The Organization of Production, Consumption and Learning

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Abstract This paper provides an extension of general equilibrium theory that incorporates the actions of individuals both as demanders and suppliers of goods and as members of firms, schools, social groups, and contractual relationships. The central notion of the paper is a *group*: a collection of individuals associated with one another for some purpose. The model takes as primitive an exogenous set of group types, interpretable as (potential) firms, schools, social groups, contracts etc. The types of schools and firms that materialize in equilibrium, as well as the way that agents acquire skills, are determined endogenously in a competitive market, as are the contracts they enter into, and the production and consumption of private commodities. Equilibrium exists and the core coincides with the set of equilibrium states. Examples and Applications illustrate the flexibility and power of the framework.[5]

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1 Introduction

In the usual general equilibrium model, individuals interact with the market but not with each other. In this paper we develop a model in which individuals interact both with the market and with each other — in firms, schools, social groups, and contractual relationships.

The central notion of this paper is a *group*: a collection of individuals who interact with one another for some purpose. (We could also call a group a *relationship* or an *organization*.) A group is determined by the characteristics of its members, by the inputs it uses and the outputs it produces, by the services it provides to its members, its infrastructure, and its governance or organizational structure. Some groups are productive (firms), some are educational (schools), some are social, some are contractual, and some have all of these aspects. Individuals typically belong to many groups.

In our model, there are two broad classes of commodity: the standard commodities of general equilibrium theory (private goods for short) and memberships in groups. Both private goods and group memberships enter into preferences, are objects of choice, and are priced. Private goods (but not group memberships) may be used as inputs or produced as outputs by a group. Within a group, memberships are distinguished by their *membership characteristic*. The prices of memberships within a group may be interpreted as the sharing of costs and revenues and as transfers among members. At equilibrium, the markets for private goods clear, taking into account the inputs and outputs of the groups that form, and membership choices are consistent across the population.

With standard assumptions (including a continuum of agents), equilibrium exists and passes a test of perfect competition (coincidence of core states and equilibrium states). The patterns of consumption and groups that emerge — the firms, schools, organizations and social and contractual relationships that form — are determined endogenously in a competitive market.

The model presented here modifies the clubs model presented in Ellickson, Grodal, Scotchmer and Zame [6] in two ways:

- We allow groups to produce outputs of standard commodities.
- Characteristics are associated with memberships, not individuals, and we allow for joint restrictions on choices of private goods and group memberships.

The first modification allows us to model the production of physical outputs. Groups can also produce services, but a service is not a standard commodity, and hence not an output of the group. The second modification allows much more flexibility in how we interpret memberships and groups. Characteristics can describe both roles within groups and attributes of individuals required to fill those roles. These attributes can be either acquired or innate.

The changes to the formal model required to accommodate these extensions are quite modest. The first change requires only that we replace the input vector of our earlier model with an input-output vector. The second change requires only that we drop the assumption in our earlier model that agents must have the same characteristic in every group to which they belong, instead allowing agents to choose different membership characteristics in different groups.

Although the modifications to the formal structure are modest, they greatly extend the scope of our framework. Groups and memberships can be interpreted in many ways. When the group is a "firm" it is natural to interpret members as "workers" or "supervisors," according to their roles, and to interpret (negative) prices for memberships as "wages." When the group is a "school" it is natural to interpret members as "teachers" or "students" and to interpret (negative) prices for teacher memberships as "salaries" and (positive) prices for student memberships as "tuition." When the group is a service, it is natural to interpret members as either "providers" or "clients" and to interpret prices as "fees", which are negative for providers and positive for clients. In all these situations, membership prices will reflect the market values of the inputs used and outputs produced, but membership prices will also reflect externalities within the group. As we stress below, firms that offer disagreeable working conditions or uncongenial co-workers will be forced to pay higher wages in order to attract employees. The examples and applications we present in Section 5 illustrate these and other instances of our model.

Membership characteristics embody both roles in groups and the skills necessary to qualify for (fulfill) those roles. Some skills are innate, and others must be learned – in private study, in schools, or in apprenticeships. We model learning as the ways agents qualify for memberships. For example, for skills that are not innate:

- Agents can qualify for a membership by consuming certain standard goods such as a home computer or a textbook.
- Agents can qualify for a membership by choosing other memberships such as attending school or serving an apprenticeship.

These possibilities are built into consumption sets. Consumption sets will typically differ across agents to reflect different capabilities to qualify for memberships, and for some agents some memberships may be simply beyond their reach. In the capitalist example of Section 6.3, for instance, the capitalist member must bring capital to the group.

There is no standard model of learning in general equilibrium theory, so we cannot compare our model of learning to a standard model. However there are standard models of production, and we should say something about what sets our treatment apart.

- In McKenzie's [14] formulation, there are no firms — only an aggregate constant-returns-to-scale aggregate production technology — and no profits. In the Arrow-Debreu [1] formulation, there are firms and profits, but the firms have no members — only shareholders.

- In both the McKenzie and Arrow-Debreu formulations, labor is a commodity just like every other commodity, and is priced just like every other commodity. In our model, labor is not a commodity. Workers are members of a firm, and their membership (which is a commodity) is priced in the same way as any other membership with compensation varying according to the role the member plays and the skills he brings to the job.
- In the standard model, there is no sense in which workers belong to a firm or care about working conditions. In our model, members of a group might care about every aspect of the group environment.

As in our paper [6], the economies considered here have a continuum of agents. This framework handles smoothly the difficulties that arise because club memberships are indivisible and because membership choices must be consistent across the population. For instance, if a type of firm requires two programmers and a lawyer, then there must be twice as many agents who choose to be programmers in that type of firm as agents who choose to be lawyers in that type of firm. The consistency problem must be solved in a context where agents can belong to many groups, as may be necessary for acquiring skills in some groups and using those skills in other groups.

Our main formal results are that equilibrium exists, that equilibrium states belong to the core, and that core states can be decentralized by prices (core equivalence). Because the present model is closely related to our earlier model, the proofs of these results require only small changes from the proofs of the corresponding results in [6], and so are omitted. (For economies with a finite number of agents, the techniques of Ellickson, Grodal, Scotchmer and Zame [7] could be adapted to prove that approximate equilibria exist, that approximate equilibrium states belong to the core, and that approximate core states can be approximately decentralized by prices.) We view the framework — rather than the theorems or the proofs — as the most important contribution of the present paper.

The theory we develop has some features in common with the competitive theory of labor management set forth by Keiding [11] and Drèze [5], but there are many important differences. As do we, Keiding [11] works in a continuum model and accommodates differentiated labor skills. However, in Keiding's model, labor skills are given exogenously and fixed (rather than acquired), there are no firms (agents have access to a single production technology), and agents care only about their own consumption of private goods and the fraction of their labor endowment delivered to the firm. As a result, there is no matching problem. Private goods are traded in competitive markets and labor is supplied cooperatively, but in equilibrium, it is "as if" there are competitive prices for the non-marketed labor services. In Drèze [5], labor skills are differentiated, and each group of agents has access to an exogenously given production set, but agents do not care about the composition of the groups in which they work, and again there is no matching problem.

Our model also has something in common with cooperative theories of coalition production, but again there are many important differences. Böhm [3], for instance specifies a production possibility set for every coalition of agents but focuses on the core, where the coalition of the whole forms and only the production possibility set of the coalition of the whole is used. In this sense, at least, there are no firms and no matching problem. Our model is closer in spirit to Ichiishi [10], in the sense that he focuses on the core of a coalition structure, so individuals really do "belong" to firms, and treats labor as supplied through membership in firm (rather than as a standard commodity as Böhm [3] does. However, Ichiishi's emphasis is on a cooperative solution, which reflects his purpose, rather than on a competitive price-taking solution, as in our model. Equally importantly, Ichiishi's model reflects a view of the world in which each agent chooses to belong to a single productive coalition, whereas our model reflects a view of the world in which each agent may choose to belong to many productive groups (indeed, it may be necessary to belong to one group in order to acquire the skills necessary to belong to another group). In Ichiishi's model, therefore, it is appropriate to consider firm structures that are partitions of the set of agents, while in our model it is appropriate to consider firm structures that are *not* partitions of the set of agents.

Several other papers bear a closer relationship to the present paper. Makowski [12] interprets clubs as organizations formed by entrepreneurs. Cole and Prescott [4] provides an integration of club theory and general equilibrium theory, which accomplishes some of the same things as [6], but their approach is to view the objects of choice as divisible lotteries over club memberships and consumption bundles, rather than as indivisible club memberships and divisible consumption bundles. Closest in spirit is Prescott and Townsend [16], which views clubs as productive units and focuses on moral hazard.

In Section 2 we preview our model with a whimsical example. The model is presented in Section 3, and Section 4 presents the main Theorems. Section 5 presents examples that show some of the power of our model. The first example, which formalizes the whimsical example of Section 2, shows how learning can be modeled through apprenticeship, and how groups can be interpreted as firms producing services. The second example, which concerns the transition in the industrial revolution from home production to factory production during the industrial revolution, shows how agents' preferences over working conditions influence the nature of equilibrium. The third example, which builds on the second, shows how our model can be used to articulate the organization of the firm. The fourth example shows how contractual issues can be represented in our framework and how competition influences the contracts chosen in equilibrium.

