second, economic viability requires that the initially determined price of domestic bonds be positive $Q_o > 0$. Assuming that nonnegative stocks of domestic bonds are always held, (36) implies that this condition will be met if and only if $0 \leq n_k/\omega \equiv n_k + n_f \leq 1$; i.e. the share of traded assets in the household's portfolio is positive.

3 *Numerical Analysis of Tax Effects*

In this section we investigate the effect of changes in capital income taxes on four key variables: portfolio shares; the mean growth rate; the variance of the growth rate and welfare. [16] We are not able to solve the core equilibrium analytically so we consider some simple analytical comparative statics and then we illustrate the effects with specific numerical examples.

We carry out the numerical analysis for the following range of parameter values:

- Production parameters: the marginal product of capital $\alpha = 0.06 - 0.12$. The variance of output, $\sigma_y^2 = 0.001 - 0.01$.
- Government: the fraction of mean government spending $g = 0.10 - 0.30$. The variance of government spending $\sigma_z^2 = 0.001 - 0.1$.
- External parameters: Foreign interest rates $i^\star = 0.04 - 0.10$. The rate of change of relative prices $\epsilon = 0.07 - 0.12$. The volatility of relative prices. $\sigma_\epsilon^2 = 0.0012$.
- Taste parameters: The rate of time preference $\rho = 0.04 - 0.07$. The coefficient of relative risk aversion $\gamma = 1 - 4$. The share of domestic goods in consumption $\theta = 0.2 - 0.6$.

We also impose the following constraints on the numerical solutions: $\alpha > i^\star$ and $\sigma_y^2\alpha^2 > \sigma_e^2$ to guarantee non–trivial solutions. We use a range of parameter values for the exogenous

[16] It is clear that any residual change in government revenue resulting from these tax changes are financed by an appropriate accommodating adjustment in the stock of bonds.

variables rather than a single parameter because empirically plausible values of many of these parameters are unknown for middle–income countries such as South Africa. The numerical examples are therefore not to be construed as providing solid quantitative evidence as to the effect of taxes on the endogenous variables but rather as a broad measure of the qualitative response of the endogenous variables to taxes.

The base parameter values are the first figures in the range. For brevity only solutions with the base parameter values are reported in the figures below. The figures are produced from an output matrix that describes the values of an endogenous variable for a given set of exogenous variables and for given τ_f and τ_k. The τ_f increase by 1 per cent per column and τ_k increase by the same amount per row.

3.1 *Portfolio Shares*

Differentiating equation (37) implies the following

$$\frac{\partial \omega}{\partial \tau_k} = -\frac{\alpha}{\alpha^2(1-\tau_k)\sigma_y^2 + (1-\tau_f)\sigma_e^2}\left[1-\alpha(1-\gamma)\omega\sigma_y^2\right] < 0, \tag{42}$$

$$\frac{\partial \omega}{\partial \tau_f} = \frac{\alpha}{\alpha^2(1-\tau_k)\sigma_y^2 + (1-\tau_f)\sigma_e^2}\left(\frac{1-\tau_k}{1-\tau_f}\right)\left[1-\alpha(1-\gamma)\omega\sigma_y^2\right] > 0. \tag{43}$$

The signs in (42) and (43) depend on whether $1 > \alpha(1-\gamma)\omega\sigma_y^2$ which in turn depends on the variance of the productivity shocks. The interpretation of this condition is as follows. On the one hand an increase in the tax on capital income τ_k reduces the after–tax mean return to capital, thereby inducing investors to shift away from capital in their portfolios. At the same time, the tax reduces the associated risk which investors "price" at $1-\gamma$ and this encourages the holding of capital. The restriction we have imposed assumes that the former effect dominates and in this case an increase in the tax rate on capital will reduce the share of capital in the traded portion of the households portfolio. An increase in the tax on foreign bonds τ_f has precisely the opposite effect.

[PLACE FIGURE 1 ABOUT HERE.]

Figure 1 illustrates the response of ω, the share of capital in the traded portion of the portfolio for base parameter values. Increasing τ_f increases the share of capital in the traded portion of the portfolio ω. Increasing τ_k decreases the share of capital in the traded portion of the portfolio. Interestingly, an increase in τ_f, *ceteris paribus*, has a greater effect on ω than an increase in τ_k does. However, increasing both τ_f and τ_k by the same proportion seems to change ω by an amount too insignificant to be detected. Indeed if $\tau_f \equiv \tau_k$ the ω is independent of the common tax rate, see (33).

3.2 *Growth Rate*

The effect of increases in the tax rate on the mean equilibrium growth rate can be expressed as:

$$\frac{\partial \psi}{\partial \tau_k} = \left[\frac{\partial \psi}{\partial \omega} + \alpha^2(1-\tau_k)\sigma_y^2 + (1-\tau_f)\sigma_e^2\right]\frac{\partial \omega}{\partial \tau_k}, \tag{44}$$

$$\frac{\partial \psi}{\partial \tau_f} = \frac{\partial \psi}{\partial \omega}\frac{\partial \omega}{\partial \tau_k}. \tag{45}$$

An increase in either tax has two effects on the growth rate. First, given ω, an increase in τ_k reduces the risk adjusted after–tax return to capital thereby reducing the growth rate. Second, it reduces ω thus causing a portfolio shift from domestic capital to traded foreign bonds. To the extent that the shift towards foreign assets raises the growth rate [*i.e.*$\partial\psi/\partial\omega < 0$] this tends to offset the reduction in the growth rate due to the first effect. However, if $\partial\psi/\partial\omega > 0$ the reduction in the growth rate is accentuated. Whether an increase in the share of domestic capital in the traded portion of the portfolio raises or lowers the mean growth rate depends upon the predominant sources of risk. For example, if the only source of risk is domestic, domestic assets will tend to have higher rates of return than foreign assets to compensate. In that case an increase in the portfolio share of domestic capital will tend to raise the growth rate. Indeed it is possible for the portfolio shift to be of sufficient magnitude for this second effect to dominate so that a higher tax on domestic capital income is actually growth–enhancing, in contrast to its known adverse effect on growth under certainty.

The effect of an increase in the tax rate on foreign assets, τ_f, depends on the response of ω. Accepting the sign of (43) we see that sgn $\partial\psi/\partial\tau_f$ = sgn $\partial\psi/\partial\omega$. If sgn $\partial\psi/\partial\omega$ an increase in the foreign tax rate will reduce the growth rate. An example where this is so is provided in our numerical illustrations. Finally, it is immediately apparent that a uniform tax increase ($d\tau_k = d\tau_f > 0$) leaves the optimal portfolio share ω unchanged, so that the net effect on the growth rate depends only upon $-1 + (1-\gamma)\alpha\omega\sigma_y^2$, and provided (42) applies will be unambiguously growth reducing.

[PLACE FIGURE 2 ABOUT HERE.]

Figure 2 illustrates the response of the mean growth rate (ψ) to changes in both taxes. In the scenario depicted in this figure increasing the tax on foreign bonds (τ_f) decreases the equilibrium growth rate (ψ). However increasing the tax on domestic capital (τ_f) decreases the equilibrium growth rate (ψ). Finally, τ_f does not exert a greater effect on growth than τ_k does.

3.3 *Variance of Growth Rate*

The effects of a higher tax rate on the variance of the growth rate are given by

$$\frac{\partial\sigma_w^2}{\partial\tau_i} = 2\left(\left[\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_e^2\right]\omega - \sigma_e^2\right)\frac{\partial\omega}{\partial\tau_i} \qquad i = k, f \tag{46}$$

Thus assuming the signs of (42) and (43), an increase in the tax rate on domestic capital income τ_k will stabilize the growth rate (i.e. reduce σ_w^2) if $\omega > \bar{\omega} = (\sigma_e^2/[\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_e^2)$ and destabilize it otherwise. An increase in τ_f will have precisely the opposite effect. This is because $\bar{\omega}$ is the variance minimizing portfolio and any tax changes that shifts the portfolio towards $\bar{\omega}$ is stabilizing.

