The Limits of Ex-Post Implementation

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Abstract: Ex-post implementation is a notion that addresses "Wilson's critique" of non-robust mechanisms, and it has found numerous applications to collective decision making. We show that only trivial choice functions are ex-post implementable in generic mechanism design settings with multi-dimensional signals and interdependent valuations. In other words, ex-post implementation implies that the same alternative must be chosen irrespective of agents' signals. Hence, ex-post implementation is often too strong for practical use. The proof is based on the observation that implementation of non-trivial choice functions is only possible if some rates of information substitution (that depend on the agents' valuations) coincide for all agents. This condition amounts to a system of equations that has no solution for generic valuation functions.

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1 Introduction

The theory of mechanism design is the most powerful theoretical tool for the study of collective decision making by privately informed, strategic agents. Following Harsanyi, interactions with incomplete information have been mostly modelled in a Bayesian framework. That is, agents' choices are required to be optimal at the interim stage where agents know their own signals but only have beliefs about the distributions of others' signals. Based on her own beliefs about the distributions of signals (and possibly on beliefs and higher order beliefs of agents), the designer chooses a mechanism that optimizes her criterion, assuming that agents will subsequently play a Bayes-Nash equilibrium.

Wilson (1987) has forcefully pointed out that the success of many of the implementation schemes used in the above mentioned literature sensitively depends on the beliefs of the agents or of the mechanism designer: if the agents or the mechanism designer are mistaken in their beliefs, the actual outcome of a supposedly optimal mechanism may be very far from the intended one.

Wilson's concern suggests the use of stronger notions of implementation. A concept that has recently received a good deal of attention is *ex-post implementation*, which requires the decision of each agent to be optimal against the strategies of other agents, independently of the realized characteristics of other agents.

Ex-post implementation is an appealing notion since a social choice function that can be ex-post implemented does not require (from agents or designer) any knowledge about the distributions of types. Our main result raises serious doubts about the practical relevance of this concept by showing that in environments where signals are multi-dimensional and the (quasi-linear) valuations are interdependent and generic, only constant choice functions are ex-post implementable.

In a class of environments that includes the one we use here, Bergemann and Morris (2004) provided a strong formal argument in favor of ex-post implementation as a notion of *robust implementation* that addresses Wilson's critique: working with universal type spaces à la Harsanyi-Mertens-Zamir, they showed that a social choice function is ex-post implementable for some given, payoff-relevant types if and only it is Bayes-Nash implementable for every system of beliefs and higher order beliefs that can be associated with those payoff-types.

Ledyard (1978) and Dasgupta et. al (1979) offered early, related arguments in favor of dominant strategy implementation, which is equivalent to ex-post implementation in frameworks with private values. More recently, Chung and Ely (2004) offer a different rationale for dominant strategy mechanisms in private values frameworks: they argue that the designer's belief about the agents' beliefs is unlikely to be reliable, whereas her belief about the payoff-relevant part of the agents' signals is usually more reliable (as it can be assessed from past experiences).[2] In their framework, the worst-case scenario of any Bayes-Nash incentive compatible mechanism performs worse than the expected outcome of the dominant strategy mechanism (that is independent of agents' beliefs). Accordingly, a "maxmin" designer should choose a dominant strategy mechanism.

The interest and attractiveness of dominant strategy implementation has been dampened by the classical impossibility result (for private values environments) due to Gibbard (1973) and Satterthwaite (1975). However, their result requires "rich enough" type spaces, and positive results are possible for restricted but still interesting classes of preferences. For settings in which preferences are not necessarily single-peaked, the most celebrated result is the Vickrey-Clarke-Groves construction for private value settings with quasi-linear utility: by assigning to each agent a transfer equal to the sum of others' valuations in the chosen social alternative, all individual payoff maximization problems become identical to the maximization of social surplus, yielding the dominant strategy implementation of the efficient choice function. In particular, the efficient social choice function can be ex-post implemented irrespectively of the dimension of the private information held by the agents in the private value setting.

The assumption of private values is very restrictive, however, and recent research has focused on models with interdependent values (while still retaining the assumption of quasi-linearity), i.e. on models where agents' payoffs depend on the entire profile of private signals, and not only on their own information.[3] Many practical choice problems fit well in such a setup. For the sake of illustration, consider two examples: 1) A committee has to decide whether to hire candidate A or candidate B (think about the hiring decisions in your own department). The committee's members have private information on some aspects of the quality of the two candidates. Since there are two candidates, and since there are often several dimensions of quality (research,

[2]Note that choosing among ex-post implementable social choice functions **does require** some knowledge about the distribution of the payoff-relevant part of agents' types.

[3]Quasi-linearity is usually regarded as a legitimate assumption when the financial stakes are moderate.

teaching ability, etc.) the private information is typically multi-dimensional. Clearly, all members care about the information held by all agents, and, usually there are many different and potentially conflicting views about how to aggregate the various dimensions of quality. Thus, it is implausible to impose a-priori assumptions of specific functional relations between the preferences of the committee members. 2) International negotiations about environmental standards also involve interdependent values with multidimensional signals because each country usually has some private information, based on data from its national firms on the (partly common) cost associated with complying with the various standards, and data from its citizen about their attitudes toward pollution. Cross-subsidies between countries make the assumption of transferable utilities plausible in this application.

A substantial amount of research has been devoted to extending the Clarke-Groves-Vickrey approach to frameworks with interdependent valuations.[4] However, Jehiel and Moldovanu (2001) have shown that, if at least one agent holds a multi-dimensional signal (and distributions of signals are independent across agents), the efficient social choice function cannot generically be Bayes-Nash implemented. Thus, a fortiori, the efficient social choice function cannot be ex-post implemented. But, that result **does not** rule out the possibility that social choice functions other than the efficient one can be ex-post implemented, and it naturally raises the question about "second-best" or constrained-efficient social choice functions. By ruling out the generic implementation of **all** non-trivial choice functions, the present paper casts a much more serious doubt on the usefulness of ex-post implementation. We want to stress that our result does not say merely: "For any given social choice function, the set of valuation functions that are compatible with the ex-post implementation of that social choice function is non-generic." This would be an easy extension of the Jehiel–Moldovanu impossibility result. In contrast, we assert the much stronger statement: "For generic valuation functions, only trivial social choice functions are ex-post implementable."

The first step in our analysis derives a geometric condition on valuation functions that must be satisfied for a non-trivial choice function to be implementable. The second step shows that this condition cannot be generically

[4] Cremer and McLean (1985), Ausubel (1997), Dasgupta and Maskin (2000), Jehiel and Moldovanu (2001), Bergemann and Välimäki (2002) and Perry and Reny (2002) construct efficient, ex-post incentive compatible mechanisms for various settings with interdependent values. These results assume that agents' signals are one-dimensional, and that valuations satisfy a Single Crossing Property (SCP). Maskin (2003) offers an excellent survey of the literature. Jehiel et al. (2004) study the alignment of agents' interests by means of potential functions.

satisfied, where the notion of genericity is either topological or measure-theoretic.

The geometric condition connects the agents' rates of information substitution: these measure how marginal variations in the several dimensions of one agent's signal affect the agents' payoffs. It is derived from a *taxation principle* which states that, in an ex-post incentive compatible mechanism, agents must agree on a favorite alternative in every state of the world. By this principle, the set of indifference states - where an agent is indifferent between two given alternatives - must be the same for all agents. On this common set, marginal variations in agent i's signal must affect all agents' payoffs in the same way. But, with multidimensional signals and generic valuations it is impossible to construct transfers that equate the resulting rates of information substitution.

The condition for efficient ex-post implementation obtained in Jehiel and Moldovanu (2001) is stronger than the present condition for non-trivial implementation, yet structurally similar. Agent i's preferences must, in a sense, be aligned with social preferences in order to ensure efficient implementation. In contrast, we need here alignment between the preferences of any two agents i and j. But, whereas the social preferences are exogenously fixed by the valuation functions, agent j's preferences can be altered via an endogenous transfer. Therefore, proving the impossibility of non-trivial ex-post implementation is considerably harder than the impossibility of efficient ex-post implementation. To illustrate a notable difference: while our present impossibility result requires the presence of at least two agents having at least two-dimensional signals, the Jehiel–Moldovanu impossibility result requires only that at least one agent has a multidimensional signal (other agents need not even have private information).

The rest of the paper is organized as follows: In Section 2 we describe the mechanism design problem, we define the ex-post equilibrium concept, and we derive a helpful "taxation principle." In Section 3 we state the generic impossibility result about implementation in ex-post equilibrium, and we illustrate how the main geometric applies to a specific example. This section concludes with two remarks on dictatorial and efficient choice functions, respectively. In Section 4 we discuss the main assumptions that underlie our result. In particular, we show that non-trivial implementation is possible in settings where either only one agent has a multi-dimensional signal. We also mention several interesting, but non-generic settings where non-trivial implementation is possible even if several agents have multi-dimensional signals. In Section 5 we gather several concluding remarks. In particular, we discuss some links to weaker implementation concepts such as Bayes-Nash and posterior implementation. Most proofs are collected in an Appendix.

2 The Model

Consider a setting with $N \in \mathbb{N}$ agents $i, j \in \mathcal{N}$, who will be affected by a decision between $K \in \mathbb{N}$ alternatives $k \in \mathcal{K}$. Agent i's utility $u^i = v^i_k - t^i$ is determined by a quasi-linear utility function, taking into account the chosen alternative k and a monetary payment $t^i \in \mathbb{R}$. Her valuation $v^i_k = v^i_k(s)$ for alternative k depends on the state of the world $s \in S$.