2 A Venetian Holiday

A whimsical example may help the reader understand the formal model to come. Consider trips on a Venetian gondola. Each trip requires the services of two gondoliers: one in the front, and one in the back. Each trip can accommodate two passengers: one in the front and one in the back. (For simplicity we assume that trips actually require two passengers, although it is certainly possible to imagine trips with a single passenger, or even none.) The trip may promise silence or it may promise singing by the rear gondolier. Trips may take place in the morning or in the afternoon.

To code gondola rides as groups in our framework, we must specify inputs and outputs, and characteristics of the memberships of each group. In the present context, the input is the use of a gondola, and there is no output (because we code the gondola ride as part of the description of the group). We distinguish 4 group types: the first consists of a front and a rear gondolier and a front and a rear passenger, with a specification that the trip will take place in the morning and promise silence; the second consists of a front and a rear gondolier and a front and a rear passenger, with a specification that the trip will take place in the morning and promise singing; the third and fourth substitute "afternoon" for "morning." (Note that we follow the usual general equilibrium practice of incorporating time by dating the commodities — or services in this case.) In the first and the third group type there are the same family of membership characteristics: front passenger, rear passenger, front gondolier, and rear gondolier. In the second and fourth group type the membership characteristics are front passenger, rear passenger, front gondolier, and rear singing gondolier

Hence we in total have 5 membership characteristics. Using obvious notation, we write gm, gsm, ga, gsa for the 4 group types (writing gsa to represent a gondola ride, with singing, in the afternoon, and so forth) and we distinguish 16 (kinds of) memberships in the 4 group types:

$$\begin{aligned}&(FG, gm), (RG, gm), (FP, gm), (RP, gm)\\&(FG, gsm), (RGS, gsm), (FP, gsm), (RP, gsm)\\&(FG, ga), (RG, ga), (FP, ga), (RP, ga)\\&(FG, gsa), (RGS, gsa), (FP, gsa), (RP, gsa)\end{aligned}$$

Note that these 16 kinds of memberships are distinct: front and rear gondoliers have different responsibilities, front and rear passengers have different views of the scenery, gondoliers and/or passengers may prefer silence or singing, afternoons are different from mornings. In our framework, memberships are objects of choice and are priced, and these 16 memberships might all have different prices.

To this point, we have said nothing about the feasibility of choices for various individuals. It is probably true that no special ability is required of passengers — apart from their physical presence in Venice — but some special ability is surely required of gondoliers. (It might even be that different abilities are required of front and rear gondoliers, a possibility we ignore here.) We might therefore imagine that society consists of two sub-populations: Venetians, who are born knowing how to operate a gondola (or have acquired that skill before our story opens), and Tourists, who do not have and cannot acquire that skill. Our formalization of this distinction is that consumption sets of Tourists allow the choice of passenger memberships but not of gondolier memberships, while consumption sets of Venetians allow the choice of gondolier memberships but not of passenger memberships.[6]

Of course, some additional special ability is also required to sing. Perhaps a class of Venetians is born with this ability (or have acquired that skill before our story opens), another class is not born with it but can acquire it by serving as the front gondolier on a singing trip, and yet a third class is not born with it and cannot acquire it. Our formalization is again in terms of consumption sets. For members of the first class, consumption sets allow either gondolier choice. For members of the third class, consumption sets do not allow choice of either (RGS, gsm) or (RGS, gsa): a non-singing Venetian cannot make a choice that requires singing. For members of the second class, the consumption set precludes the choice (RGS, gsm) and only allows the choice (RGS, gsa) in conjunction with the choice (FG, gsm): a Venetian who is not born with the ability to sing cannot make a choice that requires singing in the morning, and can only make a choice that requires singing in the afternoon if s/he acquires, in the morning, the ability to sing.

In our equilibrium notion, the 16 kinds of memberships in gondola rides are priced, and agents optimize given these prices The equilibrium conditions require that prices within each type of group must sum to the cost of inputs, and that membership choices are consistent across the population. (In particular, equal numbers of Tourists (respectively, Venetians) choose front and rear memberships in the morning, and so forth.) It is natural to guess that, at equilibrium, passengers pay positive prices, gondoliers pay negative prices (that is, gondoliers are paid by the passengers), and these prices generate a net surplus that exactly covers the cost of the gondola. We verify these guesses in Section 5.1 below.

3 General Equilibrium with Groups

We first extend our club model in [6] so that it applies to many different organizations: firms, contracts, social clubs, schools, etc. In order to make the present paper as self-contained as possible, we repeat some definitions and some motivation, highlighting the differences in square brackets. Instead of the terms *club types* and *clubs* used in the previous paper, we use here the terms *group types* and *groups*.

[6] For simplicity, we assume that Venetians cannot choose to be passengers rather than gondoliers.

3.1 Private Goods

There are $N \geq 1$ divisible, publicly traded private goods. Although we allow all kinds of private goods, we typically have physical goods in mind, rather than labor or other services, because we typically model these in the description of group types.

3.2 Groups and Memberships

Groups are described by an exogenous set of *group types.* To define group types, we begin with finite sets Ω of *membership characteristics* and Γ of *organizational characteristics.* A *group type* is a triple (π, γ, y) consisting of a *profile* $\pi : \Omega \to \mathbf{Z}_+ = \{0, 1, \ldots\}$, an organizational characteristic $\gamma \in \Gamma$, and an *input-output vector* $y \in \mathbf{R}^N$. We take as given a finite set of possible group types $\mathcal{G} = \{(\pi, \gamma, y)\}$.

As usual, we interpret negative components of y as inputs and positive components of y as outputs. We allow for the possibility that $y < 0$ (so that group formation requires inputs but produces no outputs, which would typically be the case for a group whose purpose is to provide a service) and for the possibility that $y > 0$ (so that production requires only the efforts of the members). [In our earlier paper we insisted that $y \leq 0$ so that club formation might require inputs but yielded no outputs.] For $\omega \in \Omega$, $\pi(\omega)$ represents the number of members with the membership characteristic ω that the group is required to have, and so $|\pi| = \sum_{\omega \in \Omega} \pi(\omega)$ is the total number of members that the group is required to have.

A membership characteristic might be anything that matters to the individuals who comprise a group or to the activity in which the group is engaged. In particular, a membership characteristic can encompass personal qualities (intelligence, appearance, personality, etc.), roles within the group (teacher, student, supervisor, skilled laborer, unskilled laborer, etc.) and skills (singing, dancing, language, etc.). Formally, all membership characteristics are acquired; membership characteristics that are innate (height for instance) are encompassed in our specification of consumption sets below. [In our earlier paper, we insisted that membership characteristics be innate and fixed.] Importantly, membership characteristics are observable and contractible. We emphasize that Ω is simply an abstract finite set. In particular, Ω is not a vector space and need not have any linear structure.

An organizational characteristic might be anything that matters to the potential member of a group apart from the characteristics of the members in the group and the input-output vector. In particular, an organizational characteristic can encompass the activity within the group (that is, the process used to produce output), the organizational hierarchy within the group, and the duties of each potential member of the group.[7]

[7] We could probably dispense with organizational characteristics, coding everything into membership characteristics. However, distinguishing the characteristics of

A *membership* is an opening in a particular group type corresponding to a particular membership characteristic. Formally, a membership is a pair $m = (\omega, (\pi, \gamma, y))$ such that $(\pi, \gamma, y) \in \mathcal{G}$ and $\pi(\omega) \geq 1$. We write $\mathcal{M}$ for the set of memberships; because the set $\mathcal{G}$ of group types is finite so is the set $\mathcal{M}$ of memberships.

Just as an agent chooses a bundle a private goods (possibly none), so an agent chooses a bundle of memberships (possibly none); to distinguish memberships from private goods we refer to a bundle of memberships as a *list*. Formally, a membership list is a function $\ell : \mathcal{M} \to \{0, 1, \ldots\}$; $\ell(m)$ specifies the number of memberships of type $m = (\omega, (\pi, \gamma, y))$. Write **Lists** for the set of lists.

3.3 Agents

The set of agents is a nonatomic finite measure space $(A, \mathcal{F}, \lambda)$. That is, A is a set, $\mathcal{F}$ is a σ-algebra of subsets of A, and λ is a non-atomic measure on $\mathcal{F}$ with $\lambda(A) < \infty$.

A complete description of an agent $a \in A$ consists of a consumption set, an endowment vector of private goods and a utility function. Agent a's *consumption set* X_a specifies the feasible pairs of bundles of private goods and lists of memberships that the agent may choose. We assume that X_a has the following properties:

- $X_a \subset \mathbf{R}_+^N \times \mathbf{Lists}$
- if $(x_a, \mu_a) \in X_a$ and $x'_a \geq x_a$ then $(x'_a, \mu_a) \in X_a$
- there exists $M > 0$ such that for all $a \in A$ and all $(x_a, \mu_a) \in X_a$

$$\sum_{m \in \mathcal{M}} \mu_a(m) \leq M$$

The first two requirements are familiar: private good consumption must be non-negative, and increased consumption of private goods is always possible. [In our earlier paper, we insisted that non-negativity be the only constraint on consumption of private goods.] The last assumption provides a bound on the number of memberships that each agent can choose. In particular, for each agent $a \in A$ the set

$$\mathbf{Lists}_a = \{\mu_a \in \mathbf{Lists} : \text{ there exists } x_a \in \mathbf{R}_+^N \text{ s.t. } (x_a, \mu_a) \in X_a\}$$

is finite.