[PLACE FIGURE 3 ABOUT HERE.]

Figure 3 illustrates the effect of various values of τ_f and τ_k on the variance of growth.

3.4 *Welfare*

To assess the consequences of tax policy on economic welfare, a welfare criterion must be introduced. For this purpose, we consider the welfare of the representative household, as specified by the intertemporal utility function (8) evaluated along the optimal path. By definition, this equals the value function used to solve the intertemporal optimization problem.

As shown in the Appendix, for the constant elasticity utility function, the optimized level of utility starting from an initial stock of wealth $W(o)$ is

$$X(W(o)) = \delta W(o)^{\gamma} E_o^{-\gamma(1-\theta)}, \tag{47}$$

where

$$\delta = \frac{1}{\gamma}\theta^{\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}\left(\frac{\widehat{C}}{W}\right)^{\gamma-1},$$

where C/W is the equilibrium value given in (39). Using (36), the welfare criterion can be expressed as

$$X(K_0, B_o^{\star}, E_o) = \frac{1}{\gamma}\theta^{\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}\left(\frac{\widehat{C}}{W}\right)^{\gamma-1}\left(\frac{\omega}{n_k}\right)^{\gamma}(K_o + E_o B_0^{\star})^{\gamma} E_o^{-\gamma(1-\theta)}. \tag{48}$$

Assuming that the consumption–wealth ratio C/W, and portfolio share ω/n_k are positive insures that $\gamma X(K_o + E_o B_o^{\star}) > 0$, as well.

Taking the differential of (48) yields

$$\frac{dX}{X} = (\gamma - 1)\frac{d(C/W)}{C/W} + \gamma\frac{d(\omega/n_k)}{\omega/n_k}. \tag{49}$$

The key point to observe is that tax policy affects welfare only through the growth rate ψ and the portfolio share ω. Recalling (39), (40) and (49), the response of welfare can be conveniently expressed as

$$\begin{aligned}\frac{\partial X/\partial\tau_i}{\gamma X} &= \frac{1}{C/W}\left\{n_b\frac{\partial\psi}{\partial\tau_i} + (1-\gamma)\left[\alpha^2(\sigma_y^2 + \sigma_z^2) + \sigma_e^2\right](\tilde{\omega} - \omega)\frac{\partial\omega}{\partial\tau_i}\right\} \\ &+ \frac{1}{C/W}\left\{\left[(1^{\star} + \epsilon) - \alpha(1-g)\right] n_b\frac{\partial\omega}{\partial\tau_i}\right\} \qquad i = k, f\end{aligned} \tag{50}$$

where

$$\widetilde{\omega} \equiv \frac{\alpha(1-g) - (i^{\star}+\epsilon) + (1-\gamma\theta)\sigma_e^2}{(1-\gamma)[\alpha^2(\sigma_y^2+\sigma_z^2)+\sigma_e^2]}, \tag{51}$$

is the portfolio share in the first best equilibrium where the government acting as the central planner controls resources directly.

The effect of a higher tax rate on welfare thus has two effects, a growth effect and a portfolio effect. To the extent that an increase in a particular tax rate reduces the growth rate it will be welfare reducing. However, the portfolio effect may either decrease or improve welfare, depending upon the size of the existing portfolio share ω, relative to the first-best optimum $\widetilde{\omega}$. In the absence of risk, and starting from a second-best optimum it is possible for a further increase in the tax rate to be actually welfare improving. This may be the case if the higher tax shifts the portfolio in the direction of $\widetilde{\omega}$.

How sensitive are the numerical results to changes in parameter values ? To answer this question we plot the numerical results obtained for each of the parameter values in a given range to determine if there are any departures from the basic pattern depicted in Figures (1) (2) and (3). Next we increase the exogenous variables by an arbitrary amount and observe the response of the endogenous variables. To summarize the results of this exercise, raising the marginal productivity of capital by a percentage point tended to raise ω by less than a percentage point but had little qualitative effect on the other variables and the direction of change remained the same. Doubling the value of the variance of output raises ω by a trivial amount (i.e. not noticeable at less than 4 significant digits) and decreases the growth rate ψ by about a percentage and a half but again does not change the general direction of change. Raising the variance of government spending from 1 % to 10 % raises the growth rate by about half a percent. but again does not change any of the conclusions. We therefore concluded that the numerical results described above are qualitatively robust to changes in parameter values.

4 Empirical Analysis

It is well known that household portfolio choice is best modeled as involving two related decisions. The decision of what fraction of net worth to invest in different assets — the *continuous* or intensive choice—and the decision of which asset to hold — the *discrete* or extensive choice. Ignoring the simultaneity between the discrete and continuous portfolio decisions may seriously bias estimates of asset demands. However, instead of trying to specify and estimate complicated asset demand equations that are not subject to this particular form of simultaneity, we focus on the household's discrete choice problem—how do taxes affect a household's decision of which assets to hold ?[17] To do so, we characterize the household's choice problem as a reduced–form multinomial probit model (MNP). [18] A major practical limitation of the MNP model is the complex computations required to estimate choice probabilities when similarities across alternatives are taken into account. We overcome the computational drawback of the MNP model by using the method of simulated moments (MSM) which avoids direct evaluation of the likelihood function thus facilitating estimation of a large number of alternatives.

4.1 *Econometric Specification*

In this subsection we briefly describe the empirical specification and methodology that we use to estimate the parameters of our multinomial probit model of portfolio choice.[19] The multinomial probit model has the following form. Let the number of discrete choices in each time period be K $(j = 1, \dots, K)$. Suppose in each period the choices (henceforth portfolios) of N individuals $(n = 1, \dots, N)$ among K mutually exclusive alternative portfolios are

[17] Despite its drawbacks several authors have estimated separate models for the discrete and continuous portfolio choices, Hubbard (1985).

[18] We utilize the MNP model because it does not suffer from the independence of irrelevant alternatives (IIA) property, McFadden (1973) The IIA property implies that all alternatives in an individuals choice set are equally different from one another or that there is no subset of alternatives that is more similar to each other than to the remaining alternatives.

[19] For a more formal treatment of this class of simulation estimators see Hajivassiliou & McFadden (1990), McFadden & Ruud (1987) and Keane (1994).

observed together with a set of explanatory variables believed to be relevant to their choices. The latent "benefit" of portfolio j is the sum of a deterministic component — a vector of observable variables X and a parameter vector to be estimated β — and a random component given by ϵ. Formally,

$$u_j = X_j\beta + \epsilon_j \qquad j = 1, \ldots, K. \tag{52}$$

where X_j is a $1 \times S$ row vector of exogenous regressors, β is a $S \times 1$ column vector of parameters possibly specific to choice j, ϵ_j is a K–variate random disturbance term that is independent across individuals but has a multivariate normal distribution with covariance matrix Δ.[20] Henceforth the suppression of subscripts indicates that a variable is a vector.

We do not observe u_j directly, but instead only observe an indicator variable d_j where $d_j = 1$ if individual n chooses alternative j at time t and 0 otherwise.