Each agent holds private information $s^i \in S^i$ on the state of the world $s \in S$. The signal s^i results from an exogenous draw. There is no loss of generality in assuming that the agents' joint information $(s^i)_{i \in \mathcal{N}}$ completely determines the state of the world s. We thus identify states of the world with signal combinations: $S = \prod_{j \in \mathcal{N}} S^j$. As usual, the information of every agent but i is denoted by $s^{-i} \in S^{-i}$. We adopt the usual notation $s^{-i} = (s^j)_{j \in \mathcal{N}, j \neq i}$ and $s = (s^i, s^{-i})$, when we focus on agent i. We assume $S^i = [0,1]^{d^i}$, and assume v to be a smooth function on S.[5] We denote by ∇_{s^i} the d^i-dimensional vector of derivatives with respect to s^i, and by ∂_ρ the directional derivative in direction $\rho \in \mathbb{R}^{d^i}$. Two vectors $x, y \in R^d$ are *co-directional* if $x = \lambda y$ for $\lambda \geq 0$.

We are interested in choice functions $\psi : S \to K$, with the property that there are transfers functions $t^i : S \to \mathbb{R}$, such that truth-telling is an **ex-post equilibrium** in the incomplete information game that is induced by the direct revelation mechanism $\left(\psi, (t^i)_{i \in N}\right)$, i.e.

$$v^i_{\psi(s)}(s) - t^i(s) \geq v^i_{\psi(\widetilde{s}^i, s^{-i})}(s) - t^i\left(\widetilde{s}^i, s^{-i}\right) \tag{1}$$

for all $s^i, \widetilde{s}^i \in S^i$ and $s^{-i} \in S^{-i}$, where $s := (s^i, s^{-i})$. We shall call such ψ **implementable**. We call a choice function ψ **trivial,** if it is constant on the interior $\operatorname{int} S$ of the type space.[6]

By requiring optimality of i's truth-telling for every realization of other agents information s^{-i}, equation (1) treats s^{-i} as if it was known to agent i.

[5] Assuming S^i to be the closure of any open connected subset of $\mathbb{R}^{d^i}$ would suffice as well.

[6] Restricting attention to the interior of the type space is justified since the interior has full measure. This assumption is necessary since the main geometric argument in the proof fails on the boundary of the type space. Alternatively, we could have assumed open type spaces to start with.

Her incentive constraint is thus equivalent to a monopolistic screening problem for every s^{-i}. Thus, the central authority can post personalized prices $t_k^i(s^{-i})$ for the various alternatives, and let the individuals choose among them. In equilibrium all agents must agree on a most favorable alternative:

Lemma 2.1 (Ex-Post Taxation Principle) *A choice function ψ is implementable, if and only if for all $i \in \mathcal{N}$, $k \in \mathcal{K}$ and $s^{-i} \in S^{-i}$, there are transfers $(t_k^i(s^{-i}))_k \in (\mathbb{R} \cup \{\infty\})^N$ such that:*

$$\psi(s) \in \arg\max_{k \in \mathcal{K}} \left\{ v_k^i(s) - t_k^i\left(s^{-i}\right) \right\}. \tag{2}$$

Proof . See Appendix or Chung and Ely (2003). ■

3 The Impossibility Theorem

In this section we present our generic impossibility result, Theorem 3.2. For ease of exposition, we assume there are only two agents $i, j \in \{1, 2\}$ and two possible alternatives $\{k, l\}$. Because this "2 by 2" model is naturally embedded in every model with more agents and alternatives, the impossibility result for this special setting immediately generalizes to the general setting.

Because agents' incentives are only responsive to differences in payoffs, it is convenient to focus on *relative* valuations μ^i and *relative* transfers τ^i :

$$\mu^i(s) = v_k^i(s) - v_l^i(s); \; \tau^i\left(s^{-i}\right) = t_k^i\left(s^{-i}\right) - t_l^i\left(s^{-i}\right)$$

For technical simplicity, we assume that relative valuations satisfy the mild requirement $\nabla_{s^i}\mu^i(s) \neq 0$ for all $s \in S$.[7]

We use two notions of genericity. The first is topological. If E is a complete metric space, recall that every open subset $U \subset E$ also admits a complete metric. A subset $A \subset U$ is *residual in U* if A contains the countable intersection $\bigcap_{\nu \in \mathbb{N}} A_\nu$ of open and dense sets $A_\nu \subset U$. Residual sets are generally viewed as (topologically) large, and their complements as small. In particular, the Baire Category Theorem guarantees that residual sets are dense.

[7] That is, agent i's relative valuation is everywhere responsive to i's own signal. Theorem 3.2 can be adapted to allow for relative valuations that are not everywhere responsive to own signals — and in particular to allow for interior maxima — but the additional complication makes the argument less transparent without adding any useful insights.

The second notion of genericity is measure-theoretic. Let E be a complete metric topological vector space, U an open subset of E and A a Borel subset of U. We say that A is *finitely shy* in U if there is a finite dimensional subspace $F \subset E$ such that A meets every translate of F in a set of Lebesgue measure 0 (equivalently, if every translate of A meets F in a set of Lebesgue measure 0).[8]. A Borel set $A \subset U$ is *finitely prevalent* in U if the relative complement $U \setminus A$ is finitely shy in U. Hunt et. al. (1992) and Anderson and Zame (2001) have argued that finite prevalence, and prevalence, which is a generalization, provide a sensible measure-theoretic notion of "largeness" for infinite-dimensional spaces of parameters. In particular, if $E = \mathbb{R}^n$ then $B = U \setminus A$ is finitely prevalent in U if and only if the Lebesgue measure of A is 0.

In general, these two notions of genericity are different — even in finite dimensional spaces. However, aside from a technical issue of the degree of differentiability required of the relative valuation function under consideration, we show that ex-post implementation is generically impossible in **both** the topological and measure-theoretic senses.

Definition 3.1 *For each $m \geq 1$, Let $C^m(S, \mathbb{R}^2)$ be the (Banach) space of maps $S \to \mathbb{R}^2$ that admit an m-times continuously differentiable extension to an open neighborhood of S, equipped with the topology of uniform convergence of maps and m derivatives. Let $\mathcal{H}^m \subset C^m(S, \mathbb{R}^2)$ be the open subset consisting of those pairs of relative valuation functions $(\mu^1, \mu^2) \in C^m(S, \mathbb{R}^2)$ for which the partial gradients $\nabla_{s^i} \mu^i$ do not vanish anywhere on S.*

Theorem 3.2 *Assume that the individual signal spaces have dimensions $d^1 \geq 2$ and $d^2 \geq 2$, respectively. Fix an integer $r > \frac{2d^1+1}{d^1-1}$ and set $k = dr + 2d^1 + 1 - 2d^1 r$,where $d = d^1 + d^2$.*

1. *There is a residual subset $\mathcal{G}^1 \subset \mathcal{H}^1$ such that for every $(\mu^1, \mu^2) \in \mathcal{G}^1$, only trivial social choice functions are ex-post implementable.*
2. *There is a residual* ***and*** *finitely prevalent subset $\mathcal{G}^{k+1} \subset \mathcal{H}^{k+1}$ such that for every $(\mu^1, \mu^2) \in \mathcal{G}^{k+1}$, only trivial choice functions are ex-post implementable.*

[8]If F has dimension n, say, any linear isomorphism between F and $\mathbb{R}^n$ induces a measure on F. All such measures are mutually absolutely continuous, and have the same null sets. Hence, it is consistent to abuse terminology by saying that a subset of F — or any translate of F — has Lebesgue measure 0 if it has measure 0 for one — hence all — of these induced measures

The proof of the Theorem consist of two major steps. Proposition 3.5 shows that the existence of a non-trivial ex-post implementable choice function implies a geometric condition on the gradients of the relative valuation functions; Proposition 3.6 shows that this geometric condition cannot be satisfied generically.

The proof of Proposition 3.5 relies on a geometric argument on the boundary that separates the areas (in the signal space S) where alternatives k and l, respectively, are chosen.

Definition 3.3 *The indifference set I of a choice function ψ, is defined by: $I := \overline{\psi^{-1}\{k\}} \cap \overline{\psi^{-1}\{l\}} \cap \operatorname{int} S$. For an indifference signal $\widehat{s} \in I$, we define the indifference set with fixed $\widehat{s}^i$ to be $I^i(\widehat{s}) := \{s \in I : s^i = \widehat{s}^i\}$.*

For the sake of illustration, if the choice function ψ maximizes the social surplus (so that $\psi(s) \in \arg\max_{k,l}\{\sum_i v_k^i(s), \sum_i v_l^i(s)\}$) then the social surplus is the same at both alternatives k and l whenever the state of the world s lies in the indifference set I.

The taxation principle states that, in an incentive compatible mechanism, all agents agree that the chosen alternative is the most favorable one. Assuming, for now, that relative transfers τ are continuous, this implies that the indifference set of the choice function and the indifference sets of all agents must coincide. The following lemma formalizes this assertion. (The proof of this lemma and all other proofs are relegated to the Appendix.)