A choice (x_a, μ_a) is in X_a if it is possible for agent a to consume the bundle of private goods x_a and fulfill the requirements of the memberships specified by μ_a. The consumption set X_a encodes restrictions on the choices

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the organization from the characteristics of its members seems quite natural in applications, so we have chosen to build the distinction into the theory.

that a can make with respect to both private goods and memberships. For instance, we encode the information that a is short (and that this condition is immutable) by insisting that if $(x_a, \mu_a) \in X_a$ then $\mu_a(\omega, g) = 0$ whenever the characteristic ω includes being tall. We encode the information that b cannot read (but that this condition is remediable) by insisting that if $(x_b, \mu_b) \in X_b$ and $\mu_b(\omega, g) > 0$ for some characteristic ω that includes being able to read then $\mu_b(\omega', g') > 0$ for some membership (ω', g') that teaches b to read. And we encode the information that c must own a violin to be concertmaster of an orchestra by insisting that if $(x_c, \mu_c) \in X_c$ and $\mu_c(\omega, g) > 0$ for a concertmaster membership then x_c includes at least 1 violin. (It is for this reason that we do not assume that $X_a = \mathbf{R}_+^N \times \mathbf{Lists}_a$.)

Much of the flexibility of our model arises from the fact that an agent can choose (memberships with) different characteristics in different groups. In particular, we allow an agent to fill different roles in different groups — to provide barber services in one group and receive them in another. The possibility of filling different roles in different groups is essential to the way we model the acquisition of skills, which will typically be acquired in one venue and applied in another. Of course, the acquisition of skills usually precedes their application; we incorporate the temporal element by viewing the date or time period as part of the description of a group, just as traditional general equilibrium theory frequently views the date or time period as part of the description of a commodity.

Agent a's *endowment* is $(e_a, 0) \in X_a$. Note that agents are endowed with private goods but not with group memberships, and that survival without group memberships is possible.

Agent a's *utility function* $u_a : X_a \to \mathbf{R}$ is defined over private goods consumptions and lists of group membership. We assume throughout that, for each $\mu_a \in \mathbf{Lists}$

$$u_a(\cdot, \mu_a) : \{x_a | (x_a, \mu_a) \in X_a\} \to \mathbf{R}$$

is continuous and strictly monotone; i.e., utility is strictly increasing in consumption of private goods. We make no assumptions about the way in which utility depends on the choice of group memberships.

3.4 Economies

An *economy* $\mathcal{E}$ is a mapping $a \mapsto (X_a, e_a, u_a)$ for which:

- the consumption set correspondence $a \mapsto X_a$ is a measurable correspondence
- the endowment mapping $a \mapsto e_a$ is an integrable function
- the utility mapping $(a, x, \ell) \mapsto u_a(x, \ell)$ is a jointly measurable function of its arguments

For convenience, we will sometimes make the simplifying assumption that the *aggregate endowment* $\bar{e} = \int_A e_a \, d\lambda(a)$ is strictly positive, so all private goods are represented in the aggregate. (Admittedly, this is not a very satisfactory assumption in a production economy.)

3.5 States

A *state* of an economy is a measurable mapping

$$(x, \mu) : A \to \mathbf{R}^N \times \mathbf{R}^{\mathcal{M}}$$

A state specifies choices of private goods and of group memberships for each agent. Feasibility of a state of the economy is defined as feasibility of the consumption and production plans and consistent matching of agents.

Our example of singing gondoliers provides a convenient framework for understanding consistent matching of agents. A gondola ride requires a front and rear gondolier and a front and rear passenger. Consistent matching means that there are no trips with empty seats. If agents can only choose one gondola trip, then consistent matching could be expressed by the requirement that the space of agents who choose some gondola trip can be partitioned into a disjoint family of four-member sets, each containing one front gondolier, one rear gondolier, one front passenger and one rear passenger. In our example, however, some agents will choose more than one trip (because they learn to sing in the morning), and a simple description in terms of partitions will not work. Instead, we can express consistent matching by the requirement that the "number" of agents who choose to be a front gondolier on some trip equal the "number" of agents who choose to be a rear gondolier on some trip, and so forth.[8] In a continuum framework, it is not meaningful to speak of "numbers" when the sets in question are infinite, but we can express the same idea by requiring that the fraction of the entire population who choose to be a front gondolier on some trip equal the fraction of the entire population who choose to be a rear gondolier on some trip, and so forth.

It is convenient to formalize this idea in terms of the aggregate of choices. To this end, define an *aggregate membership vector* to be an element $\bar{\mu} \in \mathbf{R}^{\mathcal{M}}$. An aggregate membership vector $\bar{\mu}$ is *consistent* if for every group type $(\pi, \gamma, y) \in \mathcal{G}$, there is a real number $\alpha(\pi, \gamma, y)$ such that

$$\bar{\mu}(\omega, (\pi, \gamma, y)) = \alpha(\pi, \gamma, y)\pi(\omega)$$

for each $\omega \in \Omega$. Given a measurable set $B \subset A$ and a measurable choice function $\mu : B \to$ **Lists**, we say that μ is *consistent for* B if the aggregate membership vector $\int_B \mu_a \, d\lambda(a)$ is consistent.

The state (x, μ) is *feasible for* the measurable subset $B \subset A$ if

[8] In a finite economy, this is not quite enough, because we must be careful that no agent chooses to be both a front gondolier and a rear gondolier on the same trip, but in the continuum framework this problem does not arise; see the discussion in Ellickson, Grodal, Scotchmer and Zame [6].

(i) **Individual feasibility**

$$(x_a, \mu_a) \in X_a \quad \text{for each } a \in B$$

(ii) **Material balance**

$$\int_B x_a \, d\lambda(a) \le \int_B e_a \, d\lambda(a) + \int_B \left[\sum_{(\omega,(\pi,\gamma,y))} \mu_a(\omega, (\pi, \gamma, y)) \frac{y}{|\pi|} \right] d\lambda(a)$$

(iii) **Consistency** $\int_B \mu_a \, d\lambda(a)$ is consistent for B.

That is, (x, μ) is feasible for B if individuals choose in their consumption sets, private consumption does not exceed the sum of endowments and net production, and agents are matched consistently. The state (x, μ) is *feasible* if it is feasible for the set A itself. If (x, μ) is a state of the economy, $m = (\omega, g)$ is a membership and $\mu_a(m) > 0$, we say *a chooses* the membership m.

If (x, μ) is consistent for B and $\int_B \mu_a \, d\lambda(a) = \alpha(\pi, \gamma, y)\pi(\omega)$ for each $\omega \in \Omega$, then material balance is equivalent to the assertion that

$$\int_B x_a \, d\lambda(a) \le \int_B e_a \, d\lambda(a) + \sum_{(\pi,\gamma,y)\in\mathcal{G}} \alpha(\pi, \gamma, y)\, y$$

Because members of a group care only about the membership characteristics of other members, and not about their identities, it is not necessary to identify the agents belonging to each individual group.

3.6 Group Equilibrium

Both private goods and group memberships are priced, so prices (p, q) lie in $\mathbf{R}^N \times \mathbf{R}^{\mathcal{M}}$; p is the vector of prices for private goods and q is the vector of prices for group memberships. Because utility functions are assumed monotone in private goods, private goods prices will be positive in equilibrium, but prices of group memberships may be positive, negative or zero. Because a membership specifies both a group type and a membership characteristic, membership prices depend both on the group type and the membership characteristic.

A *group equilibrium* consists of prices $(p, q) \in \mathbf{R}^N_+ \times \mathbf{R}^{\mathcal{M}}$ with $p \ne 0$ and a feasible state (x, μ) such that

(1) **Budget balance for group types** For each $(\pi, \gamma, y) \in \mathcal{G}$:

$$\sum_{\omega\in\Omega} \pi(\omega) \;\; q(\omega, (\pi, \gamma, y)) + p \cdot y = 0$$

(2) **Budget feasibility for agents** For almost all $a \in A$:

$$(p, q) \cdot (x_a, \mu_a) \le p \cdot e_a$$

(3) **Optimization by agents** For almost all $a \in A$:

$$(x'_a, \mu'_a) \in X_a \text{ and } u_a(x'_a, \mu'_a) > u_a(x_a, \mu_a) \Rightarrow (p, q) \cdot (x'_a, \mu'_a) > p \cdot e_a$$

Thus, at an equilibrium the sum of membership prices in a given group type just balances the value of the input-output vector, and individuals optimize subject to their budget constraints. (Recall that feasibility of the state (x, μ) already entails material balance.) A *group quasi-equilibrium* satisfies (1), (2) and the weaker condition

(3′)**Quasi-Optimization** For almost all $a \in A$:

$$(x'_a, \mu'_a) \in X_a \text{ and } u_a(x'_a, \mu'_a) > u_a(x_a, \mu_a) \Rightarrow (p, q) \cdot (x'_a, \mu'_a) \geq p \cdot e_a$$

As usual, the difference between equilibrium and a quasi-equilibrium is that at the latter agents do not necessarily optimize in their budget sets but only choose consumption bundles that are not dominated by any consumption bundle that costs strictly less than their wealth. Evidently every equilibrium is a quasi-equilibrium; we give conditions in Section 4 that guarantee that every quasi-equilibrium is an equilibrium.