Let $u_1, \ldots, u_K$ denote the unobserved (latent) benefits of alternatives $1, 2, \ldots, K$. Then

$$u_1 = X_1\beta + \epsilon_1,$$

$$u_K = X_K\beta + \epsilon_K. \tag{53}$$

Since the optimal portfolio choice delivers maximum benefit, the differences in benefit levels between the best choice and any other choice, not the benefit level of the maximal choices, are relevant for the household's decision. Suppose that a household chooses portfolio 1. The differences in the benefit from portfolio $2, \ldots, K$ from that of portfolio 1 are given by

$$u_2 - u_1 = Z_1\beta + \eta_1,$$

$$u_K - u_1 = Z_{K-1}\beta + \eta_{k-1}, \tag{54}$$

The problem is to determine the probability of choosing a given portfolio. For a given portfolio to be chosen, the error differences can be at most as large as the differences in the

[20]A limitation of our approach is that we are implicitly assuming independence of choice behavior over time as well as over individuals. Due to the independence assumption over time, our analysis does not exploit the temporal dependence pattern in the data.

deterministic component. For instance, the probability that portfolio 1 is chosen is

$$\begin{aligned}\Pr[\text{ Portfolio 1 is chosen}] &= \Pr(u_j - u_1 \leq 0, \quad \forall j = 2, \ldots, K), \\ &= \Pr(\eta_j \leq -Z_j\beta \quad \forall j = 1, \ldots, K-1), \\ &= \Upsilon(-Z\beta, \Delta_{K-1}). \end{aligned} \tag{55}$$

where $\Upsilon(\cdot)$ represents the cumulative distribution function (CDF) of the $K-1$ variate normal random variable η with mean vector zero and covariance matrix Δ_{K-1} and Z is the matrix of vectors Z_j $j = 1, 2, \ldots, K-1$ while Δ_{k-1} is the covariance matrix of $\eta_1, \ldots, \eta_{K-1}$ where $eta_j = \epsilon_{j+1} - \epsilon_j$.

The CDF in (55) can be written as

$$\Upsilon(-Z\beta, \Delta_{K-1}) = \int_{[\eta_{k-1} \leq -Z_{k-1}\beta]} \cdots \int_{[\eta_1 \leq -Z_1\beta]} \vartheta(\eta_1, \ldots, \eta_{K-1}) d\eta_1 \ldots d\eta_{K-1}, \tag{56}$$

where ϑ is the density of the $K-1$ variate normal random variable. Obtaining $\Upsilon(\cdot)$ directly requires expensive *numerical* integration of the above $K-1$ variate normal density function. The advantage of MSM is that it only requires inexpensive evaluation of the $K-1$ variate normal density function. The MSM replaces $\Upsilon(-Z_{ik}\beta, \Delta_{K-1})$ in (56) by an unbiased simulation estimator. Furthermore the MSM does not require consistent estimates of $\Upsilon(\cdot)$ when there are a large number of observations in the data set. This is because simulation error is averaged out across observations due to the fact that $\Upsilon(\cdot)$ enters the moment equation linearly.

Next we describe how to construct the moment equations. Constructing the *moment equations* is an important step in method of moments estimation. [21] Consider the conditional expectation

$$\mathbb{E}(d_k|Z_k) = \Upsilon(-Z_k\beta, \Delta_{K-1}), \tag{57}$$

which can also be written as

$$\mathbb{E}[(d_k - \Upsilon(-Z_k\beta, \Delta_{K-1}))|Z_k] = 0. \tag{58}$$

[21] See Hansen's (1982) generalized method of moments estimator, another member of the class of method of moments estimators that exploits the orthogonality principle.

Substituting and manipulating the above equation yields

$$\mathbb{E}(Z_k'(d_k - \Upsilon(-Z_k\beta, \Delta_{K-1})] = 0. \tag{59}$$

Equation (59) consists of S moment equations, one for each of the S explanatory variables. Given a sample of N independent individuals, equation (59) can be replaced by the empirical moments equation

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K} Z_{ik}'(d_{ik} - \Upsilon(-Z_{ik}\beta, \Delta_{K-1})) = 0, \tag{60}$$

where d_{ik} is the observed choice of individual i for alternative K, Z_{ik} is the $1 \times S$ row vector of explanatory variables corresponding to the i'th individual and $\Upsilon(-Z_{ik}\beta, \Delta_{k-1})$ is the probit choice probability for alternative k for the i'th individual in the sample. If the number of parameters exceeds S, Z_{ik} is replaced by the $1 \times M$ vector of instruments W_{ik}

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K} W_{ik}'(d_{ik} - \Upsilon(-Z_{ik}\beta, \Delta_{K-1})) = 0, \tag{61}$$

where W_{ik} is the $1 \times M$ vector of instruments associated with choice K and individual i stacked as in d.

With K alternatives in the choice set, the parameters to be estimated are β and the parameters of the covariance matrix Δ_{K-1}. Let the parameter vector θ contain the elements of β and Δ_{K-1}. Then the MSM estimates are obtained by minimizing the left hand side of the empirical moment equation (61) expressed in quadratic form as

$$\theta_{sm} \equiv \arg\min_{\theta} = (d - f(\theta))'WW'(d - f(\theta)), \tag{62}$$

where θ_{sm} denotes the parameter estimates obtained by minimizing the quadratic function, d is a $NK \times 1$ vector of *observed* choices stacked by individual and by alternative within observation, $f(\theta)$ is a $NK \times 1$ vector of choice probabilities obtained from the $K-1$-variate normal distribution via simulation and W is the $NK \times M$ matrix of instruments, where $M \geq S - 1 + (K(K-1)/2)$.[22]

[22]The $S \times 1$ vector of covariates X_{ik} associated with alternative k and individual i and its higher order powers can be used as the instrument vector W_{ik}.

4.2 *Data, Sample Characteristics and Estimation Results*

We use data compiled from individual income tax and investment portfolio files from the Republic of South Africa.[23] Before describing the data it is useful to provide a brief description of some features of taxation in South Africa. Tax revenues in South Africa derive from four main sources: the individual income tax, a general sales tax, the direct taxation of companies other than mines and taxation of mines. Between 1980 and 1985 the contribution of individual income taxes to total tax revenues increased from 22 to 33 percent.[24] Virtually all of South Africa's income tax is paid by a quarter of the population and 70 % by the wealthiest 5%. The individual income tax on married persons was levied progressively on 15 (pre-1986) and 18 income brackets (post-1986). The income brackets start at a 15 percent marginal rate for taxable income less than 5,000 rand and end at 43 percent for incomes higher than 80,000 rand.

The basic principles of taxation for capital income passing to a South African resident are relatively simple. Investment income reaching the hands of the individual is charged to income tax in the same way as employment income. South Africa is one of the few countries in the world which does not tax capital gains. This is important because the tax treatment of different assets depends on the way in which the yield occurs, that is whether it occurs as dividend yield or capital gains. Like many countries, the tax code is based on nominal income so the effective tax rates are sensitive to the rate of inflation. Inflation affects the real value of debts stated in nominal terms, erodes depreciation allowances and understates stock valuations. In South Africa the $(20\%, 20\%, 20\%, 20\%, 20\%)$ depreciation allowance method in the hotel and manufacturing sectors reduces the effects of inflation. We are therefore careful to distinguish between real and nominal capital income in our estimation.

The dataset is unique because data on household asset/wealth holdings are rarely available. This is often attributed to the proprietary nature of the information. However, almost

[23]South Africa has a GDP of $ 120 billion with a per capita GDP of $ 2,900 in 1993 (if converted at PPP $ 4,455) making it a middle-income country in the same category as South Korea, Brazil and Argentina.

[24]South African Department of Finance, Inland Revenue *Statistical Bulletin* No. 6 (1988).

all countries have investment brokers and/or accounting firms that prepare tax returns and investment portfolios for clients — often the same clients year after year. The data we use in the present analysis were obtained from such a source— based in South Africa.[25]

The data provide detailed information on the sources of taxable income and deductible expenses of households and the value of several categories of assets included in household investment portfolios. The raw sample consists of 875 individuals. In selecting members for inclusion in the sample we chose to restrict the sample to a relatively homogeneous group of households so that attention can be focused on the tax variable.

We apply the following criteria in choosing members to include in the empirical analysis. First, the household must have filed tax/investment forms and have reliable and complete data on each of the covariates for each of the five years. Second, we restrict our analysis to White residents of South Africa. This is to provide some control on the educational and occupational backgrounds to account for possible human capital effects. The Apartheid system resulted in Blacks being discriminated against in higher education and severely constrained their occupational opportunities. The final cross–section comprises 349 individuals.