Lemma 3.4 *Let (ψ, t) be a non-trivial ex-post incentive compatible mechanism with continuous relative transfers τ^i.*

1. *For every $\widehat{s} \in \operatorname{int} S$ and $i \in \{1, 2\}$, we have*

$$\mu^i(\widehat{s}) - \tau^i\left(\widehat{s^{-i}}\right) = 0 \qquad \Leftrightarrow \qquad \widehat{s} \in I \tag{3}$$

2. *For all $\widehat{s} \in I$, $I^i(\widehat{s}) = \{s \in \operatorname{int} S : s^i = \widehat{s}^i, \mu^{-i}(s) = \mu^{-i}(\widehat{s})\}$ is a $(d^{-i} - 1)$-dimensional sub-manifold of $\operatorname{int} S$.*

If relative transfers are differentiable, the gradient of an agent's payoff function is perpendicular to her indifference set. Thus, the coincidence of the agents' indifference sets as expressed in (3) implies that the gradients of agents' payoff functions must be co-directional on the indifference set:

$$\begin{pmatrix} \nabla_{s^i}\mu^i(s) \\ \nabla_{s^{-i}}\mu^i(s) - \nabla_{s^{-i}}\tau^i(s^{-i}) \end{pmatrix} \text{ and } \begin{pmatrix} \nabla_{s^i}\mu^{-i}(s) - \nabla_{s^i}\tau^{-i}(s^i) \\ \nabla_{s^{-i}}\mu^{-i}(s) \end{pmatrix}$$

$$\text{are co-directional on } I \tag{4}$$

Equation (4) says that the payoff functions of agent i and $-i$ have the same rate of information substitution: the relative effect on payoffs of changing any two dimensions of the signal must coincide for all agents.

Proposition 3.5 *Let (ψ, t) be a non-trivial ex-post incentive compatible mechanism.*

1. *If the relative transfers τ^i are continuous on* $\operatorname{int} S^{-i}$ *for all $i \in \{1, 2\}$ then, there are an indifference signal $\widehat{s} \in I$, and a vector $y \in \mathbb{R}^{d^i}$ such that*

$$\nabla_{s^i}\mu^i(s) \text{ and } (\nabla_{s^i}\mu^{-i}(s) - y)$$

$$\text{are co-directional for every } s \in I^i(\widehat{s}) \tag{5}$$

2. *If relative transfers τ^{-i} are discontinuous at $\widehat{s}^i \in$* $\operatorname{int} S^i$ *for some $i \in \{1, 2\}$ then, locally, agent i's incentives do not depend on s^{-i}. That is, there is a vector $y \in \mathbb{R}^{d^i}$ and a non-empty open set $Q \subset S^{-i}$ such that*

$$\nabla_{s^i}\mu^i(\widehat{s}^i, q) \text{ and } y$$

$$\text{are co-directional for every } q \in Q \tag{6}$$

For differentiable relative transfer functions, a proof for Proposition 3.5 is simple: Take any $s = (\widehat{s}^i, s^{-i}) \in I^i(\widehat{s})$. By considering a slight perturbation s^i_ε in a neighborhood of $\widehat{s}^i$ such that $s_\varepsilon = (s^i_\varepsilon, s^{-i}) \in I$, equation (4) implies that $\nabla_{s^i}\mu^i(s)$ and $\nabla_{s^i}\mu^{-i}(s) - \nabla_{s^i}\tau^{-i}(\widehat{s}^i)$ must be co-directional at $s = (\widehat{s}^i, s^{-i})$. Letting $y = \nabla_{s^i}\tau^{-i}(\widehat{s}^i)$ yields equation (5). The full proof (see Appendix) is slightly more complicated in large part because the relative transfer functions are not known to be differentiable, or even continuous.

The second half of the proof of the Impossibility Theorem is that the geometric conditions (5), (6) cannot be generically satisfied. To show this, fix valuation functions μ^1, μ^2; for each $\widehat{s} \in \operatorname{int} S$ define

$$\tilde{I}^i(\widehat{s}) = \{s \in \operatorname{int} S : s^i = \widehat{s}^i, \mu^{-i}(s) = \mu^{-i}(\widehat{s})\}$$

As in Lemma 3.4, this is a manifold of dimension $d^{-i} - 1$. Moreover, for each mechanism and each $\widehat{s} \in I$, Lemma 3.4 guarantees that $\tilde{I}^i(\widehat{s}) = I^i(\widehat{s})$. Hence the following Proposition is enough to complete the proof of the Impossibility Theorem.

Proposition 3.6 *There is a residual set $\mathcal{G}^1 \subset \mathcal{H}^1$ and a residual* and *finitely prevalent subset $\mathcal{G}^{k+1} \subset \mathcal{H}^{k+1}$ such that if $(\mu^1, \mu^2) \in \mathcal{G}^1$ or $(\mu^1, \mu^2) \in \mathcal{G}^{k+1}$ then*

(1) there do not exist $\widehat{s} \in \operatorname{int} S$ and $y \in \mathbb{R}^{d^1}$ such that $\nabla_{s^1}\mu^2(s) - y$ and $\nabla_{s^1}\mu^1(s)$ are co-directional for every $s \in \tilde{I}^1(\widehat{s})$

(2) there do not exist $\widehat{s} \in \operatorname{int} S$ and $y \in \mathbb{R}^{d^2}$ such that $\nabla_{s^2}\mu^1(s) - y$ and $\nabla_{s^2}\mu^2(s)$ are co-directional for every $s \in \tilde{I}^2(\widehat{s})$

(3) there do not exist $\widehat{s} \in \operatorname{int} S$, $y \in \mathbb{R}^{d^1}$, and a non-empty open set $Q \subset S^2$ such that y and $\nabla_{s^1}\mu^1(\widehat{s}^1, q)$ are co-directional for every $q \in Q$

(4) there do not exist $\widehat{s} \in \operatorname{int} S$, $y \in \mathbb{R}^{d^2}$, and a non-empty open set $Q \subset S^1$ such that y and $\nabla_{s^2}\mu^2(\widehat{s}^2, q)$ are co-directional for every $q \in Q$

Fix an indifference signal $\widehat{s} \in I$ and a vector $y \in \mathbb{R}^{d^i}$. If $\nabla_{s^i}\mu^i(s)$ and $\nabla_{s^i}\mu^{-i}(s) - y$ are co-directional for every $s \in I^i(\widehat{s}) = \{s \in S : s^i = \widehat{s}^i, \mu^{-i}(s) = \mu^{-i}(\widehat{s})\}$, then the valuation functions μ^1, μ^2 satisfy a certain set of first-order differential equations. It is not hard to see that generic valuations functions do not satisfy these equations. But the actual argument is substantially more complicated since the assertion of Proposition 3.6 is that generic valuation functions do not satisfy these equations for **any** $\widehat{s}$ and **any** y. Intuitively, varying $\widehat{s}$ and y only do not offer enough degree of freedom to guarantee that $\nabla_{s^i}\mu^i(s)$ and $\nabla_{s^i}\mu^{-i}(s) - y$ are co-directional all over $I^i(\widehat{s})$.

By way of illustration, we apply now Proposition 3.5 to a setting with bi-linear valuations and 2-dimensional signals $s^i = (s^i_k, s^i_l) \in [0,1]^2$. In this case, non-trivial implementation implies a simple algebraic condition (that cannot be satisfied generically) on the coefficients of the valuation functions (See Proposition 6.3 of the Appendix for a generalization to the class of all polynomials of degrees less than a sufficiently large K.)

Example 3.7 *Define valuations v by:*

$$\begin{aligned} v_k^i(s) &= a_k^i s_k^i + b_k^i s_k^i s_k^{-i} = s_k^i \left(a_k^i + b_k^i s_k^{-i}\right) \\ v_l^i(s) &= a_l^i s_l^i + b_l^i s_l^i s_l^{-i} = s_l^i \left(a_l^i + b_l^i s_l^{-i}\right) \end{aligned}$$

where $a_k^i, b_k^i, a_l^i, b_l^i \neq 0$. *Thus,*

$$\mu^i(s) = a_k^i s_k^i - a_l^i s_l^i + b_k^i s_k^i s_k^{-i} - b_l^i s_l^i s_l^{-i}.$$

For a vector $y = \begin{pmatrix} y_k \\ y_l \end{pmatrix}$, *we have*

$$\begin{aligned} \nabla_{s^i} \mu^i(s) &= \begin{pmatrix} a_k^i + b_k^i s_k^{-i} \\ -a_l^i - b_l^i s_l^{-i} \end{pmatrix} \\ \left(\nabla_{s^i} \mu^{-i}(s) - y\right) &= \begin{pmatrix} b_k^{-i} s_k^{-i} - y_k \\ -b_l^{-i} s_l^{-i} - y_l \end{pmatrix} \end{aligned}$$

It is readily verified that $b_l^1 b_k^2 - b_k^1 b_l^2 = 0$ *is necessary for such vectors to remain co-directional when we vary* s_k^{-i} *and* s_l^{-i} *(see Appendix for details). It follows from Proposition 3.5 that a non-trivial choice function* ψ *is implementable only if*

$$b_l^1 b_k^2 - b_k^1 b_l^2 = 0. \tag{7}$$

The above condition is obviously non-generic: the set of parameters where it is satisfied has zero Lebesgue-measure in the 8-dimensional space of coefficients that parameterize the bi-linear valuations in this example.

We conclude this section with two important remarks:

Remark 3.8 *Our main result rules out implementation of dictatorial choice functions. First, note that "dictatorship" is ambiguous with interdependent values: 1) If* ψ *chooses the alternative for which dictator i has the highest valuation, then* $\psi(s)$ *depends on everybody's information* s*, as the dictator does not know her favorite alternative. Point 1 of Proposition 3.5 shows that the agents' incentive constraints cannot be simultaneously satisfied. 2) A dictatorial rule in the sense that* $\psi(s) = \psi(s^i)$ *depends only on the dictator's information* s^i *is generically not implementable either: The relative transfers to the other agent* $\tau^{-i}(s^i)$ *has to be discontinuous, and point 2 of the Proposition shows that, generically, i's incentive constraint cannot be satisfied for all* s^{-i}.