3.7 Pricing Relevant Characteristics

The description of a membership in a group is very detailed: it includes the characteristics of the given membership and of the other memberships in the group, the purpose and organization of the group, and the input-output vector. Because we allow individuals to care about all these aspects, we must allow prices to depend on all these aspects as well; if we did not, we could easily find examples in which some core states could not be decentralized by prices and indeed in which equilibrium did not exist. However, in certain circumstances, it may happen that some aspects of membership are irrelevant to agents in the economy, and in those circumstance we can conclude that prices do not distinguish between such aspects. Indeed, if, in equilibrium, there is a set of positive measure of agents, each of whom chooses (in equilibrium) the membership m and finds the membership m' to be a perfect substitute for m, then m' must be at least as expensive as m: $q(m') \geq q(m)$. And if there is also a set of positive measure of agents, each of whom chooses the membership m' and finds the membership m to be a perfect substitute for m', then m, m' must have the same price: $q(m') = q(m)$.

The pricing of irrelevant aspects highlights one of the distinctions between the present framework, in which memberships chosen by a particular individual may display different characteristics in different groups, and the framework in [6], in which individuals bring the same characteristics to each group to which they belong. As a consequence, in our earlier framework irrelevant aspects of an individual may seem relevant to membership prices, when in fact they are not. Our present framework facilitates a much more direct connection between membership prices and the attributes of memberships that matter.

To illustrate, consider writing a paper with two coauthors, one Danish and the other English, and imagine the only thing that matters is that both authors speak the same language. In the present framework, we need only consider two types of group, d (Danish-speaking) and e (English-speaking), and two types of membership, D (Danish-speaking) and E (English speaking). Assuming coauthorship requires no inputs, we would, with the obvious notation, identify the group types as $((D,D),d,0)$, $((E,E),e,0)$ and the memberships as $(D,((D,D),d,0))$, $(E,((E,E),e,0))$. In the framework of [7], however, we must formally distinguish individuals who speak both Danish and English from individuals who speak only one language or the other, and must then consider six kinds of partnership and eight kinds of membership; with the obvious notation the partnerships would be

$$((D,D),d,0) \qquad ((E,E),e,0) \qquad ((DE,D),d,0)$$

$$((DE,E),e,0)) \qquad ((DE,DE),d,0)) \qquad ((DE,DE),e,0)$$

and the memberships would be

$$(D,((D,D),d,0)) \qquad (E,((E,E),e,0)) \qquad (DE,((DE,D),d,0))$$

$$(D,((DE,D),d,0)) \qquad (DE,((DE,E),e,0))) \qquad (E,((DE,E),e,0))$$

$$(DE,((DE,DE),d,0)) \qquad (DE,((DE,DE),e,0))$$

Of course, if there are eight kinds of memberships there must formally be eight membership prices. However, if no one cares whether their partner can speak two languages, many of these memberships will be perfect substitutes and the corresponding equilibrium membership prices will coincide: $q(DE,((DE,D),d,0)) = q(D,((D,D),d,0))$ (assuming both partnerships are chosen in equilibrium), etc.

4 Theorems

We say the feasible state (x,μ) is *Pareto optimal* if there is no feasible state (x',μ') such that $u_a(x'_a,\mu'_a) > u_a(x_a,\mu_a)$ for almost every $a \in A$; we say (x,μ) is *strongly Pareto optimal* if there is no feasible state (x',μ') such that $u_a(x'_a,\mu'_a) \geq u_a(x_a,\mu_a)$ for almost every $a \in A$ and $u_b(x'_b,\mu'_b) > u_b(x_b,\mu_b)$ for all b in some subset $B \subset A$ of positive measure. Similarly, we say the feasible state (x,μ) is in the *core* if there is no subset $B \subset A$ of positive measure and state (x',μ') that is feasible for B such that $u_b(x'_b,\mu'_b) > u_b(x_b,\mu_b)$ for almost every $b \in B$; we say (x,μ) is in the *strong core* if there is no subset $B \subset A$ of positive measure and state (x',μ') that is feasible for B such that $u_b(x'_b,\mu'_b) \geq u_b(x_b,\mu_b)$ for almost every $b \in B$ and $u_c(x'_c,\mu'_c) > u_c(x_c,\mu_c)$ for every c in some subset $C \subset B$ of positive measure.

Of course the Pareto set contains the strong Pareto set and the core contains the strong core. For pure exchange economies in which consumption sets are the positive orthant and preferences are strictly monotone, the Pareto set and strong Pareto set coincide and the core and strong core coincide. In our context, however, the strong Pareto set may be a proper subset of the Pareto set and the strong core may be a proper subset of the core.[9] However a natural assumption provides a simple way around this problem.

Say that *endowments are desirable* if for every agent a and every consumption choice $(x_a, \mu_a) \in X_a$ for which $u_a(x_a, \mu_a) > u_a(e_a, 0)$, there exists $x'_a \leq x_a, x'_a \neq x_a$ such that $(x'_a, \mu_a) \in X_a$.[10]

Proposition 1. *If endowments are desirable then the strong Pareto set coincides with the Pareto set and the strong core coincides with the core.*

The First Welfare Theorem follows by the usual straightforward argument (but the Second Welfare Theorem may fail; see [6]).

Theorem 1. *Every group equilibrium state belongs to the core and in particular is Pareto optimal. If endowments are desirable then every group equilibrium state belongs to the strong core and in particular is strongly Pareto optimal.*

As in the exchange case, a quasi-equilibrium (x, μ) need not be an equilibrium if a positive measure set of agents B are in the minimum expenditure situation. (That is, for agents $b \in B$ there is no bundle of private commodities x'_b such that $x'_b \leq x_b, x'_b \neq x_b$ and $(x'_b, \mu_b) \in X_b$.) In the exchange case, irreducibility rules out this possibility; a similarly-motivated condition will rule it out in our setting also.

Let $\mathcal{E}$ be a group economy and let (x, μ) be a feasible state. Let $I \subset \{1, \cdots, N\}$ be a non-empty set of private goods. Say that the feasible state (x, μ) is a *minimum consumption configuration for good* i if for almost all agents $a \in A$ there does not exist a bundle x'_a of private goods such that $x'_a \leq x_a, x'_{ai} < x_{ai}$ and $(x'_a, \mu_a) \in X_a$. (If $(0, \mu_a) \in X_a$ then a feasible state is a minimum consumption figuration for good i only if the entire social endowment of i is used in group formation.) Say that (x, μ) is *group linked* if for every partition $\{1, \cdots, N\} = I \cup J$ of the set of consumption goods for which (x, μ) is a minimum expenditure configuration for each good $i \in I$, then for

[9] In the exchange case, the arguments for equality of the Pareto set and strong Pareto set, and for equality of the core and strong core, are familiar. The essential point is that if we are given allocations x, x' such that x is weakly preferred to x by some set of agents and strictly preferred by some subset of these agents, then we can tax the latter agents and redistribute the proceeds, obtaining an allocation x'' that is strictly preferred to x by all agents in the set. In our context, however, given states $(x, \mu), (x', \mu')$ such that (x', μ') is weakly preferred to (x, μ) by some set of agents and strictly preferred by some subset of those agents, we may find that in the state (x', μ') the latter group of agents consumer no private goods — or consume a bundle that is minimal in their consumption set, given the group membership choices — and hence cannot be taxed.

[10] The reader familiar with [6] will note that we have adapted this definition to allow general consumption sets.

almost every $a \in A$ there is a real number $r \in \mathbf{R}$ and an index $j \in J$ such that

$$u_a(e_a + r\delta_j, 0) > u_a(x_a, \mu_a)$$

(As usual, we write δ_j for the consumption bundle consisting of one unit of the private good j and nothing else.) We say that $\mathcal{E}$ is *group irreducible* if every feasible allocation is group linked.[11]

Proposition 2. *If $\mathcal{E}$ is group irreducible then every quasi-equilibrium is an equilibrium.*

In our continuum framework, equilibrium exists and passes a familiar test of perfect competition: coincidence of the core with the set of equilibrium states.

Theorem 2. *If $\mathcal{E}$ is group irreducible and endowments are desirable and uniformly bounded above then it admits a group equilibrium.*

Theorem 3. *If $\mathcal{E}$ is group irreducible and endowments are desirable and uniformly bounded above then the core coincides with the set of group equilibrium states.*

The (omitted) proofs of these results follow closely the proofs of the corresponding results in [6]; the only changes necessary are the very minor ones necessary to incorporate the small differences in formal structures.