Table 1 provides descriptive statistics on the five groups of financial assets in the sample. We are interested in the *financial* assets of households and therefore exclude real assets (such as houses) from our analysis. By only focusing on financial assets we are implicitly assuming separability between services rendered by financial and real assets. In other words the effect that real assets have on choice of financial assets is assumed to be random, Uhler & Cragg (1971).

[PLACE TABLE 1 ABOUT HERE.]

The econometric specification requires that the individual choose between mutually exclusive *portfolios* of assets in terms of the relative riskiness of the portfolio. We must therefore

[25]We are grateful to Pater Salazar for making it possible for us to collect the data. It took almost one year for a team of research assistants to compile the data under the onerous confidentiality restrictions imposed by the investment firm.

decide on a rule for ranking the portfolios in terms of which is "more" or "less" risky. This is important if we are to make meaningful conclusions about how changes in taxes influence risk–taking. However, such a ranking is difficult to make precise unless some restrictions are placed on the probability distribution of returns.

To rank the portfolios, we start with the assumption used in the theoretical model that returns on risky financial assets evolve as Brownian motion. This assumption implies that asset returns are normally distributed, such that the (unconditional) mean and variance fully characterize the distribution of returns.[26] Next, we compute the mean and variance–covariance matrix of annual returns from April 1970–April 1985.[27] Then given the historical distribution of returns we use a hill–climbing algorithm to identify the portfolio that achieves the lowest variance for each given mean return. We repeat this process for a range of mean returns. The procedure yields a set of asset allocations that correspond to points on the mean–variance efficient frontier.[28]

Having computed the universe of mean–variance efficient portfolios we then rank the portfolios such that portfolio 1 (denoted P1) is the least risky portfolio and portfolio 7 (denoted P7) is the most risky.[29] Next, we compute actual portfolio returns for each household for each year of the sample period. Then we allocate actual household portfolios to optimal portfolios (P1–P7) according to the following simple rule. A household's actual portfolio is allocated to one of the seven portfolios if it is within ± 50 basis points of a mean–variance efficient portfolio. This rule enables actual household portfolios to deviate from the mean–variance efficient frontier.

[26]This assumption should be considered an approximation as actual return data for most assets have been found to be leptokurtic i.e. peaks are higher than the normal distribution and the tails fatter.

[27]When calculating the return to equities we ignore dividends and therefore understate the pre–tax returns that an investor would have received. However, most risk in equity investment comes from capital gains and losses (which are not taxed in South Africa) so the omission of the dividend component of returns will not produce much bias in the measurement of risk.

[28]The relative proportion of risky assets in the optimal portfolios is not identical because we retain the assumption (used in the theoretical model) that there is no riskless asset. It is well known that without a riskless asset, optimal portfolios will not necessarily contain the same relative proportions of risky assets.

[29]Our choice of the number of portfolios (seven) is arbitrary. However, the empirical results are robust to the choice of five, six and eight portfolios.

The mean–variance efficiency criterion is only valid if returns are Gaussian or utility is quadratic. However, as mentioned above it is likely that returns are not Gaussian. Furthermore, in our theoretical analysis we assumed a constant elasticity utility function. Therefore, a valid concern is how robust our portfolio rankings are to the mean–variance criterion. To address this concern we computed optimal portfolios using a hill–climbing algorithm that chooses the portfolio that maximizes expected utility for various values of the risk–aversion parameter. Expected utility was computed based on the historical distribution of returns.[30] Little difference was found between the set of optimal portfolios generated according to the expected utility criterion and the mean–variance efficient portfolios.

Next, we construct an instrument for each individuals marginal tax rate. This is necessary because there is a potential problem with using actual marginal tax rates in our specification since returns to different assets have different tax treatments. This implies that an individual's marginal tax rate depends on the portfolio of assets the individual chooses to hold, Feldstein (1976). To avoid this potential simultaneity problem we use an instrumental variable in place of the actual marginal tax rate.

We construct the instrument for the household's marginal capital income tax rate in the following way. First, we calculate the mean annual return to all portfolios (1–7). Then we assume that all households hold all their wealth in a hypothetical portfolio that is an average of the portfolio returns calculated above. Finally, using this imputed figure for capital income we arrive at an estimate of the household's marginal tax rate on capital income by applying the appropriate personal income tax rate structure in each year after deducting the labor income of the household head.[31]

Columns 1 and 2 of Table 2 provide the parameter estimates and standard errors obtained from estimating the multinomial probit model with an instrument for marginal tax rates.

[30]This method of computing expected utility requires the strong assumption that all the moments of the subjective distribution are exactly equal to the historical distribution.

[31]Agell & Edin (1989) use a similar method of constructing an instrument for marginal tax rates in their cross–sectional study of Swedish households.

The dependent variables are the households portfolio choice (P1–P7). The explanatory variables are TAX, AGE INCOME and macroeconomic controls. It is important to include the macroeconomic controls because over the sample period the price of gold tumbled, the exchange rate fell from 3.28 to 1.52 and the CPI almost doubled in South Africa. Estimated coefficients for the macroeconomic controls are not reported here to conserve on space but are available on request from the corresponding author.

For each explanatory variable, the estimated coefficient measures the relative influence of that variable on the likelihood of the household choosing each portfolio relative to the base (least risky) portfolio. For example, TAX·P2 refers to the likelihood that the household's marginal capital income tax influences (positively or negatively) the choice of portfolio 2 relative to the choice of portfolio 1, holding all other variables constant. Recall that portfolio 1 is the relatively safe portfolio.

We restrict our comments to the influence of taxes on portfolio choice. From Table 2 it is evident that increased taxes decrease the probability of holding any of the "riskier" portfolios relative to the probability of holding the "safe" portfolio, holding all other variables constant. However, the overall effect is unclear because relative values of the tax coefficients among risky assets matter. To see this note that the tax coefficients increase as you move from portfolio 2 to portfolio 7. This pattern implies that as the marginal tax rate increases, agents hold more risky assets within relatively risky asset groups. Considering these two effects together (i) P1 vs P2–P7 comparison; (ii) P2 vs P3 ... vs P7, it is not immediately clear whether increased taxes discourages holdings of P7 overall.

To investigate the overall effect of increased taxes on portfolio choice we grouped the assets into "more risky" assets (equity and foreign asset) and "less risky" (bank savings and bonds). Then, we estimated a regression of the ratio of amounts invested in the two groups on the marginal tax rate. The coefficient estimate from this regression is negative and precisely measured at the 5 percent level which suggests that increased taxes discourages holdings of riskier portfolios. As an additional check on the net effect of taxes on portfolio choice we allowed the base portfolio to change for each regression. That is we estimated the MNP specification with a different base portfolio (P2–P7). The pattern that emerges from these 7 MNP estimates (with different base portfolios) confirm the results from Table 2.[32]

The empirical results are striking and differ from the conventional view that increasing the tax on a risky asset increases holdings of the risky asset. The conventional view was based on a situation in which loss–offsets are fully allowed such that taxation gives rise only to an income effect resulting in an increase in risk–taking. The estimates reported in Table 2 are consistent with a situation in which households discount very heavily the possibility of loss. If losses are not fully deductible against other income, such that the tax lessens the advantage of holding risky portfolios there will likely be a switch from more risky to less risky investments. A progressive personal income tax such as that in South Africa with no loss–offsets provision will almost certainly have a net disadvantageous effect on investment in risky ventures.