Remark 3.9 *The role played by the dimension of the signals has been emphasized by Jehiel and Moldovanu (2001) in the context of efficient Bayes-Nash implementation. The generic impossibility of ex-post efficient implementation is obviously a corollary of Theorem 3.2, but we now offer an independent argument in order to highlight both the similarities and the differences between the two results. For the sake of illustration, assume here that only one agent i holds private information, and that $\nabla_{s^i}\mu^{\mathcal{N}} \neq 0$ where $\mu^{\mathcal{N}} = \sum_i \mu^i$. Efficient ex-post implementation implies that there is a difference in transfers $\Delta = \tau^i$, such that society is indifferent between the alternatives if and only if this is the case for agent i. Mathematically, the level set $\left(\mu^i\right)^{-1}(\Delta)$ must coincide with the indifference set of the efficient choice function $I^{eff} := \left(\mu^{\mathcal{N}}\right)^{-1}(0)$. Therefore, we obtain that:*

$$\nabla_{s^i}\mu^i(s) \text{ and } \nabla_{s^i}\mu^{\mathcal{N}}(s) \text{ are co-directional for all } s \in I^{eff}. \tag{8}$$

Efficient implementation is only possible if there is a congruence between the private and social rates of information substitution. In contrast, our present condition for non-trivial implementation requires a congruence of private rates for any two agents i and $-i$. Whereas the social preference is ***exogenously*** *fixed by the agents' valuations, agent $-i$'s preferences can be easily altered via the* ***endogenous*** *transfer τ^{-i}. Thus, we needed to show that no transfer whatsoever achieves the required congruence. In particular, we show below that the presence of at least two agents, each equipped with a multidimensional signal, is crucial for the present result, whereas, as just explained, it is not crucial for Jehiel-Moldovanu's result.*

4 The Limits of the Impossibility Result

We now discuss the role played by the following three assumptions :

1) There are multiple (strategic) agents.

2) The signal of at least two agents has multiple dimensions.

3) Valuation functions are generic.

4.1 One strategic agent

Suppose that only agent i is strategic about the release of her information, whereas the information held by all agents j, $j \neq i$ is freely available to the designer. Let $t^i_k = t^i_k(s^{-i})$ be any transfer to agent i in alternative k (this may be constantly zero). The non-trivial social choice function that implements any outcome $k \in \arg\max_k v^i_k(s) - t^i_k(s^{-i})$ for every signal profile $s = (s^i, s^{-i})$ is ex-post implementable. This simple finding should be contrasted with the impossibility of efficient implementation in this setting (see Jehiel and Moldovanu (2001), or the previous section).

4.2 One-dimensional signals

Efficient, ex-post implementation is possible if all agents have one-dimensional signals, and if a single crossing property holds.[9] Our present impossibility result requires that at least two agents have multi-dimensional signals, and thus it is silent about what happens if all but one agent have one-dimensional signals. Consider the following example that shows how positive results can be obtained in this case:

Example 4.1 *There are two agents $i = 1, 2$ and two alternatives k, l. Agent 1 has a one-dimensional signal $s^1 \in [0, 1]$. Agent 2 has a two-dimensional signal, $s^2 = (s^2_k, s^2_l) \in [0, 1]^2$. Assume that the relative valuation μ^1 satisfies the condition: :*

$$\frac{\partial}{\partial s^1}\mu^1(s) > 0 \tag{9}$$

(Note that the set of valuations satisfying this condition is open.) We show how to implement a social choice function ψ that chooses alternative k for high values of s^1 and chooses alternative l for low values of s^1. Set first transfers $t^2(s^1)$ such that

$$\frac{\partial}{\partial s^1}\left(\mu^2(s) - \tau^2(s^1)\right) > 0 \tag{10}$$

and

$$\mu^2(s) - \tau^2(s^1) \text{ takes on values above and below zero on } S \tag{11}$$

Consider now the choice function

$$\psi(s) = \begin{cases} k & \text{if } \mu^2(s) - \tau^2(s^1) \geq 0 \\ l & \text{if } \mu^2(s) - \tau^2(s^1) < 0 \end{cases} \tag{12}$$

[9] This condition is determined by strict inequalities, and it is satisfied for an open set of valuations.

By condition (10), for a fixed s^2 *there is* $\overline{s}^1(s^2)$ *such that*

$$\psi(s) = \begin{cases} k & \text{if } s^1 \geq \overline{s}^1(s^2) \\ l & \text{if } s^1 < \overline{s}^1(s^2) \end{cases} \tag{13}$$

For agent 1 *we apply the standard technique from the literature with one-dimensional signals, and we set transfer* $\tau^1(s^2) = \mu^1(\overline{s}^1(s^2))$. *Using the monotonicity assumption in equation (9) we get that*

$$\psi(s) = \begin{cases} k & \text{if } \mu^1(s) - \tau^1(s^2) \geq 0 \\ l & \text{if } \mu^1(s) - \tau^1(s^2) < 0 \end{cases} . \tag{14}$$

By equations (14) and (12) (ψ, t) *is incentive compatible. It is non-trivial by equation (11). For generic* μ, *the choice function* ψ *is non-dictatorial.*

4.3 Non-generic valuation functions

Generic impossibility allows for large classes of interesting (yet non-generic) valuation functions for which non-trivial ex-post implementation is feasible. For example, this is the case for separable valuation functions, i.e. v for which there are functions $f_k^i : S^i \to \mathbb{R}$, $h_k^i : S^{-i} \to \mathbb{R}$, with:

$$v_k^i(s) = f_k^i(s^i) + h_k^i(s^{-i}).$$

Condition 4 requires that

$$\begin{pmatrix} \nabla_{s^i}(f_k^i - f_l^i)(s^i) \\ \nabla_{s^{-i}}(h_k^i - h_l^i)(s^{-i}) - \nabla_{s^{-i}}\tau^i(s^{-i}) \end{pmatrix}$$

is co-directional on I with

$$\begin{pmatrix} \nabla_{s^i}\left(h_k^{-i} - h_l^{-i}\right)(s^i) - \nabla_{s^i}\tau^{-i}(s^i) \\ \nabla_{s^{-i}}\left(f_k^{-i} - f_l^{-i}\right)(s^{-i}) \end{pmatrix}$$

As the upper (resp. lower) half of the condition is independent of s^{-i} (resp. s^i) the two gradients can be equalized everywhere by setting, for example, $\tau^{-i}(s^i) := \left(h_k^{-i} - h_l^{-i}\right)(s^i) - (f_k^i - f_l^i)(s^i)$, and similarly for $\tau^i(s^{-i})$. These transfers implement the choice function $\psi(s) \in \arg\max_k \left\{\sum_i f_k^i(s^i)\right\}$.

In a recent paper, Bikchandani (2004) studies multi-object auction problems where social alternatives are partitions of goods. The agents do not care about the exact partition among others. In other words, at each possible signal, each agent is indifferent between many alternatives. Bikchandani shows that non-trivial ex-post implementation is possible in this obviously non-generic framework where many of the relevant rates of information substitution are constant. The needed mechanisms are not "intuitive": for example, in an one-object auction with two buyers, the object is sold if one buyer has a low valuation while another has a high valuation, but the object must remain in the hands of the seller if both buyers have high valuations. In an auction setting, the indifference among alternatives arises naturally if there are no allocative externalities among agents. But such an indifference seems strong and less appealing in social choice settings where information can affect the relative value of any two alternatives.

5 Conclusion

We have established the impossibility of non-trivial ex post implementation in generic quasi-linear environments with interdependent preferences and multidimensional signal spaces. The main motivation for considering ex-post implementation (as opposed to Bayes-Nash implementation) is the idea that the agents or the designer may have insufficient information about relevant features of the environment.

Our result **does not** imply that Bayes-Nash implementation is impossible for any given, specific beliefs. But, in conjunction with the results in Bergemann and Morris (2004), it implies that, for any non-trivial mechanism (social choice rule + transfers), there are beliefs for which this mechanism is not Bayes-Nash incentive compatible. Thus, robust implementation a la Bergemann-Morris is generically impossible. At least three significant tasks remain for future research:

1)Identify additional interesting models with restricted classes of preferences where ex-post implementation is possible (i.e. besides frameworks with single-dimensional signals, or with separable valuations, or auctions of private goods without allocative externalities).

2)Focus on the possibly weaker conditions needed for robust implementation of social choice correspondences: suppose that we can establish the existence of a Bayes-Nash equilibrium for a family of priors, and suppose it is known that the priors of agents must belong to this family. Then the social choice **correspondence** that associates to each signal profile s the set of alternatives that may emerge in a Bayes-Nash equilibrium (for some of these priors) can be robustly implemented in the sense that, whatever the effective prior, there is an equilibrium outcome that belongs to the social choice correspondence.

3) Identify a fruitful way to address Wilson's critique in frameworks where ex-post implementation is not possible.

6 Appendix

Proof of Lemma 2.1. "if": Given $t_k^i\left(s^{-i}\right)$ such that equation (2) holds, define $t^i\left(s\right) := t_{\psi(s)}^i\left(s^{-i}\right)$. Agent i's problem in the game induced by (ψ, t) is to choose $\widehat{s}^i$ in order to maximize $v_{\psi(\widehat{s}^i, s^{-i})}^i\left(s^i, s^{-i}\right) - t_{\psi(\widehat{s}^i, s^{-i})}^i\left(s^{-i}\right)$. By equation (2), it is optimal for her to report truthfully $\widehat{s}^i = s^i$, and let the choice function ψ pick her most preferred alternative.