- In our earlier work we allow for inputs to group formation; here we allow for inputs and outputs. This difference requires only that we extend our accounting to keep track of inputs and outputs.
- In our earlier work we insist that consumption sets be of the form $X_a = \mathbf{R}^\mathbf{L} \times \mathbf{Lists_a}$; here we allow for general consumption sets. This difference requires only that we be more careful about the distinction between quasi-equilibrium and equilibrium.
- In our earlier work we insist that agents choose only memberships corresponding to a particular (given and immutable) external characteristic; here the characteristics are attached to the memberships instead of to the agents, and the agents are allowed to assume different characteristics in different groups. However, aside from allowing more general consumption sets, this difference requires no changes in the argument.

[11] The reader familiar with [6] will note that we have adapted the definitions of club linked and club irreducible to take into account that we allow general consumption sets.

5 Examples and Applications

In this Section, we give a series of examples to illustrate some of the flexibility of our model: groups can be interpreted as apprenticeships, as firms producing personal services that are not traded on the market, as firms producing physical goods that are traded on the market, or as relationships governed by contracts — and some groups possess several of these aspects. The flexibility of our model relies heavily on the possibility that an agent can belong to several groups.

Example 5.1 elaborates our whimsical Venetian holiday (Section 2). The example illustrates that some skills are innate but that others can be acquired, and that skills can be acquired in one group (an apprenticeship, in this case) and applied in another group. Of course, the acquisition of skills typically precedes the application of those skills — a feature that is modeled by dating the groups.

In Example 5.1, the groups produce services which are not traded. In Examples 5.2 and 5.3, the groups produce physical goods which are traded. These examples illustrate how our model of production differs from the standard general equilibrium model (see our discussion in the Introduction). In the setting considered here, which is motivated by Mokyr's [15] description of the historical shift of production from homes to factories during the industrial revolution 1760-1830, the difference is crucial because it enables us to incorporate the tension between the unpleasantness of working conditions and the productivity gains that characterized factory production. As Example 5.2 shows, the resolution of this tension depends on parameters of the economy: if factory production is sufficiently more efficient then it may drive out home production — but for an open set of parameters, factory production and home production can co-exist.

Example 5.3 provides another difference between our model and the standard model. In the standard model, capital is purchased on the market just as is any other input, so there is no role for "capitalists." Example 5.3 provides such a role by differentiating between a worker-managed firm, in which capital is provided equally by all the members, and a capitalist-managed firm, in which capital is provided only by the lone capitalist. In our example, there is again a tension between the unpleasantness of working for a capitalist and the productivity gains possible in that organizational form. As before, the resolution of this tension depends on parameters of the economy: if capitalist-managed firms are sufficiently more efficient they may drive out worker-managed firms — but for an open set of parameters, capitalist-managed firms and worker-managed firms can co-exist.

Finally, Example 5.4 shows how a group can be interpreted as a contract, and why that is useful. Contracts are typically cast as bargaining problems, with the influence of the market appearing only in reservation payoffs. As this example illustrates, however, our framework permits us to analyze contracting directly within a competitive environment. In particular, we show how the terms of contracts will be determined by competitive market forces, and contrast the competitive outcome with the outcomes possible when the terms of contracts are determined by bargaining among the parties.

5.1 Venetian Holiday

As a simple illustration of the way our model works — especially the intertemporal acquisition of skills — and of the computation of equilibrium, we flesh out our whimsical example of Venetian gondola rides; see Section 2 and Subsection 3.7.

We identify 5 membership characteristics: front and rear passengers, front and rear gondoliers, and rear gondoliers who sing: FP, RP, FG, RG, RGS.[12] Similarly, we identify 4 group types: non-singing gondola trips in the morning and afternoon, singing gondola trips in the morning and afternoon. For simplicity, we assume gondola rides require no inputs (ignoring the necessary gondola), so with the obvious abuse of notation we have:

$$\begin{aligned} gm &= ((FP, RP, FG, RG), m, 0) \\ ga &= ((FP, RP, FG, RG), a, 0) \\ gsm &= ((FP, RP, FG, RGS), m, 0) \\ gsa &= ((FP, RP, FG, RGS), a, 0) \end{aligned}$$

Note that morning and afternoon trips are distinguished only by the organizational characteristic m, a.

There are a continuum of agents, of three kinds: Tourists T, Venetians who can sing VS, and Venetians who cannot sing V; with population masses $\lambda(T), \lambda(VS), \lambda(V)$ respectively. All agents are endowed with four units of the single consumption good.

Tourists can choose any non-negative quantity of the private good, and at most one gondola trip — as either a front or rear passenger — in the morning and at most one gondola trip in the afternoon. Write

$$\begin{aligned} \mathbf{Lists_T} = \Big\{ \ell : \ell(FP, gm) + \ell(RP, gm) + \ell(FP, gsm) + \ell(RP, gsm) \leq 1, \\ \ell(FP, ga) + \ell(RP, ga) + \ell(FP, gsa) + \ell(RP, gsa) \leq 1 \Big\} \end{aligned}$$

so that $X_t = \mathbf{R}_+ \times \mathbf{Lists_T}$ for each $t \in T$. Tourists care about consumption during the day, and about gondola rides; a gondola ride (as front or rear passenger, in the morning or afternoon) without singing doubles utility, a gondola ride with singing quadruples utility, but additional rides are of no value. Hence for each $t \in T$ and $(x, \ell) \in X_t$:

[12] Although it seems natural to distinguish rear gondoliers who sing, it is not really necessary, since membership in the group type which promises singing would distinguish them equally well. This and other modeling choices are largely matters of convenience and taste.

$$u_t(x,\ell) = \begin{cases} x \text{ if } & \sum_{\omega,g} \ell(\omega,g) = 0 \\ 2x \text{ if } & \sum_{\omega}[\ell(\omega,gm)+\ell(\omega,ga)] \geq 1 \\ \quad \text{but } & \sum_{\omega}[\ell(\omega,gsm)+\ell(\omega,gsa)] = 0 \\ 4x \text{ if } & \sum_{\omega}[\ell(\omega,gsm)+\ell(\omega,gsa)] \geq 1 \end{cases}$$

Venetians who can sing can choose any non-negative quantity of the private good, and can choose at most one gondola trip — as a front or rear gondolier, but not as a passenger[13] — in the morning and one in the afternoon. Write

$$\mathbf{Lists_{VS}} = \Big\{\ell : \ell(FG,gm)+\ell(RG,gm)+\ell(FG,gsm)+\ell(RGS,gsm) \leq 1, \\ \ell(FG,ga)+\ell(RG,ga)+\ell(FG,gsa)+\ell(RGS,gsa) \leq 1\Big\}$$

so that $X_v = \mathbf{R}_+ \times \mathbf{Lists_{VS}}$ for each $v \in VS$.

Venetians who cannot sing can choose any non-negative quantity of the private good, and can choose at most one gondola trip — as a front or rear gondolier — in the morning and one in the afternoon, but cannot choose to be a singing gondolier in the morning, and cannot choose to be a singing gondolier in the afternoon unless they have chosen to be a front gondolier in a singing gondola trip in the morning. Write

$$\mathbf{Lists_{V}} = \Big\{\ell : \ell(FG,gm)+\ell(RG,gm)+\ell(FG,gsm)+\ell(RGS,gsm) \leq 1, \\ \ell(FG,ga)+\ell(RG,ga)+\ell(FG,gsa)+\ell(RGS,gsa) \leq 1, \\ \ell(RGS,gsm) = 0,\ \ell(RGS,gsa) \leq \ell(FG,gsm)\Big\}$$

so that $X_v = \mathbf{R}_+ \times \mathbf{Lists_{V}}$ for each $v \in V$.

Venetians care about consumption but suffer disutility from providing services:

$$u_v(x,\ell) = x - A[\ell(FG,gm)+\ell(FG,gsm)+\ell(FG,ga)+\ell(FG,gsa)] \\ - B[\ell(RG,gm)+\ell(RG,ga)] \\ - C[\ell(RGS,gsm)+\ell(RGS,gsa)]$$

We take $0 \leq A \leq B \leq C$: the front of the gondolier is a less difficult post than the rear, and singing is additionally onerous.

The equilibrium prices and choices will depend on the proportion of each subpopulation and on the disutility parameters A, B, C. We assume here that $\lambda(T) > 2\lambda(V \cup VS)$ (which guarantees that some tourists obtain no rides), that $\lambda(V) > \lambda(VS)$ (which guarantees that not all Venetians who cannot sing can learn how), and that $0 \leq A \leq 1$, $A \leq B \leq A+1$ and $B \leq C \leq B+2$ (which

[13] Alternatively, we could allow Venetians to choose to be passengers but to derive no utility from such a choice; this would complicate notation, but lead to the same equilibrium outcomes.

guarantees that Venetians have the proper incentives to provide singing and non-singing trips, in the morning and in the afternoon, and to learn to sing).