The estimates reported in Table 2 provide a broader test of the central proposition in the theory of taxation and risk–taking because we have several assets and not a single risky asset and single risk–free asset as in classic treatments, Domar & Musgrave (1944). However, Sandmo (1989) has shown that the central proposition generalizes directly to the case of many risky assets with substitution away from the safe asset reflected in equal percentage increases in holdings of all risky assets—providing the composition of the risky asset portfolio stays the same. Interestingly, this theoretical result goes through even in the absence of a safe return on one of the assets.

[PLACE TABLE 2 ABOUT HERE.]

Several shortcomings in the econometric analysis deserve comment. First, why do we use a reduced form specification? The main advantage of the reduced form approach is it

[32]The estimates with different base values are available on request from the corresponding author.

enables us to study and predict the complex dynamic interrelationships illustrated in the theoretical analysis through estimation of a relatively small number of unknown parameters. The obvious alternative to the reduced form is a structural model. However, we are unable to obtain a closed form expression for the underlying stochastic optimization problem. When closed forms do exist for such problems they are often derived from highly simplified parsimoniously parameterized optimization problems which place severe restrictions on the ability of the structural models to represent the data.

Second, the instrument vector we use provides parameter estimates that are consistent but not asymptotically efficient. In principle such estimates can be obtained—McFadden (1989) provides a derivation of the efficient instrument vector which would provide asymptotically efficient parameter estimates. Third, conventional measures of goodness–of–fit — such as the U^2 statistic— are not applicable to the simulation estimator used here. Despite these shortcomings we believe the benefits of being able to estimate multi-period probit models with a large number of alternatives open up a number of promising research avenues in diverse areas of economics.

5 Concluding Remarks

The share of private investment in GDP is arguably the single most important determinant of differences in cross–country growth performance. Private investment requires entrepreneurs to bear risk. Taxes affect the propensity of individuals to take risks and thus to accumulate capital through investment. However, the relationship between taxation and risk–taking (at the micro-level) and growth (at the macro–level) is not well understood. This paper provides a first attempt at addressing this intellectually challenging problem.

What lessons can we draw from the present analysis ? First, the theoretical analysis emphasizes the importance of studying the effects of risk–taking and taxes in a stochastic general equilibrium setting. Our analysis highlights two factors that are important in determining the optimal portfolio: (i) a speculative component which depends upon the differential after–tax real rate of return; (ii) a hedging component which depends on the relative tax–adjusted variances associated with the returns on the two traded assets.

Second, the empirical analysis demonstrates that taxation of income from risky assets discourages risk–taking in our sample. Given the casual evidence on the importance of entrepreneurship and risk–taking in recent growth "miracles" the evidence that increases taxation of risky assets discourages risk–taking is important news for scholars and policy-makers seeking answers to pressing questions on growth.

In future work we intend to incorporate the household's labor–leisure choice in the analysis. Accounting for labor income would introduce a role for nontradable human capital in consumption and investment decisions. Such an analysis has the potential to provide empirically testable predictions that we can confront with data on household portfolio choice to shed more light on the relationship between taxes, risk–taking and growth.

Appendix A

The consumer's stochastic optimization problem is to choose the consumption–wealth ratio and portfolio shares to

$$\max \mathbb{E}_0 \int_0^\infty \frac{1}{\gamma}(C_d^\theta C_m^{1-\theta})^\theta e^{-\rho t} dt \qquad -\infty < \gamma < 1; \quad 0 \le \theta \le 1$$

subject to the stochastic wealth accumulation and the evolution of the real exchange rate

$$\frac{dW}{W} = \psi dt + dw,$$

$$\frac{dE}{E} = \epsilon dt + de,$$

and portfolio adding up condition

$$n_k + n_b + n_f = 1$$

where for notational convenience

$$\psi \equiv n_k(1-\tau_k)r_k + n_b(1-\tau_b)r_b + n_f(1-\tau_f)r_f - \frac{C_d}{W} - \frac{EC_m}{W},$$

$$dw \equiv n_k(1-\tau_k)du_k + n_b(1-\tau_b)du_b + n_f(1-\tau_f)du_f,$$

We define the differential generator of the value function $V(W, E, t)$ by

$$\begin{aligned}\Phi[V(W,E,t)] \equiv\ & \frac{\partial V}{\partial t} + \psi W\frac{\partial V}{\partial W} + \epsilon E\frac{\partial V}{\partial E} + \frac{1}{2}\sigma_w^2 W^2\frac{\partial^2 V}{\partial W^2} + \\ & \frac{1}{2}\sigma_e^2 E^2\frac{\partial^2 V}{\partial E^2} + \sigma_{we} W E\frac{\partial^2 V}{\partial W \partial E},\end{aligned}$$

Given the exponential time discounting, V can be assumed to be of the time separable form

$$V(W,E,t) = e^{-\rho t}X(W,E).$$

The formal optimization problem is now to choose C_d, C_m, n_k, n_b, n_f to maximize the Lagrangean expression

$$e^{-\rho t}\frac{1}{\gamma}(C_d^\theta C_m^{1-\theta})^\gamma + \Phi\left[e^{-\rho t}X(W,E)\right] + e^{-\rho t}\lambda\left[1 - n_k - n_b - n_f\right].$$

Taking partial derivatives of this expression and canceling $e^{-\rho t}$ yields

$$\theta\left(C_d^\theta C_m^{1-\theta}\right)^{\gamma-1} C_d^{\theta-1}C_m^{1-\theta} = X_W, \tag{A. 1}$$

$$1-\theta\left(C_d^\theta C_m^{1-\theta}\right)^{\gamma-1} C_d^{\theta-1}C_m^{-\theta} = EX_W, \tag{A. 2}$$

$$(1-\tau_k)r_k W X_W + \mathrm{cov}(dw,(1-\tau_k)du_k)X_{WW}W^2 + WEX_{WE}\mathrm{cov}(de,(1-\tau_k)du_k) = \frac{\lambda}{\rho} \tag{A. 3}$$

$$(1-\tau_b)r_b W X_W + \mathrm{cov}(dw,(1-\tau_b)du_b)X_{WW}W^2 + WEX_{WE}\mathrm{cov}(de,(1-\tau_b)du_b) = \frac{\lambda}{\rho} \tag{A. 4}$$

$$(1-\tau_f)r_f W X_W + \mathrm{cov}(dw,(1-\tau_f)du_f)X_{WW}W^2 + WEX_{WE}\mathrm{cov}(de,(1-\tau_f)du_f) = \frac{\lambda}{\rho} \tag{A. 5}$$

$$n_k + n_b + n_f = 1 \tag{A. 6}$$

These equations determine the optimal values for C_d, C_m, n_k, n_b, n_f, as functions of X_W, X_{WW}, X_{EW} of the value function. In addition, the value function must satisfy the Bellman equation

$$\max_{\{c_d, c_m, n_k, n_b, n_f\}}\left\{\frac{1}{\gamma}(C_d^\theta C_m^{1-\theta})^\gamma e^{-\rho t} + \Phi\left[e^{-\rho t}X(W,E)\right]\right\} = 0.$$

This involves substituting for the optimized values obtained from (A.4) and solving the resulting differential equation for $X(W,E)$, namely

$$\frac{1}{\gamma}(\widehat{C}_d^{\theta}\widehat{C}_m^{1-\theta})^{\gamma} - \rho X(W,E) + \widehat{\psi} W X_W = \epsilon E X_E + \frac{1}{2}\sigma_w^2 W^2 X_{WW} + \frac{1}{2}\sigma_e^2 E^2 X_{EE} + \sigma_{we} W E X_{WE} = 0, \tag{A. 7}$$

where $\widehat{}$ denotes optimized value.