"only if": Let (ψ, t) be an ex-post incentive compatible mechanism. We define

$$t_k^i\left(s^{-i}\right) := \begin{cases} t^i\left(s^i, s^{-i}\right) & \text{if } \psi\left(s^i, s^{-i}\right) = k \\ \infty & \text{if } \psi\left(s^i, s^{-i}\right) \neq k \text{ for all } s^i \in S^i. \end{cases} \tag{15}$$

Note that $t_k^i\left(s^{-i}\right)$ is well-defined. By i's incentive constraint

$$\psi\left(s^i, s^{-i}\right) = \psi\left(s'^i, s^{-i}\right) = k \text{ implies } t^i\left(s^i, s^{-i}\right) = t^i\left(s'^i, s^{-i}\right).$$

By i's incentive constraint again, she always reports in order to maximize her payoff. Thus, with $t_k^i\left(s^{-i}\right)$ as defined in (15), condition (2) is satisfied. ∎

Proof of Lemma 3.4. 1) $\mu^i\left(\widehat{s}\right) - \tau^i\left(\widehat{s}^{-i}\right) = 0$ and $\nabla_{s^i}\mu^i\left(\widehat{s}\right) \neq 0$ imply that there are s'^i, s''^i arbitrarily close to $\widehat{s}^i$ such that $\mu^i\left(s''^i, \widehat{s}^{-i}\right) - \tau^i\left(\widehat{s}^{-i}\right) < 0 < \mu^i\left(s'^i, \widehat{s}^{-i}\right) - \tau^i\left(\widehat{s}^{-i}\right)$. Applying the taxation principle to agent i yields $\psi\left(s'^i, \widehat{s}^{-i}\right) = k$ and $\psi^1\left(s''^i, \widehat{s}^{-i}\right) = l$. Hence $\widehat{s} \in I$.

For the converse, assume that $\mu^i\left(\widehat{s}\right) - \tau^i\left(\widehat{s}^{-i}\right) > 0$, say. By continuity, we have $\mu^i\left(s\right) - \tau^i\left(s\right) > 0$, and thus $\psi\left(s\right) = k$, for all s in a neighborhood of $\widehat{s}$. Thus, $\widehat{s} \notin I$.

2) is immediate from the above.

3) Since we assumed that $\nabla_{s^{-i}}\mu^{-i}$ is non-vanishing, we can apply the implicit function theorem to conclude. ∎

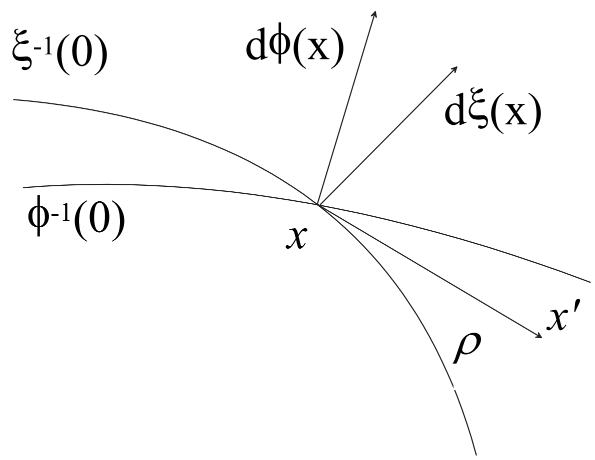

Figure 1: If the gradients of ϕ and ξ are not co-directional at x, the functions disagree at some x', i.e. $\phi(x') < 0 < \xi(x')$.

To prove Proposition 3.5, we first state a simple Lemma.

Lemma 6.1 *Let ϕ and ξ be smooth functions on an open set $X \subset \mathbb{R}^N$. Assume that there exists $x \in X$ such that $\phi(x) = \xi(x) = 0$, but $\nabla\phi(x)$ and $\nabla\xi(x)$ are not co-directional. Then there exists x' arbitrarily close to x such that $\phi(x') < 0 < \xi(x')$.*

Proof. As $\nabla\phi(x)$ and $\nabla\xi(x)$ are not co-directional, there exists a direction $\rho \in \mathbb{R}^N$ with $\rho \cdot \nabla\phi(x) < 0 < \rho \cdot \nabla\xi(x)$. For $x' = x + \varepsilon\rho$, with $\varepsilon > 0$, we get $\phi(x') < 0 < \xi(x')$, as desired. This argument is illustrated in Figure 1. ■

Proof of Proposition 3.5. Consider an ex-post incentive compatible mechanism (ψ, t) and the associated relative valuations and transfers.

1) If τ is differentiable, the discussion preceding the Proposition together with Lemma 6.1 completes the proof.

More generally, we need to deal with two sub-cases:

1.a) The direction of the gradient $\nabla_{s^i}\mu^i(s)$ does not depend on $s \in I^i(\widehat{s})$. Instead of showing that τ^{-i} is differentiable, we directly construct the vector y. Denote the orthogonal complement of $\nabla_{s^i}\mu^i(s)$ by $(\nabla\mu^i)^{\perp} \subset \mathbb{R}^{d^i}$ and let $\rho \in (\nabla\mu^i)^{\perp}$. Fix for a moment s^{-i} with $(\widehat{s}^i, s^{-i}) \in I^i(\widehat{s})$. By

Lemma 3.4, $\mu^{-i}(\cdot, s^{-i}) - \tau^{-i}(\cdot)$ must equal zero on the sub-manifold $\{s^i : \mu^i(s^i, s^{-i}) = \mu^i(\widehat{s}^i, s^{-i})\}$. Thus, restricted to that manifold, τ^{-i} is differentiable and we have $\partial_\rho \mu^{-i}(\widehat{s}^i, s^{-i}) = \partial_\rho \tau^{-i}(\widehat{s}^i)$. Therefore, $\rho \cdot \nabla_{s^i} \mu^{-i}(\widehat{s}^i, s^{-i}) = \partial_\rho \mu^{-i}(\widehat{s}^i, s^{-i})$ is independent of $(\widehat{s}^i, s^{-i}) \in I^i(\widehat{s})$.Set now $y := \nabla_{s^i} \mu^{-i}(\widehat{s}) + \lambda \nabla_{s^i} \mu^i(\widehat{s})$. By construction, we have $\rho \cdot (\nabla_{s^i} \mu^{-i}(\widehat{s}^i, s^{-i}) - y) = 0$ for $\rho \in (\nabla \mu^i)^\perp$. By choosing λ sufficiently large, $\nabla_{s^i} \mu^i(s)$ and $(\nabla_{s^i} \mu^{-i}(\widehat{s}^i, s^{-i}) - y)$ must be co-directional, and condition (5) is satisfied.

1.b) The direction of the gradient $\nabla_{s^i} \mu^i(s)$ varies in $s \in I^i(\widehat{s})$. In this case we will show that τ^{-i} is differentiable at some $\widetilde{s}^i$ close to $\widehat{s}^i$.

As a first step, we show that the directional derivatives $\partial_\rho \tau^{-i}(\widehat{s}^i)$ in directions $\rho \in \nabla_{s^i} \mu^i(\widehat{s}^i, s^{-i})^\perp$ exist. Fix $s \in I^i(\widehat{s})$ and $\rho \in \nabla_{s^i} \mu^i(s)^\perp$ such that there are $\underline{s}, \overline{s} \in I^i(\widehat{s})$ close to s with $\rho \cdot \nabla_{s^i} \mu^i(\overline{s}) > 0 > \rho \cdot \nabla_{s^i} \mu^i(\underline{s})$. By agent i's incentive constraint, we have $\psi\left(\widehat{s}^i + \varepsilon\rho, \overline{s}^{-i}\right) = k$ and $\psi(\widehat{s}^i + \varepsilon\rho, \underline{s}^{-i}) = l$ for small enough $\varepsilon > 0$ (compare this argument to the one for Lemma 6.1). In turn, agent $(-i)$'s incentive constraint implies $\partial_\rho \mu^{-i}(\overline{s}) \geq -\frac{\tau^{-i}\left(\widehat{s}^i + \varepsilon\rho\right) - \tau^{-i}\left(\widehat{s}^i\right)}{\varepsilon} \geq \partial_\rho \mu^{-i}(\underline{s})$. As $\underline{s}^{-i}$ and $\overline{s}^{-i}$ approach s^{-i},and ε approaches zero, this entails $\partial_\rho \tau^{-i}(\widehat{s}^i) = \partial_\rho \mu^{-i}(\widehat{s}^i, s^{-i})$.

By assumption, $\nabla_{s^i} \mu^i(\widehat{s}^i, s^{-i})^\perp$ varies (continuously) in s^{-i}. Therefore, $\partial_\rho \tau^{-i}(\widehat{s}^i)$ exists for an open set of directions $\rho \in \Lambda \subset \mathbb{R}^{d^i}$. In order to conclude, we need to show that these directional derivatives are continuous in s^i.