To solve for equilibrium, we rely on two observations: a) in the relevant range, this is a transferable utility economy, so the equilibrium state maximizes social welfare, and b) at equilibrium, agents who are *ex ante* identical must obtain the same utility.

Taking the consumption good as numeraire, with price 1, let q be the equilibrium membership price function. Budget balance for group types entails:

$$\begin{aligned} q(FG, gm) + q(RG, gm) + q(FP, gm) + q(RP, gm) &= 0 \\ q(FG, gsm) + q(RGS, gsm) + q(FP, gsm) + q(RP, gsm) &= 0 \\ q(FG, ga) + q(RG, ga) + q(FP, ga) + q(RP, ga) &= 0 \\ q(FG, gsa) + q(RGS, gsa) + q(FP, gsa) + q(RP, gsa) &= 0 \end{aligned}$$

Some Venetians who cannot sing will not be able to learn; these Venetians provide non-singing services in the afternoon and in the morning, in the front or in the rear; Venetians who cannot sing but do learn provide services in the front of a singing ride in the morning and in the rear of a singing ride in the afternoon. All these must obtain the same utility, so:

$$\begin{aligned} &[-q(FG, gm) - A] + [-q(FG, ga) - A] \\ &= [-q(FG, gm) - A] + [-q(FG, gsa) - A] \\ &= [-q(FG, gm) - A] + [-q(RG, ga) - B] \\ &= [-q(RG, gm) - B] + [-q(FG, ga) - A] \\ &= [-q(RG, gm) - B] + [-q(FG, gsa) - A] \\ &= [-q(RG, gm) - B] + [-q(RG, ga) - B] \\ &= [-q(FG, gsm) - A] + [-q(RGS, gsa) - C] \end{aligned}$$

Some tourists obtain no rides, consume their endowments and obtain utility 4, so all tourists obtain utility 4. Keeping in mind that tourists are indifferent between morning and afternoon rides and front and rear seating, it follows that

$$\begin{aligned} q(FP, gm) = q(RP, gm) = q(FP, ga) = q(RP, ga) &= 2 \\ q(FP, gsm) = q(RP, gsm) = q(FP, gsa) = q(RP, gsa) &= 3 \end{aligned}$$

¿From the equations above, a little straightforward algebra yields the remaining membership prices (remember that negative prices are wages):

$$\begin{aligned} q(FG, gm) &= -2 + \frac{1}{2}(B - A) \\ q(RG, gm) &= -2 - \frac{1}{2}(B - A) \\ q(FG, ga) &= -2 + \frac{1}{2}(B - A) \end{aligned}$$

$$q(RG, ga) = -2 - \frac{1}{2}(B - A)$$
$$q(FG, gsm) = -(C - B) + \frac{1}{2}(B - A)$$
$$q(RGS, gsm) = -6 + (C - B) - \frac{1}{2}(B - A)$$
$$q(FG, gsa) = -2 + \frac{1}{2}(B - A)$$
$$q(RGS, gsa) = -4 - \frac{1}{2}(B - A)$$

(Our assumptions about the disutility parameters guarantee that providing gondolier services is preferred to not working.)

Note that if $B > \frac{1}{3}(2C + A)$ then $q(FG, gsm) > 0$: learning to sing is so valuable that some Venetians pay for singing lessons when they join the crew in the morning. Because the equilibrium is Pareto optimal, this human capital is efficiently acquired.

5.2 The Factory System

In this application we address the rise of the factory system, using the stimulating discussion in Mokyr [15] as motivation. According to Mokyr, when the site of production shifted from homes to factories during the industrial revolution of 1760 –1830, the consequences for the worker were profound. Other things equal, workers preferred working at home, but factories offered productivity gains through on-site training and team production.

We assume a continuum of agents and 2 commodities. The set of membership characteristics is $\Omega = \{W, M\}$ where W represents a worker and M a manager. There are two types of productive enterprise:

- In *domestic production* a firm consists of 2 workers, each working at home, and a manager. The group type is $g_1 = (\pi_1, \gamma_1, y_1)$ where $\pi_1 = (2, 1)$ and $y_1 = (-3, 3)$.
- In *factory production* a firm consists of two workers, working in a centralized factory, and an on-site manager. The group type is $g_2 = (\pi_2, \gamma_2, y_2)$ where $\pi_2 = (2, 1)$ and $y_2 = (-6, \alpha)$.

The 2-worker factory requires twice the input of the 2-worker firm, 3 units of materials (as with domestic production) and 3 units for building and equipment. Each domestic firm produces 3 units of output; each factory produces output $\alpha > 0$. We will refer to the parameter α as factory productivity.

All agents are *ex ante* identical. Each has endowment $e = (k, 0)$. We refer to k as the (per capita) wealth of the economy. No agent can join more than one firm. Preferences are described by the utility function:[14]

[14] Interpretation: workers do not like the regimen of factory life; managers have to appear on the factory floor to monitor, whereas with domestic production

$$u(x,\ell) = \begin{cases} 8\sqrt{x_1x_2} & \text{if the agent chooses no memberships} \\ 6\sqrt{x_1x_2} & \text{if he chooses a membership of type } (M,g_1) \\ 4\sqrt{x_1x_2} & \text{if he chooses membership } (W,g_1) \text{ or } (M,g_2) \\ 2\sqrt{x_1x_2} & \text{if he chooses membership } (W,g_2) \end{cases}$$

Equilibrium is described by prices p_1, p_2 for inputs and outputs and q for memberships, and choices for all agents; there is no loss in normalizing so that $p_1 = 1$. Since agents are *ex ante* identical, choices can be described by the fractions ρ_0 of agents choosing no membership and ρ_j of agents choosing a membership in a firm of type g_j $(j = 1, 2)$. We defer the calculations, and first describe equilibrium.

The nature of equilibrium depends on the endowment parameter k and the productivity parameter α. Figure 1 which subdivides the parameter space

$$\{(k,\alpha) \in \mathbf{R}^2 \mid (k,\alpha) > 0\}$$

into a number of regions, captures the most important features of the equilibrium correspondence.

The curve labeled B_1 is the graph of the function

$$\alpha = B_1(k) := \frac{54+63k}{9+7k} \qquad k > 0 \tag{1}$$

Similarly, the curve labeled B_2, which intersects the graph of B_1 at the point $(k,\alpha) = (9, 69/8)$, is the graph of the function

$$\alpha = B_2(k) := \frac{33k-21}{4k-4} \qquad k \geq 9 \tag{2}$$

(Note that B_2 is defined only for $k \geq 9$.)

Below the curve $\min\{B_1, B_2\}$ (Regions 2 and 3), where factory productivity is low, there is no factory production: agents either engage in domestic production or choose not to work. Above the curve B_1 (Region 4), where factory productivity is high, there is no domestic production: agents either engage in factory production or choose not to work.

On the curve B_1 (Region 1) and, for wealth $k > 9$, in the area bounded by the curves B_1 and B_2 (Region 5) domestic production and factory production coexist. According to Mokyr, such a split between working in factories and working at home was characteristic of most industries in which factories appeared during the Industrial Revolution. The most detailed evidence he presents comes from a 1906 census in France. Of workers working either in

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monitoring is not as demanding. This is simply an interpretation. One of the virtues of our approach is that what goes on inside a firm does not have to be modeled explicitly. Of course, it would be very interesting to connect our approach to the extensive literature on firm organization in which the monitoring and team production technology is made quite explicit. But that is for another time.

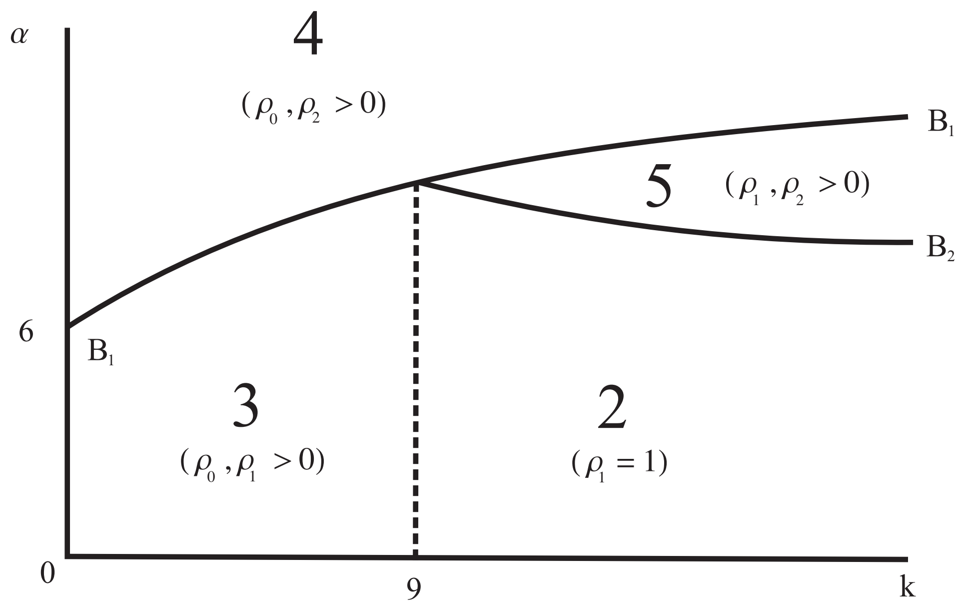

Fig. 1. The factory system

factories or at home, the per cent working in factories varied across industries as follows:[15]

- > 90 %: Chemicals, glass and pottery, iron and steel, printing, rubber and paper.
- 80 % – 89 %: Food processing, fine metals and jewelry, metalwork, textiles.
- 60 % – 69 %: Stone-cutting, wood and carpentry.
- 50 % – 59 %: Leather, straw and baskets.
- 30 % – 39 %: Apparel making.