The solution is by the method of undetermined coefficients. This is a standard procedure for solving partial differential equations. That is we postulate a functional form for $X(W,E)$ and determine the conditions under which the proposed solution satisfies both the optimality conditions and the Bellman equation. The solution postulated is of the form

$$X(W,E) = \delta W^{\gamma} E^{x} \tag{A. 8}$$

where the coefficients δ and x are to be determined. For the present problem this solution turns out to be fully general and unique. This equation immediately implies

$$X_W = \delta\gamma W^{\gamma-1}E^{x}, \qquad X_E = \delta x W^{\gamma} E^{x-1}, \qquad X_{WW} = \delta\gamma(\gamma-1)W^{\gamma-2}E^{x}, \tag{A. 9}$$

$$X_{EE} = \delta x(x-1)W^{\gamma}E^{x-2}, \qquad X_{WE} = \delta\gamma x W^{\gamma-1}E^{x-1}, \tag{A. 10}$$

To solve we begin by dividing (A. 1) by (A. 2). Combining the resulting expression with the definition of expenditure $C \equiv C_d + EC_m$, yields the expenditure shares

$$C_d = \theta C \qquad EC_m = (1-\theta)C,$$

so that

$$C_d^{\theta}C_m^{1-\theta} = \theta^{\theta}(1-\theta)^{1-\theta} C E^{-(1-\theta)}, \tag{A. 11}$$

Substituting the expressions for C_d, C_m and X_W back into (A. 1) yields

$$C = \left(\delta\gamma\theta^{-\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}E^{x+\gamma(1-\theta)}\right)^{\frac{1}{\gamma-1}} W. \tag{A. 12}$$

Next substituting from (A. 9), (A. 10), (A. 11) and (A. 12) into the Bellman equation (A. 7) we obtain

$$\frac{1}{\gamma}\theta^{-\frac{\gamma\theta}{\gamma-1}}(1-\theta)^{-\frac{\gamma(1-\theta)}{\gamma-1}}(\delta\gamma)^{\frac{\gamma}{\gamma-1}}W^{\gamma}E^{[x+(1-\theta)]\frac{\gamma}{\gamma-1}} - \rho\delta W^{\gamma}E^{x} + \widehat{\psi}\delta\gamma W^{\gamma}E^{x}$$

$$+\epsilon\delta x W^\gamma E^x + \frac{1}{2}\sigma_w^2\delta\gamma(\gamma-1)W^\gamma E^x + \frac{1}{2}\sigma_e^2\delta x(x-1)W^\gamma E^x$$
$$+\,\delta\gamma x\sigma_{we}W^\gamma E^x = 0. \qquad \text{(A. 13)}$$

This equation consists of terms involving W and E raised to constant powers.

The function (A. 8) will be a viable solution if and only if

$$x = -\gamma(1-\theta),$$

in which case (A. 12) reduces to

$$C = \left(\delta\gamma\theta^{\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}\right)^{\frac{1}{\gamma-1}} W.$$

Canceling the terms $W^\gamma E^x$ (recalling that $x = -\gamma(1-\theta)$ in (A. 13), and noting the definition of $\widehat{\psi}$, we find that the optimal solution for C/W and the undetermined coefficient δ are given by

$$\begin{aligned} \frac{C}{W} &= \left(\delta\gamma\theta^{-\gamma\theta}(1-\theta)^{-\gamma(1-\theta)}\right)^{\frac{1}{\gamma-1}} \\ &= \frac{1}{1-\gamma}\left\{\rho + \epsilon\gamma(1-\theta) - \gamma\beta - \frac{1}{2}\gamma(\gamma-1)\sigma_w^2 - \frac{1}{2}\gamma(1-\theta)[\gamma(1-\theta)+1]\sigma_e^2\right. \\ &\quad \left. +\gamma^2(1-\theta)\sigma_{ew}\right\}, \qquad \text{(A. 14)} \end{aligned}$$

The solution for the value function is therefore

$$X(W,E) = \delta W^\gamma E^{-\gamma(1-\theta)}.$$

where δ, obtained from the first equation in (A. 14), can be written as

$$\delta = \frac{1}{\gamma}\theta^{\gamma\theta}(1-\theta)^{\gamma(1-\theta)}\left(\frac{\widehat{C}}{W}\right)^{\gamma-1},$$

and the optimal consumption-wealth ratio is obtained from the second equation in (A. 14). This is equation (17) in the text. Note the fact that the equilibrium $C/W > 0$ implies $\beta\gamma > 0$. Equation (47) in the text corresponds to welfare starting from initial values. Finally, substituting for X_W X_{WW} X_{WE} into (A. 3)-(A. 5) and subtracting yields the optimality conditions (18) and (19). ∎

References

Agell, J. and P. Edin, 1990, Marginal Taxes and the Asset Portfolios of Swedish Households, Scandinavian Journal of Economics, 92 (1), 47–64.

Asea, P.K., E. Mendoza, E. and G.M. Milesi–Ferretti, On the Ineffectiveness of Tax Policy in altering Long–run Growth? Harberger's Superneutrality Conjecture," forthcoming, Journal of Public Economics.

Barro, R. 1990, Government Spending in a Simple Model of Endogenous Growth, Journal of Political Economy 98, S103–S125.

Bovenberg, L.A. 1986, Capital Income Taxation in Growing Open Economies, Journal of Public Economics 31, 347–376.

Corsetti, G. 1992, Taxation, Risk–taking and Optimal Savings: The Role of the Budget Process, Yale University, Economic Growth Center Discussion Paper No.678.

Domar, E.D. and R. A. Musgrave, 1944, Proportional Income Taxation and Risk–taking, Quarterly Journal of Economics 58, 388–422.

Eaton, J. 1981, Fiscal Policy, Inflation, and the Accumulation of Risky Capital, Review of Economic Studies 48, 435–445.

Feldstein, M. 1976, Personal Taxation and Portfolio Composition: An Econometric Analysis, Econometrica 44, 631–650.

Frenkel, J.A., A. Razin and E. Sadka, 1991 International Taxation in an Integrated World, (MIT Press, Cambridge, MA).

Giovannini, A. 1990, International Capital Mobility and Capital Income Taxation: Theory and Policy, European Economic Review 34, 480–488.

Grinols, E.L., and S.J. Turnovsky, 1994, Exchange Rate Determination and Asset Prices in a Stochastic Small Open Economy, Journal of International Economics 36, 75–97.

Hajivassiliou, V. and D. McFadden, 1990, The Method of Simulated Scores for the Estimation of LDV Models with an Application to the External Debt Crisis, Cowles Foundation Discussion Paper 967.

Hansen, L.P. 1982, Large Sample Properties of the Generalized Method of Moments Estimator, Econometrica 50, 4, 1029–1054.

Hubbard, G. 1985, Personal Taxation, Pension Wealth and Portfolio Composition, Review of Economics and Statistics 67, 53–60.

Jones, L.E. and R.E. Manuelli, 1990, A Convex Model of Equilibrium Growth: Theory and Policy Implications, Journal of Political Economy 98, 1008–1038.

Keane, M. 1994, A Computationally Practical Simulation Estimator for Panel Data, Econometrica 62 (1) 95–116.

Lent, G. E. 1967, Tax Incentives for Investment in Developing Countries, International Monetary Fund Staff Papers 14, 249–323.

Levine, R. 1991 Stock Markets, Growth and Tax Policy, Journal of Finance 56 (4) 1445–1465.

Malliaris, A.G. and W.A. Brock, 1982, Stochastic Methods in Economics and Finance (North-Holland, Amsterdam).

McFadden, D. 1973, Conditional Logit Analysis of Qualitative Choice Behavior, in: P. Zarembka ed., Frontiers of Econometrics, New York, Academic Press.

McFadden, D. 1989, A Method of Simulated Moments for the Estimation of Discrete Response Models Without Numerical Integration, Econometrica 57, 5, 995–1026.

McFadden, D. and P. Ruud, 1987, Estimation of Limited Dependent Variable Models from the Regular Exponential Family by the Method of Simulated Moments, Working Paper University of California, Berkeley.

Merton, R.C. 1969, Lifetime Portfolio Selection under Uncertainty: The Continuous–time Case, Review of Economics and Statistics 51, 247–257.