Consider $\widetilde{s}^i = \widehat{s}^i + \varepsilon\rho$ for some $\rho \in \Lambda$ and $\varepsilon \in \mathbb{R}$ sufficiently small. By the above argument, there is a neighborhood U of $\widetilde{s}^i$, such that the directional derivatives $\partial_\rho \tau^{-i}(s^i)$ for $\rho \in \Lambda \subset \mathbb{R}^{d^i}$ and $s^i \in U$ exist and are continuous in s^i. Thus, τ^{-i} is differentiable for $s^i \in U$ and, after replacing $\widehat{s}^i$ by $\widetilde{s}^i$, we can conclude. For an intuition consider Figure 2. **2)** Assume now that the relative

transfer τ^{-i} is discontinuous at some $\widehat{s}^i \in \operatorname{int} S^i$. We can assume w.l.o.g. that $\tau^{-i}(s^i) \in T^{-i}(s^i) := [\inf_{s^{-i}} \{\mu^{-i}(s^i, s^{-i})\}, \sup_{s^{-i}} \{\mu^{-i}(s^i, s^{-i})\}]$ for all s^i.[10] By assumption, there is a sequence of i's signals $(s^i_n)_{n \in \mathbb{N}}$ such that $\lim_n s^i_n = \widehat{s}^i$ but such that $\tau^{-i}(s^i_n)$ does not converge to $\tau^{-i}(\widehat{s}^i)$. Modulo taking a subsequence, we can assume that $\lim_n \tau^{-i}(s^i_n) = \tau^{-i}(\widehat{s}^i) + \varepsilon$, for $\varepsilon > 0$, say. Consider $\widetilde{S}^{-i} := \{s^{-i} \in S^{-i} : \mu^{-i}(\widehat{s}^i, s^{-i}) \in \left(\tau^{-i}(\widehat{s}^i) + \frac{\varepsilon}{4},\ \tau^{-i}(\widehat{s}^i) + \frac{\varepsilon}{2}\right)\}$.[11]

[10] If $\tau^{-i}\left(s^i\right) < \inf_{s^{-i}} \left\{\upsilon^{-i}\left(s^i, s^{-i}\right)\right\}$, say, we have $0 < \upsilon^i\left(s^i, s^{-i}\right) + \tau^{-i}\left(s^i\right)$ for all s^{-i}, and agent $-i$ will "choose" outcome k, no matter what her signal s^{-i} is. This is still the case, after we change $\tau^{-i}\left(s^i\right)$ to $\inf_{s^{-i}} \left\{\upsilon^{-i}\left(s^i, s^{-i}\right)\right\}$.

[11] Note that $S^i(\varepsilon)$ is not empty. Taking $\tau^{-i}\left(s^i_n\right) \in T^{-i}\left(s^i_n\right)$ to the limit, yields that $\tau^{-i}\left(\widehat{s}^i\right) + \varepsilon \in T^{-i}\left(\widehat{s}^i\right)$. Together with $\tau^{-i}\left(\widehat{s}^i\right) \in T^{-i}\left(\widehat{s}^i\right)$, this yields

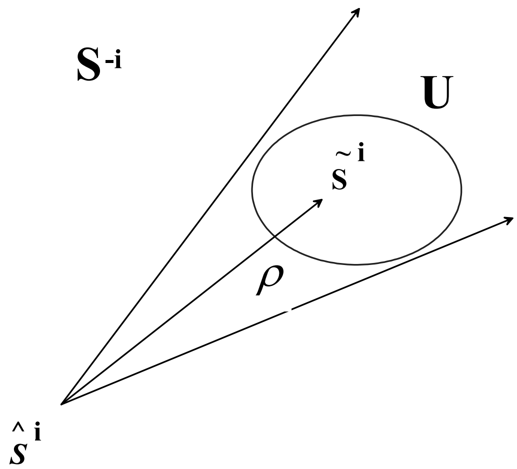

Figure 2: An illustration of $S^i \subset \mathbb{R}^2$: the directional derivatives $\partial_\rho \tau^j(\widehat{s}^i)$ exist for directions ρ inside the cone. As $\nabla_{s^i} \upsilon^i(s^i, s^j)$ is continuous in s^i, these directional derivatives also exist in a neighborhood U of $\widetilde{s}^i$ and are continuous.

These types $s^{-i} \in \widetilde{S}^{-i}$ of agent $-i$ prefer k when the relative payment is $\tau^{-i}(\widehat{s}^i)$, but prefer l when the relative payment is $\tau^{-i}(\widehat{s}^i) + \varepsilon$. Therefore, $\psi(\widehat{s}^i, s^{-i}) = k$, but $\psi(s^i_n, s^{-i}) = l$ for large enough n.[12] As $\lim_n s^{-i}_n = \widehat{s}^{-i}$, we can apply the taxation principle to agent i to obtain $\mu^i(\widehat{s}^i, s^{-i}) - \tau^i(s^{-i}) = 0$, for all $s^{-i} \in \widetilde{S}^{-i}$ (recall that μ^i is continuous).

We now show that the gradients $\nabla_{s^i} \mu^i(\widehat{s}^i, s^{-i})$ are co-directional for all $s^{-i} \in \widetilde{S}^{-i}$. This proves the desired result since $\widetilde{S}^{-i}$ is open, and since it contains the manifolds $\left\{s = (\widehat{s}^i, s^{-i}) : \mu^{-i}(s) = \tau^{-i}(\widehat{s}^i) + \frac{\varepsilon}{3}\right\}$.

Assume that this is not the case for $s'^{-i}, s''^{-i} \in \widetilde{S}^{-i}(\varepsilon)$. We assume w.l.o.g. that $\mu^{-i}(\widehat{s}^i, s'^{-i}) < \mu^{-i}(\widehat{s}^i, s''^{-i})$. By Lemma 6.1, there is $\widetilde{s}^i$, arbitrarily close to $\widehat{s}^i$, with $\mu^i(\widetilde{s}^i, s''^{-i}) + \tau^i(s''^{-i}) < 0 < \mu^i(\widetilde{s}^i, s'^{-i}) + \tau^i(s'^{-i})$. Thus, $\psi(\widetilde{s}^i, s'^{-i}) = k$ and $\psi(\widetilde{s}^i, s''^{-i}) = l$. However, for $\widetilde{s}^i$ close enough to $\widehat{s}^i$, continuity of μ^i yields $\mu^i(\widetilde{s}^i, s'^{-i}) < \mu^i(\widetilde{s}^i, s''^{-i})$. This yields a contradiction to the monotonicity of ψ and concludes the argument. ■

We turn now to the proof of the genericity assertion, Proposition 3.6. Write $d = d^1 + d^2$, and let $\mathcal{P}^{2dr}$ be the space of polynomials on $\mathbb{R}^d$ of degree at most $2dr$. We need the following lemma, whose simple proof is left to the

$\left[\tau^{-i}(\widehat{s}^i), \tau^{-i}(\widehat{s}^i) + \varepsilon\right] \subset T^{-i}(\widehat{s}^i) = [\inf_{s^{-i}} \left\{\upsilon^{-i}(s^i, s^{-i})\right\}, \sup_{s^{-i}} \left\{\upsilon^{-i}(s^i, s^{-i})\right\}]$.

[12]Specifically, n such that: $\upsilon^{-i}(s^i_n, s^{-i}) < \upsilon^{-i}(\widehat{s}^i, s^{-i}) + \frac{\varepsilon}{4} \leq \tau^{-i}(\widehat{s}^i) + \frac{3\varepsilon}{4} < \tau^{-i}(s^i_n)$

reader.

Lemma 6.2 *Let $s_1, \ldots, s_r$ be distinct points in $\mathbb{R}^d$ and let $\{a_i : 1 \le i \le r\}$ and $\{a_{ij} : 1 \le i \le r, 1 \le j \le d\}$ be families of real numbers. There is a polynomial $P \in \mathcal{P}^{2dr}$ such that for all i, j:*

$$
\begin{aligned}
P(s_i) &= a_i \\
\frac{\partial P}{\partial x^j}(s_i) &= a_{ij}
\end{aligned}
$$

Recall that we fixed $r > \frac{2d^1+1}{d^1-1}$ and defined $k = dr + 2d^1 + 1 - 2d^1 r$. For each i, let $\pi^i : \mathbb{R}^d \to \mathbb{R}^{d^i}$ be the projection. We will derive both parts of Proposition 3.6 from the following finite dimensional Proposition.

Proposition 6.3 *Let $\mathcal{L} \subset C^{k+1}(S, \mathbb{R}^2)$ be any finite-dimensional subspace that contains $\mathcal{P}^{2dr} \times \mathcal{P}^{2dr}$, let $\mathcal{M}$ be any translate of $\mathcal{L}$ in $C^{k+1}(S, \mathbb{R}^2)$, and let $\mathcal{M}_0 = \mathcal{H}^{k+1} \cap \mathcal{M}$. There are subsets $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4 \subset \mathcal{M}_0$ that are residual and have full Lebesgue measure in $\mathcal{M}_0$ and enjoy the following properties.*

(1) If $(\mu^1, \mu^2) \in \mathcal{M}_1$ then there do not exist $\widehat{s} \in \operatorname{int} S$ and $y \in \mathbb{R}^{d^1}$ such that $\nabla_{s^1}\mu^1(s)$ and $\nabla_{s^1}\mu^2(s) - y$ are collinear for every $s \in \tilde{I}^1(\widehat{s})$.

(2) If $(\mu^1, \mu^2) \in \mathcal{M}_2$ then there do not exist $\widehat{s} \in \operatorname{int} S$ and $y \in \mathbb{R}^{d^2}$ such that $\nabla_{s^2}\mu^2(s)$ and $\nabla_{s^2}\mu^1(s) - y$ are collinear for every $s \in \tilde{I}^2(\widehat{s})$.

(3) If $(\mu^1, \mu^2) \in \mathcal{M}_3$ then there do not exist $\widehat{s} \in \operatorname{int} S$, $y \in \mathbb{R}^{d^1}$, and a non-empty open set $Q \subset S^2$ such that $\nabla_{s^1}\mu^1(\widehat{s}^1, q)$ and y are collinear for every $q \in Q$.