Apparently even as late as the beginning of the Twentieth Century, the tug of working in the home was still powerful enough to offset the advantages of working in a factory.

To calculate equilibrium, it is convenient to work with indirect utilities. Recalling that $p_1 = 1$, indirect utility takes the form

$$v(p, q, k, \omega) = \begin{cases} V(0) & := 4k/\sqrt{p_2} & \text{if no memberships} \\ V(M, g_1) & := 3(k - q(M, g_1))/\sqrt{p_2} & \text{if } \omega = (M, g_1) \\ V(W, g_1) & := 2(k - q(W, g_1))/\sqrt{p_2} & \text{if } \omega = (W, g_1) \\ V(M, g_2) & := 2(k - q(M, g_2))/\sqrt{p_2} & \text{if } \omega = (M, g_2) \\ V(W, g_2) & := (k - q(W.g_2))/\sqrt{p_2} & \text{if } \omega = (W, g_2) \end{cases}$$

15 See Mokyr [15], p. 151, Table 2.

Recall that ρ_0 is the fraction of agents choosing no membership and ρ_j is the fraction choosing a membership in a firm of type g_j $(j = 1, 2)$. Budget balance requires

$$2q(W, g_1) + q(M, g_1) + 3p_2 - 3 = 0 \tag{3}$$

and

$$2q(W, g_2) + q(M, g_2) + \alpha p_2 - 6 = 0 \tag{4}$$

for firms of type g_1 and g_2 respectively. Market clearing for private commodity 1 requires

$$\begin{aligned} k = \rho_0 \left[\frac{k}{2}\right] &+ \rho_1 \left[\frac{2}{3}\left(\frac{k - q(W, g_1)}{2}\right) + \frac{1}{3}\left(\frac{k - q(M, g_1)}{2}\right)\right] \\ &+ \rho_2 \left[\frac{2}{3}\left(\frac{k - q(W, g_2)}{2}\right) + \frac{1}{3}\left(\frac{k - q(M, g_2)}{2}\right)\right] \\ &+ \rho_1 + 2\rho_2 \end{aligned}$$

To the left of the equality sign is the per capita endowment of commodity 1. On the right of the equality sign, the first two lines represent per capita demand by consumers and the third line per capita demand for inputs by firms. Substituting $\rho_0 + \rho_1 + \rho_2 = 1$ and the budget-balance equations, this market-clearing equation simplifies to

$$(p_2 + 1)\rho_1 + \left(\frac{\alpha p_2 + 6}{3}\right)\rho_2 = k \tag{5}$$

If $\rho_1 > 0$, then agents must be indifferent between working for or managing firms of type g_1. Setting $V(W, g_1) = V(M, g_1)$ implies

$$-2q(W, g_1) + 3q(M, g_1) = k \tag{6}$$

Similarly, if $\rho_2 > 0$, then equating $V(W, g_2) = V(M, g_2)$ yields

$$-q(W, g_2) + 2q(M, g_2) = k \tag{7}$$

Equations 3–7 provide the main ingredients for characterizing the equilibrium correspondence mapping the wealth-productivity parameters (k, α) to prices

$$(p_2, q(W, g_1), q(M, g_1), q(W, g_2), q(M, g_2))$$

and to the fraction ρ_0 of agents choosing not to work, the fraction ρ_1 engaging in domestic production as a worker or manager, and the fraction ρ_2 engaging in factory production as a worker or manager.

To derive the equilibrium correspondence, we begin with the case in which $\rho_0, \rho_1, \rho_2 > 0$: a positive fraction of agents belong to each type of firm and a positive fraction choose leisure. As we now show, this case corresponds to the curve B_1, the graph of equation 1, which we call Region 1. Agents are indifferent between leisure, working for or managing a firm of type g_1, and working for or managing a firm of type g_2. Solving

$$V(0) = V(W, g_1) = V(M, g_1) = V(W, g_2) = V(M, g_2)$$

yields equilibrium membership prices

$$q(W, g_1) = -k \quad q(M, g_1) = -\frac{k}{3} \quad q(W, g_2) = -3k \quad q(M, g_2) = -k$$

Equations 3 and 4 (budget balance for firms of type g_1 and g_2) imply

$$p_2 = \frac{9 + 7k}{9} = \frac{6 + 7k}{\alpha}$$

and hence

$$\alpha = B_1(k) := \frac{54 + 63k}{9 + 7k}$$

which is equation 1. From the market-clearing equation 5, the ρ_j must satisfy

$$(18 + 7k)\rho_1 + (36 + 21k)\rho_2 = 9k$$

as well as $\rho_0 + \rho_1 + \rho_2 = 1$. Solving these two equations for ρ_1 and ρ_2 as functions of ρ_0 yields

$$\rho_1 = \frac{12k + 36 - (21k + 36)\rho_0}{18 + 14k}$$
$$\rho_2 = \frac{2k - 18 + (7k + 18)\rho_0}{18 + 14k}$$

Imposing the restrictions $0 \leq \rho_1, \rho_2 \leq 1$ implies that

$$\rho_0 \in \left[\frac{18 - 2k}{18 + 7k}, \frac{12 + 4k}{12 + 7k}\right] \qquad \text{if} \quad k \leq 9$$

and

$$\rho_0 \in \left[0, \frac{12 + 4k}{12 + 7k}\right] \qquad \text{if} \quad k > 9$$

This characterizes the set of assignments (ρ_0, ρ_1, ρ_2) along the curve B_1.

Below $\min\{B_1, B_2\}$ no agents are engaged in factory production: either all are engaged in domestic production (Region 2) or they are split between engaging in domestic production and not working (Region 3). Suppose first that all agents are engaged in domestic production: $\rho_1 = 1; \rho_0 = \rho_2 = 0$. The market-clearing equation 5 implies $p_2 = k - 1$. Equations 3 and 6 imply

$$q(W, g_1) = \frac{9 - 5k}{4} \qquad q(M, g_1) = \frac{3 - k}{2}$$

The inequalities $V(W, g_1) \geq V(W, g_2)$ and $V(M, g_1) \geq V(M, g_2)$ imply

$$q(W,g_2) \geq \frac{9-7k}{2} \qquad q(M,g_2) \geq \frac{9-5k}{4}$$

Since $V(W,g_1) \geq V(0)$ implies $k \geq 9$, combining the inequalities above with equation 4 implies that

$$\alpha \leq B_2(k) := \frac{33k-21}{4k-4}, \qquad k \geq 9$$

which confirms that this case corresponds to Region 2.

Suppose instead that $\rho_2 = 0$ and $\rho_0, \rho_1 > 0$: some agents are engaged in domestic production and the rest choose leisure. The equalities $V(W,g_1) = V(M,g_1) = V(0)$ imply that

$$q(W,g_1) = -k \qquad q(M,g_1) = -\frac{k}{3}$$

while the inequalities $V(0) \geq V(W,g_2)$ and $V(0) \geq V(M,g_2)$ imply

$$q(W,g_2) \geq -3k \qquad q(M,g_2) \geq -k$$

Equation 3 implies

$$p_2 = \frac{9+7k}{9}$$

Combining equation 4 with the above inequalities, we conclude that

$$\alpha \leq B_1(k)$$

Since we must have $k \leq 9$ to rule out the preceding case, this case coincides with Region 3. Equation 5 yields the equilibrium fraction of agents belonging to firms of type 1,

$$\rho_1 = \frac{9k}{18+7k}$$

ρ_1 increases as agent wealth k increases, reaching 1 when $k = 9$.

Above the curve B_1, no agents are engaged in domestic production. We consider first the case in which a positive fraction work or manage factories and a positive fraction do not work: $\rho_1 = 0$ and $\rho_0, \rho_2 > 0$. The equalities $V(W,g_2) = V(M,g_2) = V(0)$ imply

$$q(W,g_2) = -3k \qquad q(M,g_2) = -k$$

The inequalities $V(W,g_2) \geq V(W,g_1)$ and $V(M,g_2) \geq V(M,g_1)$ imply

$$q(W,g_1) \geq -k \qquad q(M,g_1) \geq -\frac{k}{3}$$

Equation 4 implies

$$p_2 = \frac{6+7k}{\alpha}$$

When factory productivity α increases, per capita wealth k held fixed, the equilibrium price of the produced good falls. Equation 5 implies

$$\rho_2 = \frac{3k}{12+7k}$$

so that, as per capita wealth k increases, factory employment increases. From equation 3 and the above equalities and inequalities for prices, we conclude this case applies if

$$\alpha > B_2(k)$$

corresponding to Region 4 in Figure 1.