Merton, R.C. 1971, Optimum Consumption and Portfolio Rules in a Continuous-time Model, Journal of Economic Theory 3, 373–413.

Mossin, J. 1968, Taxation and Risk–Taking: An Expected Utility Approach, Economica, 35, 74–82.

Nielsen, S.B. and P.B. Sorensen 1991, Capital Income Taxation in a Growing Open Economy, European Economic Review 34, 179–197.

Rebelo, S. 1991, Long–run Policy Analysis and Long–run Growth, Journal of Political Economy 99, 500–521.

Sandmo, A. 1989, Differential Taxation and the Encouraging of Risk–Taking, Economics Letters 31, 55–59.

Schumpeter, J. 1934, The Theory of Economic Growth, Harvard University Press.

Sen, P. and S. J. Turnovsky, 1990, Investment Tax Credit in an Open Economy, Journal of Public Economics 42, 277–299.

Shah, S. and J. Toye, 1978, Fiscal Incentives for Firms in some Developing Countries: Survey and Critique, in: J.F.J. Toye ed; Taxation and Economic Development, London, Frank Cass.

Sibert, A. 1990 Taxing Capital in a Large, Open Economy, Journal of Public Economics 41, 277–309.

Sinn, H. 1990, Tax Harmonization and Tax Competition in Europe, European Economic Review 34, 489–504.

Slemrod, J. 1988 Effects of Taxation with International Capital Mobility, in: H. Aaron, H. Galper, and J.A. Pechman eds., Uneasy Compromise: Problems of a Hybrid Income–Consumption Tax, Brookings Institution, Washington D.C..

Stiglitz, J. 1969, The Effects of Income Wealth and Capital Gains Taxation on Risk–Taking, Quarterly Journal of Economics 83, 262–283

Stulz, R. 1981, A Model of International Asset Pricing, Journal of Financial Economics 9, 383–406

Turnovsky, S. J. and M. Bianconi, 1992, The International Transmission of Tax Policies in a Dynamic World Economy, Review of International Economics 1, 49–72.

Turnovsky, S.J. 1993, Macroeconomic Policies, Growth, and Welfare in a Stochastic Economy, International Economic Review 34, 953–981.

Uhler, R. and J.G. Cragg, 1971 The Structure of the Asset Portfolios of Households, Review of Economic Studies 38(115) 341–57.