(4) If $(\mu^1, \mu^2) \in \mathcal{M}_4$ then there do not exist $\widehat{s} \in \operatorname{int} S$, a vector $y \in \mathbb{R}^{d^2}$, and a non-empty open set $Q \subset S^1$ such that $\nabla_{s^2}\mu^2(\widehat{s}^2, q)$ and y are collinear for every $q \in Q$.

Moreover, the intersection $\mathcal{M}^* = \bigcap \mathcal{M}_i$ is also residual and has full Lebesgue measure in $\mathcal{M}_0$, and every pair $(\mu^1, \mu^2) \in \mathcal{M}^*$ enjoys the four properties above.

Proof. Write $(\operatorname{int} S)^{(r)}$ for the open subset of $(\operatorname{int} S)^r$ consisting of distinct r-tuples. To construct $\mathcal{M}_1$, write

$$V = (\operatorname{int} S)^{(r)} \times \mathbb{R}^r \times \mathbb{R}^{d^1} \times \mathbb{R}^{d^1} \times \mathbb{R}$$

For each $n = 1, \ldots, r$ define

$$\phi_n : \mathcal{M}_0 \times V \to \mathbb{R}^{d^1} \times \mathbb{R}^{d^1} \times \mathbb{R} = \mathbb{R}^{2d^1+1}$$

by

$$\begin{aligned} &\phi_n(\mu^1, \mu^2; s_1, \ldots, s_r; \lambda_1, \ldots, \lambda_r; y, w, c) \\ &= \big(\nabla_{s^1}\mu^1(s_n) - \lambda_n[\nabla_{s^1}\mu^2(s_n) - y],\ \pi 1(s_n) - w, \mu^2(s_n) - c\big) \end{aligned}$$

Finally, write

$$\Phi = (\phi_1, \ldots, \phi_r) : \mathcal{M}_0 \times V \to \mathbb{R}^{(2d^1+1)r}$$

Because the components of Φ are either linear functions or evaluations of first derivatives of $(k+1)$-times continuously differentiable functions, Φ itself is k-times continuously differentiable. Using Lemma 6.2, it is easy to check that for every $(\mu^1, \mu^2; v) \in \mathcal{M}_0 \times V$ the directional derivatives of Φ in directions in $\mathcal{P}^{2dr} \times \mathcal{P}^{2dr} \times V$ span $\mathbb{R}^{(2d^1+1)r}$. In particular, for each $(\mu^1, \mu^2; v) \in \mathcal{M}_0 \times V$ the differential $D\Phi$ is onto. Hence, the transversality theorem (see Mas-Colell (1985) for instance) provides a subset $\mathcal{M}_1 \subset \mathcal{M}_0$ that is residual and of full measure such that, for each $(\mu^1, \mu^2) \in \mathcal{M}_1$, the set

$$J(\mu^1, \mu^2) = \{v \in V : \Phi(\mu^1, \mu^2; v) = 0\}$$

is either empty or is a manifold of dimension

$$dr + r + d^1 + d^1 + 1 - (2d^1 + 1)r = dr + 2d^1 + 1 - 2d^1 r$$

To see that $\mathcal{M}_1$ has the desired property (1), suppose not, so that there exist $\widehat{s} \in \operatorname{int} S$ and $y \in \mathbb{R}^{d_1}$ such that $\nabla_{s^1}\mu^1(s)$ and $\nabla_{s^1}\mu^2(s) - y$ are collinear for each $s \in \tilde{I}^1(\widehat{s})$. If $z_1, \ldots, z_r$ are distinct points of $\tilde{I}^1(\widehat{s})$ then we can find $\lambda_1, \ldots, \lambda_r \in \mathbb{R}$ such that

$$\nabla_{s^1}\mu^1(z_i) = \lambda_i \left(\nabla_{s^1}\mu^2(z_i) - y\right)$$

Hence

$$\big(z_1, \ldots, z_r; \lambda_1, \ldots, \lambda_r; y, \pi^1(\widehat{s}), \mu^2(\widehat{s})\big) \in J(\mu^1, \mu^2)$$

Equivalently, $\tilde{I}^1(\widehat{s})^{(r)}$ is a subset of the projection of $J(\mu^1, \mu^2)$ into $(\operatorname{int} S)^{(r)}$. Because $\tilde{I}^1(\widehat{s})$ has dimension $d2 - 1$ and projection does not raise the dimension of a manifold, it follows that $J(\mu^1, \mu^2)$ must have dimension at least $(d^2 - 1)r$. However, our computation of the dimension of $J(\mu^1, \mu^2)$ implies that

$$dr + 2d^1 + 1 - 2d^1 r \geq (d^2 - 1)r$$

and equivalently, that

$$\frac{2d^1 + 1}{d^1 - 1} \geq r$$

Since this contradicts our choice of r, we conclude that $\mathcal{M}_1$ has the desired property.

To construct $\mathcal{M}_2$ we proceed exactly as above, except that we reverse the roles of μ^1, μ^2 . The constructions of $\mathcal{M}_3, \mathcal{M}_4$ use a variant of this same construction. For $\mathcal{M}_3$, write

$$V = \operatorname{int} S^1 \times (\operatorname{int} S^2)^{(r)} \times \mathbb{R}^{d^1} \times \mathbb{R}^r$$

For each $n = 1, \ldots, r$ define

$$\phi_n : \mathcal{M}_0 \times V \to \mathbb{R}^{d^1}$$

by

$$\phi_n(\mu^1, \mu^2; \widehat{s}^1; q_1, \ldots, q_r; y; \lambda_1, \ldots, \lambda_r) = \nabla_{s^1} \mu^1(\widehat{s}^1, q_n) - \lambda_n y$$

Finally, write

$$\Phi = (\phi_1, \ldots, \phi_r) : \mathcal{M}_0 \times V \to \mathbb{R}^{d^1 r}$$

As above, we use the transversality theorem to find a residual set of full measure $\mathcal{M}_3 \subset \mathcal{M}_0$ such that if $(\mu^1, \mu^2) \in \mathcal{M}_3$ then

$$J(\mu^1, \mu^2) = \{v : (\widehat{s}^1, q_1, \ldots, q_r; y; \lambda_1, \ldots, \lambda_r) : \Phi(\mu^1, \mu^2; v) = 0\}$$

is a manifold of dimension

$$2d^1 + (d^2 - d^1)r + r - d^1 r = 2d^1 + r + (d^2 - d^1)r$$

We claim that if $(\mu^1, \mu^2) \in \mathcal{M}_3$ then there does not exist $\widehat{s}^1 \in \operatorname{int} S^1$, $y \in \mathbb{R}^{d^1}$ and an open set $Q \subset \operatorname{int} S^2$ such that $\nabla_{s^1} \mu^1(\widehat{s}^1, q)$ and y are collinear for each $q \in Q$. To see this, we argue exactly as before: if such existed then the dimension of $J(\mu^1, \mu^2)$ would be at least as large as rd^2, whence

$$2d^1 + r + (d^2 - d^1)r \geq r$$

and

$$r \leq \frac{2d^1}{d^1 - 1} < \frac{2d^1 + 1}{d^1 - 1}$$

This contradicts our choice of r, so we conclude that $\mathcal{M}_3$ has the desired property. To construct $\mathcal{M}_4$ we proceed exactly as above, except that we reverse the roles of μ^1, μ^2.

Finally, $\mathcal{M}^*$ is residual and of full measure because it is the intersection of a finite number of sets with these properties. ■

With Proposition 6.3 in hand, we turn to the proof of Proposition 3.6.

Proof of Proposition 3.6. We begin by constructing $\mathcal{G}^{k+1}$ as the intersection of four sets $\mathcal{W}_1, \ldots, \mathcal{W}_4$, corresponding to the various properties, and then use Proposition 6.3 to show that $\mathcal{G}^{k+1}$ has the desired properties.

To construct $\mathcal{W}_1$ and $\mathcal{W}_2$ we proceed in the following way. First choose and fix an increasing sequence of compact sets $L_1, L_2, \ldots$, whose union is $\operatorname{int} S^1$. For each index m, let $C(m)$ be the set of pairs $(\mu^1, \mu^2) \in \mathcal{H}^{k+1}$ for which there exist $\widehat{s} \in L_m$, $y \in \mathbb{R}^{d^1}$ with $|y| \leq m$, and a subset $Z \subset \tilde{I}^1(\widehat{s})$ such that:

- for every $z \in Z$ there is a $\lambda \in \mathbb{R}$ such that $\mu^1(z) - \lambda[\mu^2(z) - y] = 0$ and $|\lambda| \leq m$
- the projection of Z into some $d^2 - 1$-dimensional subspace of $\mathbb{R}^d$ contains a ball of radius at least $1/m$

It is straightforward to check that each $C(m)$ is a closed subset of $\mathcal{H}^{k+1}$, so the complement $\mathcal{H}^{k+1} \setminus C(m)$ is open. Set

$$\mathcal{W}_1 = \bigcap_{m=1}^{\infty} \left[\mathcal{H}^{k+1} \setminus C(m)\right]$$

We construct $\mathcal{W}_2$ in exactly the same way, except that the roles of μ^1, μ^2 are reversed.