Figure 1: Share of Capital in the Portfolio

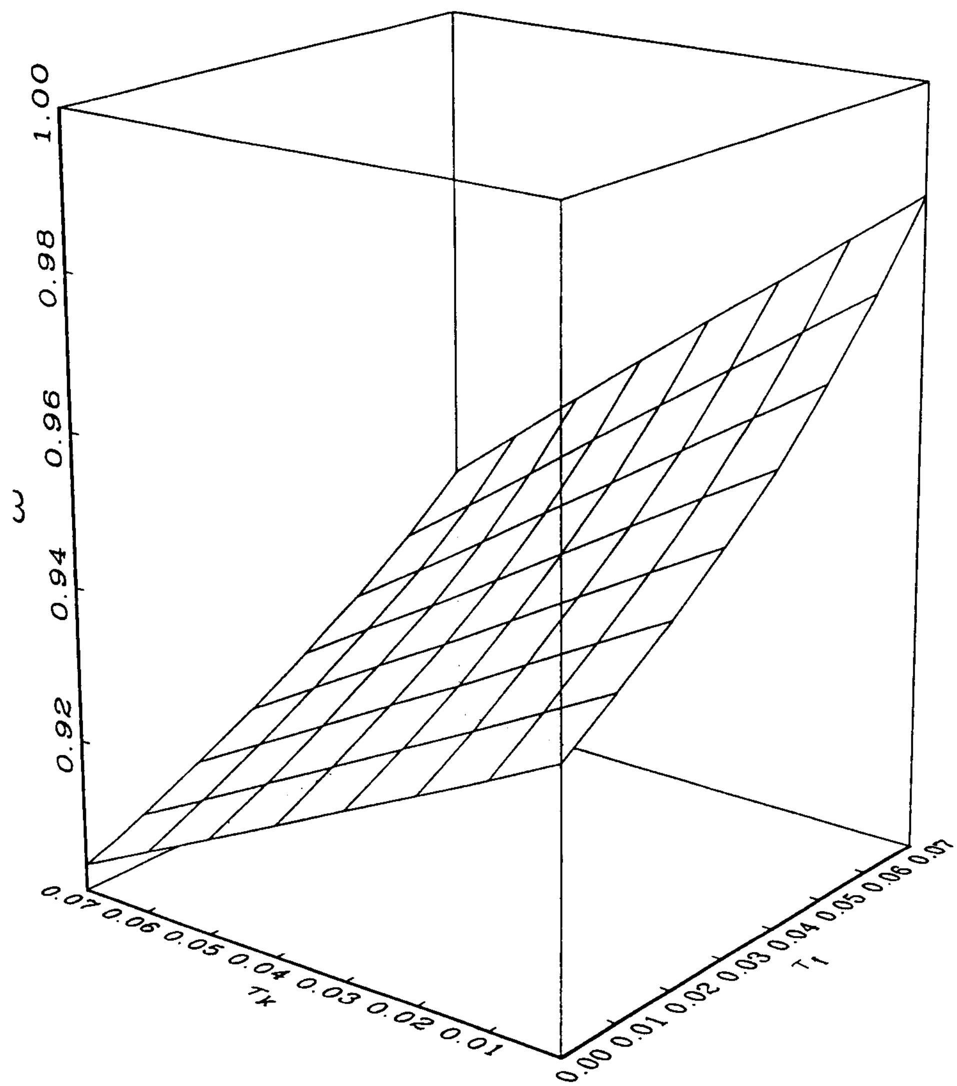

Figure 2: Equilibrium Growth Rate

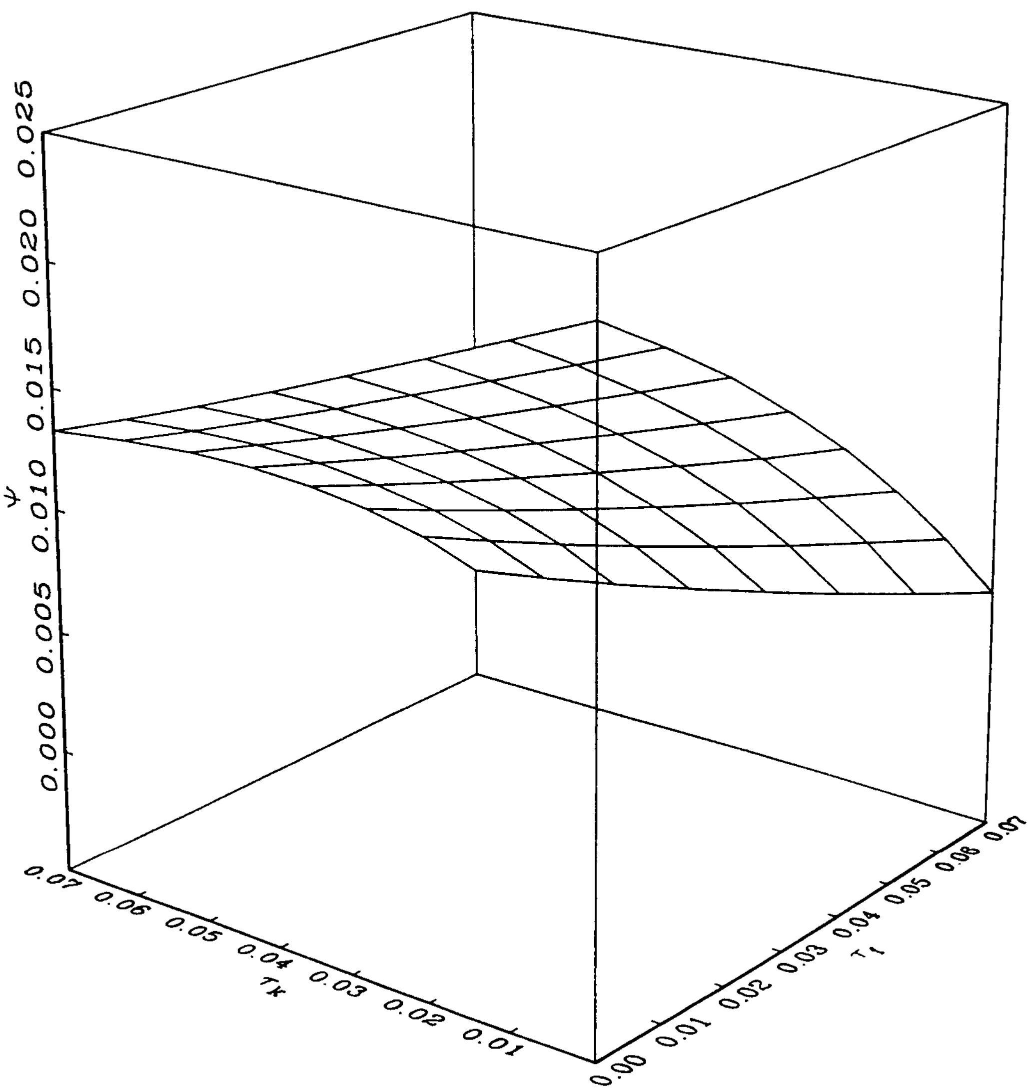

Figure 3: Variance of Growth

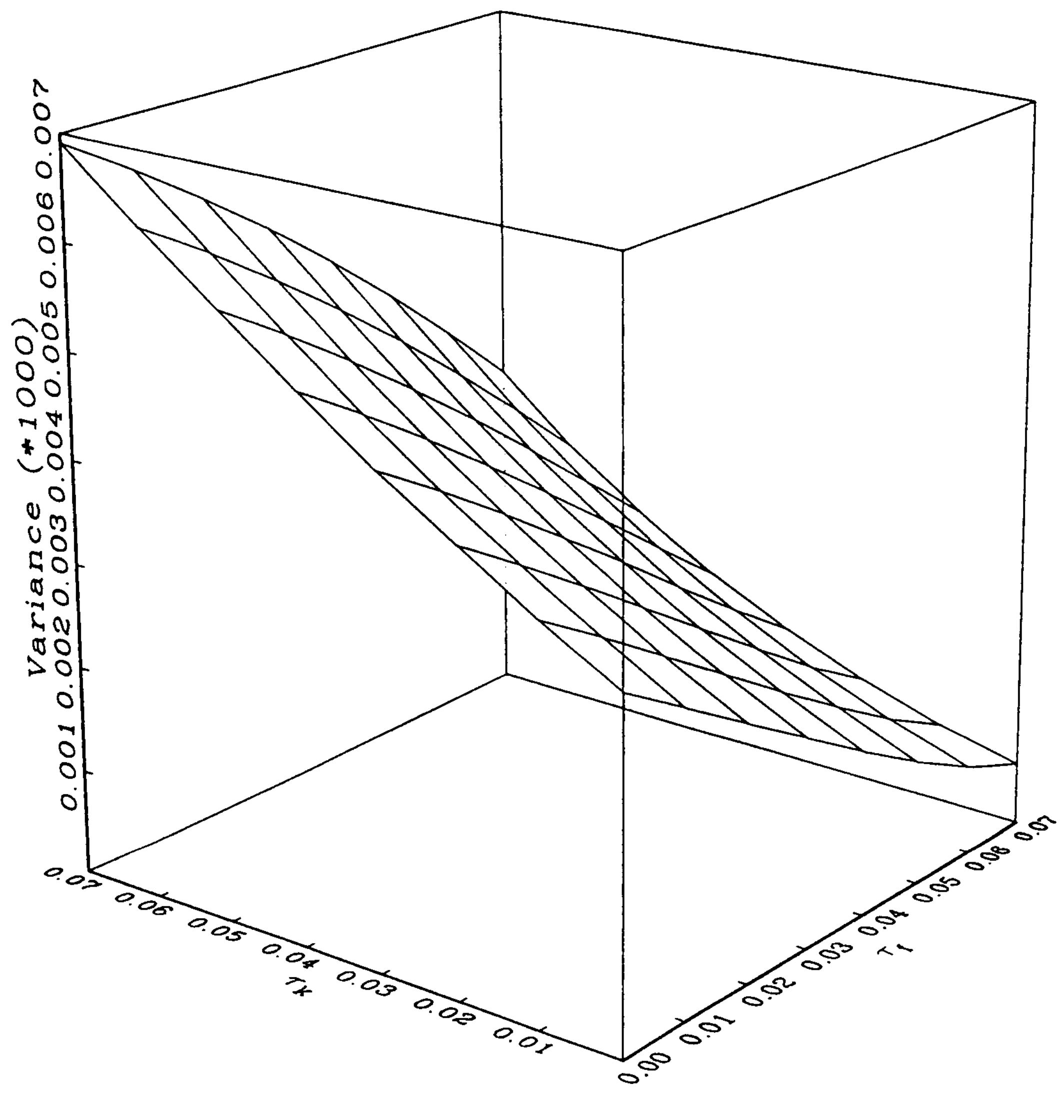

Table 1
SAMPLE STATISTICS

Financial Assets					
	Percentage holding the asset				
Asset	1981	1982	1983	1984	1985
Bank Savings[a]	.79	.75	.67	.64	.72
Government bonds	.33	.39	.26	.29	.35
Domestic Equities	.53	.51	.37	.32	.35
Foreign Assets[b]	.12	.19	.36	.52	.54
Liabilities[c]	.66	.55	.54	.66	.53

[a]Includes Bank checking and savings accounts.
[b]Includes foreign currency holdings.
[c]Excludes mortgage debt.

Table 2
MULTINOMIAL PROBIT ESTIMATES
OF THE INFLUENCE OF TAXES ON PORTFOLIO CHOICE[a]

Variable	MSM Estimate[b]	Standard Error
AGE·P2	0.161*	[1.217]
AGE·P3	0.719	[0.108]
AGE·P4	0.432	[0.160]
AGE·P5	0.547*	[1.118]
AGE·P6	-0.214	[0.105]
AGE·P7	-0.114	[0.044]
TAX·P2	-0.828	[0.141]
TAX·P3	-0.764	[0.101]
TAX·P4	-0.656	[0.276]
TAX·P5	-0.640	[0.118]
TAX·P6	-0.586	[0.169]
TAX·P7	-0.214	[0.060]
INCOME·P2	0.214	[0.060]
INCOME·P3	0.247	[0.118]
INCOME·P4	0.314*	[1.160]
INCOME·P5	0.547	[0.118]
INCOME·P6	0.674	[0.160]
INCOME·P7	0.694*	[1.260]
No. of observations	1745	

[a]The data is for 349 individuals over the period 1981-1985. The choice variables are the seven mutually exclusive portfolios: P1–P7. Portfolio 1 is the least risky and portfolio 7 the most risky. Each explanatory variable is interacted with each portfolio choice except portfolio 1 which is the base portfolio. The specification includes a constant and macoreconomic controls that are not reported. AGE is the age of the head of household. TAX is an instrumental variable for household's marginal tax rate on capital income. INCOME is real annual disposable income in 1981 South African Rand. The instrument vector is the $S \times 1$ vector of covariates associated with each choice and individual and its higher order powers.

[b]MSM denotes the method of simulated moments. As the MSM is not a maximum likelihood procedure so no log likelihood value is provided. A star, *, indicates *not* measured accurately at the 5 per cent level.