To construct $\mathcal{W}_3$ and $\mathcal{W}_4$, we proceed as follows. For each index m, let $K(m)$ be the set of pairs $(\mu^1, \mu^2) \in \mathcal{H}^{k+1}$ for which there exist $\widehat{s} \in L_m$, $y \in \mathbb{R}^{d^1}$ with $|y| \leq m$, and a ball $B \subset S^2$ such that:

- for every $b \in B$ there is a $\lambda \in \mathbb{R}$ such that $\mu^1(\widehat{s}^1, b) - \lambda y = 0$ and $|\lambda| \leq m$
- the radius of B is at least $1/m$

It is easy to see that $K(m)$ is closed, and hence that $\mathcal{H}^{k+1} \setminus K(m)$ is open. Set

$$\mathcal{W}_3 = \bigcap_{m=1}^{\infty} \left[\mathcal{H}^{k+1} \setminus K(m)\right]$$

We construct $\mathcal{W}_4$ in exactly the same way, except that the roles of μ^1, μ^2 are reversed.

Set $\mathcal{G}^{k+1} = \bigcap \mathcal{W}_i$. By definition, $\mathcal{G}^{k+1}$is the countable intersection of open sets, and, in particular, is a Borel set.

To see that $\mathcal{G}^{k+1}$ is finitely prevalent in $\mathcal{H}^{k+1}$, define $\mathcal{L} = \mathcal{P}^{2dr}$ and let $\mathcal{M}$ be any translate of $\mathcal{L}$. The construction of $\mathcal{G}^{k+1}$ and Proposition 6.3 guarantee that

$$\left(\mathcal{H}^{k+1} \setminus \mathcal{G}^{k+1}\right) \cap \mathcal{M} \subset \mathcal{M}^*$$

Hence Proposition 6.3 implies that $\mathcal{H}^{k+1} \setminus \mathcal{G}^{k+1}$ meets every translate of $\mathcal{L}$ in a set of Lebesgue measure 0. By definition, therefore, $\mathcal{H}^{k+1} \setminus \mathcal{G}^{k+1}$ is finitely shy in $\mathcal{H}^{k+1}$, and $\mathcal{G}^{k+1}$ is finitely prevalent in $\mathcal{H}^{k+1}$.

To see that $\mathcal{G}^{k+1}$ is residual in $\mathcal{H}^{k+1}$, let $F \subset C^{k+1}(S, \mathbb{R}^2)$ be any finite dimensional subspace that contains $\mathcal{P}^{2dr}$. It follows from Proposition 6.3 that $\mathcal{G}^{k+1} \cap F$ has full Lebesgue measure in $\mathcal{H}^{k+1} \cap F$; in particular, $\mathcal{G}^{k+1} \cap F$ is dense in $\mathcal{H}^{k+1} \cap F$. Because $C^{k+1}(S, \mathbb{R}^2)$ is the union of finite dimensional subspaces that contain F_0 , we conclude that $\mathcal{G}^{k+1}$ is dense in $\mathcal{H}^{k+1}$. Because our construction guarantees that $\mathcal{G}^{k+1}$ is the countable intersection of open sets, we conclude that it is residual in $\mathcal{H}^{k+1}$, as desired.

To construct $\mathcal{G}^1$ we proceed in almost the same way, except that we work in $\mathcal{H}^1$ instead of in $\mathcal{H}^{k+1}$. For each index m, let $C(m)$ be the set of pairs $(\mu^1, \mu^2) \in \mathcal{H}^1$ for which there exist $y \in \mathbb{R}^{d^1}$ with $|y| \leq m$ and a subset $Z \subset L_m$ such that

- for every $z \in Z$ there is a $\lambda \in \mathbb{R}$ such that $\mu^1(z) - \lambda[\mu^2(z) - y] = 0$ and $|\lambda| \leq m$
- the projection of Z into some d^2-1-dimensional subspace of $\mathbb{R}^d$ contains a ball of radius at least $1/m$.

It is straightforward to check that each $C(m)$ is a closed subset of $\mathcal{H}^1$, so the complement $\mathcal{H}^1 \setminus C(m)$ is open. Set

$$\mathcal{V}_1 = \bigcap_{m=1}^{\infty} \left[\mathcal{H}^1 \setminus C(m)\right]$$

We construct $\mathcal{V}_2$ in exactly the same way, except that the roles of μ^1, μ^2 are reversed. For each index m, let $K(m)$ be the set of pairs $(\mu^1, \mu^2) \in \mathcal{H}^1$ for which there exist $\widehat{s}^1 \in L_m$, $y \in \mathbb{R}^{d^1}$ with $|y| \leq m$, and a ball $B \subset S^2$ such that

- for every $b \in B$ there is a $\lambda \in \mathbb{R}$ such that $\mu^1(\widehat{s}^1, b) - \lambda y = 0$ and $|\lambda| \leq m$
- the radius of B is at least $1/m$

It is easy to see that $K(m)$ is closed, and hence that $\mathcal{H}^1 \setminus K(m)$ is open. Set

$$\mathcal{V}_3 = \bigcap_{m=1}^{\infty} \left[\mathcal{H}^1 \setminus K(m)\right]$$

We construct $\mathcal{V}_4$ in exactly the same way, except that the roles of μ^1, μ^2 are reversed.

Now set $\mathcal{G}^1 = \bigcap \mathcal{V}_i$. By definition, $\mathcal{G}^1$ is the countable intersection of open sets. In order to show that it is residual in $\mathcal{H}^1$, we need only show it is dense. To this end, view $C^{k+1}(S, \mathbb{R}^2)$ as a subset of $C^1(S, \mathbb{R}^2)$, and note that $\mathcal{G}^{k+1} \subset \mathcal{G}^1$ and $\mathcal{H}^{k+1} \subset \mathcal{H}^1$. Malgrange (1966) shows that $C^{k+1}(S, \mathbb{R}^2)$ is dense in $C^1(S, \mathbb{R}^2)$. Because $\mathcal{H}^{k+1}$ is open in $C^{k+1}(S, \mathbb{R}^2)$, it follows that $\mathcal{H}^{k+1}$ is dense in $\mathcal{H}^1$. Our above construction shows that $\mathcal{G}^{k+1}$ is dense in $\mathcal{H}^{k+1}$, and hence in $\mathcal{H}^1$. Because $\mathcal{G}^{k+1} \subset \mathcal{G}^1$ it follows that $\mathcal{G}^1$ is dense in $\mathcal{H}^1$. By construction, $\mathcal{G}^1$ is the countable intersection of open sets, so that, as asserted, it is residual in $\mathcal{H}^1$. ■

Remark 6.4 *The relevant property required of the space of polynomials of degree at most* $2dr$ *is embodied in Lemma 6.2: given* r *distinct points, we can find polynomials whose values and first partials can be specified arbitrarily at those points. Any other space with this property would do as well. Note, however, that the space of separable relative valuation functions does* ***not***

have this property: If μ is a separable relative valuation function and the first d^1 coordinates of s_1 and s_2 coincide then

$$\frac{\partial\mu}{\partial x^i}(s_1) = \frac{\partial\mu}{\partial x^i}(s_2)$$

for $1 \le i \le d^1$.

Proof for Example 3.7. If (ψ, t) is a non-trivial incentive compatible ex-post mechanism with continuous relative transfers τ^i, condition (5) must be satisfied: there is $\widehat{s} \in I$, $\Delta := \mu^i(\widehat{s})$ and $(y_k, y_l)^T \in \mathbb{R}^2$, such that for all $s \in \tilde{I}^i(\widehat{s})$

$$\begin{pmatrix} a_k^i + b_k^i s_k^{-i} \\ -a_l^i - b_l^i s_l^{-i} \end{pmatrix} \text{ and } \begin{pmatrix} b_k^{-i} s_k^{-i} - y_k \\ -b_l^{-i} s_l^{-i} - y_l \end{pmatrix} \text{ are collinear}$$

For this to be true at some s, the cross product of these vectors must vanish, implying the following condition :

$$\left(a_k^i + b_k^i s_k^{-i}\right)\left(-b_l^{-i} s_l^{-i} - y_l\right) - \left(-a_l^i - b_l^i s_l^{-i}\right)\left(b_k^{-i} s_k^{-i} - y_k\right) = 0. \qquad (16)$$

We now argue, that the above condition can be satisfied for all s in the set $I^i(\widehat{s})$ only if the coefficients a, b satisfy the algebraic condition (7).

The one-dimensional indifference set $I^i(\widehat{s})$ can be parametrized by $s_k^{-i} = \frac{\Delta}{a_k^{-i} + b_k^{-i}\widehat{s}_k^i} + \frac{a_l^{-i} + b_l^{-i}\widehat{s}_l^i}{a_k^{-i} + b_k^{-i}\widehat{s}_k^i} s_l^{-i}$, where $\Delta = \mu^{-i}(\widehat{s})$. As $a_k^{-i}, b_k^{-i}, a_l^{-i}, b_l^{-i} \neq 0$, we can assume w.l.o.g. that $a_k^{-i} + b_k^{-i}\widehat{s}_k^i \neq 0$, and $a_l^{-i} + b_l^{-i}\widehat{s}_l^i \neq 0$. Substituting for s_k^{-i} in condition (16), we see that this equation can only hold on all of $I^i(\widehat{s})$ if the coefficient of the quadratic term in s_l^{-i} vanishes, i.e. if $\frac{a_l^{-i} + b_l^{-i}\widehat{s}_l^i}{a_k^{-i} + b_k^{-i}\widehat{s}_k^i}\left(-b_k^i b_l^{-i} + b_l^i b_k^{-i}\right) = 0$. This implies condition (7). Finally, for the case of discontinuous transfers τ^i, condition (6) reduces here to $b_k^i = b_l^i = 0$, so that condition (7) is satisfied. ■

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