Because ρ_2 approaches 3/7 in the limit as $k \to \infty$, full employment in factory production cannot occur. It is also easy to verify this directly. If $\rho_2 = 1$ (and so $\rho_0 = \rho_1 = 0$), then equation 5 requires

$$p_2 = \frac{3k-6}{\alpha}$$

Equations 4 and 7 imply

$$q(W,g_2) = \frac{24-7k}{5} \qquad q(M,g_2) = \frac{12-k}{5}$$

But $V(W,g_2) = V(M,g_2) \geq V(0)$ implies $k \leq -3$, which is impossible.

The final possibility is that agents engage simultaneously in factory and domestic production.[16] We already know this occurs along the curve B_1, but along that line a positive fraction of agents also choose not to work. Suppose $\rho_0 = 0$ but $\rho_1, \rho_2 > 0$. Equating $V(W,g_1) = V(M,g_2)$ implies

$$q(W,g_1) = q(M,g_2)$$

Combining this equality with equations 3–7 yields

$$q(W,g_1) = \frac{(9-k)\alpha - 18k - 54}{8\alpha - 45} \qquad q(M,g_1) = \frac{(6+2k)\alpha - 27k - 36}{8\alpha - 45}$$

$$q(W,g_2) = \frac{(18-10k)\alpha + 9k - 108}{8\alpha - 45} \qquad q(M,g_2) = \frac{(9-k)\alpha - 18k - 54}{8\alpha - 45}$$

$$p_2 = \frac{3+21k}{8\alpha - 45}$$

and

$$\rho_1 = \frac{90 - 45k - 17\alpha + k\alpha}{48 + 21k - 9\alpha - 7k\alpha} \qquad \rho_2 = \frac{-42 + 66k + 8\alpha - 8k\alpha}{48 + 21k - 9\alpha - 7k\alpha}$$

The restriction $0 \leq \rho_1, \rho_2 \leq 1$ implies that $k \geq 9$ and $\alpha \in [B_1(k), B_2(k)]$, so this case corresponds to Region 5 of Figure 1.

[16] There is one other logical possibility, $\rho_0 = 1$ and $\rho_1 = \rho_2 = 0$, but it is easy to show this can never occur.

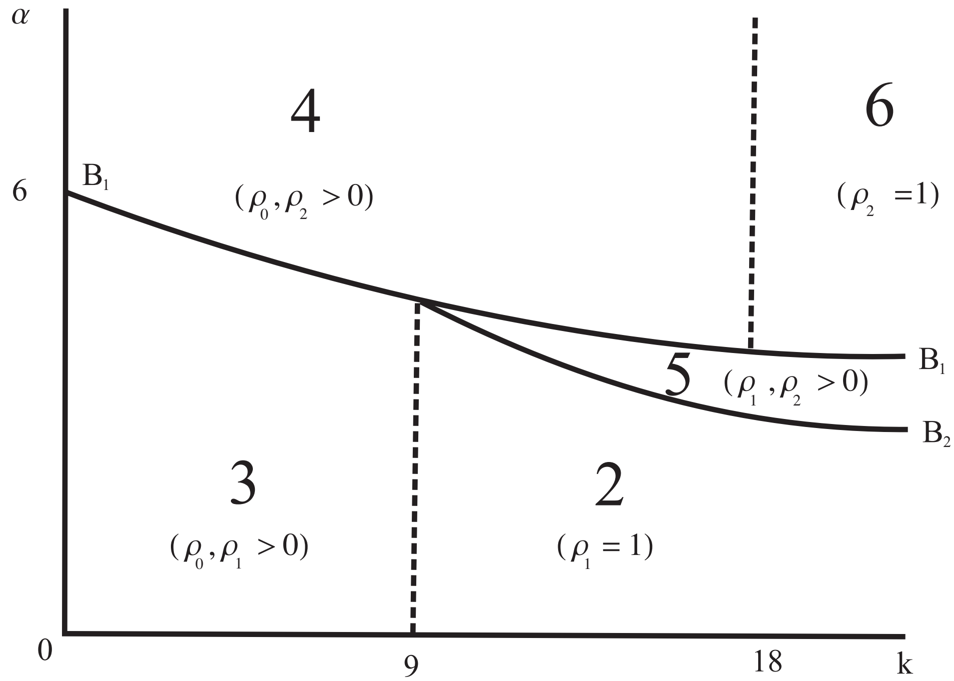

Fig. 2. No preference distinctions

To this point, we have assumed — as in Mokyr — that agents care about where they work. What difference does it make if we eliminate this assumption? Suppose preferences are described by the utility function

$$u(x,\ell) = \begin{cases} 8\sqrt{x_1x_2} & \text{if the agent chooses no memberships} \\ 6\sqrt{x_1x_2} & \text{if he chooses a membership of type } (M,g_1) \text{ or } (M,g_2) \\ 4\sqrt{x_1x_2} & \text{if he chooses membership } (W,g_1) \text{ or } (W,g_2) \end{cases}$$

Utility depends on whether you are a manager or a worker, but not on whether the productive activity takes place at home or on the factory floor.

Notice that, in contrast to the preceding example, firms acquire no inputs from the market. Instead capital is provided by the workers (in firms of type g_1) or by an entrepreneur (in firms of type g_2), an obligation we build into consumption sets. An agent choosing a membership (W, g_1) is required to choose a nonnegative consumption vector (x_1, x_2) such that $x_1 \geq 1$ — but the first unit consumed of commodity 1 contributes nothing to utility. Similarly, an agent choosing a membership (M, g_2) must consume at least 3 units of commodity 1, but the first three units contribute nothing to utility.

Many qualitative features of the equilibrium correspondence are preserved. Proceeding just as before leads to the phase diagram illustrated in Figure 2. Once again the fundamental feature of the phase diagram is the boundary defined by the graphs of two functions,

$$\alpha = B_1(k) := \begin{cases} \frac{21k+54}{7k+9} & \text{if } 0 \leq k \leq 18 \\ \frac{3k-6}{k-3} & \text{if } k > 18 \end{cases}$$

and

$$\alpha = B_2(k) := \frac{3k}{k-1} \qquad k \geq 9$$

As before, there is no factory production below $\min\{B_1, B_2\}$ and no domestic production above B_1. The regions of this figure have the same interpretation as the corresponding regions of Figure 1, but there is now a Region 6 corresponding to full employment in factory work ($\rho_2 = 1$, $\rho_0 = \rho_1 = 0$). (The boundary between Regions 5 and 6 is the vertical dotted line at $k = 18$.) Details of the equilibrium correspondence are left to the interested reader.

Nevertheless, caring about where you work does make a qualitative impact on the nature of equilibrium. Consider equilibria in which domestic and factory production coexist (corresponding to wealth-productivity parameters (k, α) in region 5 or on B_1). As the reader can easily verify, under the initial specification of preferences — where agents care about their workplace — managers and workers are compensated for working in factories:

$$q(W, g_1) > q(W, g_2) \qquad \text{and} \quad q(M, g_1) > q(M, g_2)$$

In contrast, when agents do not care about their workplace, there are no compensating differentials:

$$q(W, g_1) = q(W, g_2) \qquad \text{and} \quad q(M, g_1) = q(M, g_2)$$

Caring about where you work has other consequences as well. In Figure 2 the curve B_1 slopes downward: the threshold level for shifting from domestic to factory production decreases as economies acquire additional wealth for building the factories. This reflects the effect of indivisibilities at the plant level. Each factory requires 6 units of commodity 1 as input; as k increases, this indivisibility matters less and less.

In Figure 1, on the other hand, B_1 has an upward slope. With greater wealth, economies require more productivity from factories to compensate for inferior working conditions, and this trumps the influence of indivisibility.

More subtly, in Figure 1 the separation between curves B_1 and B_2 remains in the limit as k approaches infinity,

$$\lim_{k \to \infty} B_1(k) = 9 > \frac{33}{4} = \lim_{k \to \infty} B_2(k)$$

but in Figure 2 these curves converge:

$$\lim_{k \to \infty} B_1(k) = \lim_{k \to \infty} B_2(k) = 3$$

In the latter case, the gap disappears as the difference in material input cost between factory and domestic production becomes negligible relative to per capita wealth. But in the former case, where agents care about where they work, not only the cost of material inputs but also the cost of workers and of managers differs between domestic firms and factories. In contrast to material inputs, the difference in the cost of human resources does not become negligible as wealth increases — wealthy agents demand better working conditions.

5.3 Capitalists

In the preceding application no one owns a firm. Capital, the material inputs a manager and his workers require if they are to form a going concern, is acquired from the "market," not from any specific agent. In this application we articulate a role for the "capitalist," an agent who supplies capital to a particular firm.

All agents are *ex ante* identical with endowment $e = (k, 0)$. As in application 5.2, we distinguish between workers and managers: $\Omega = \{W, M\}$. There are two types of firm. A firm of type g_1 is worker managed. There are 3 workers, each contributing 1 unit of capital (commodity 1) as well as his labor; the firm is managed cooperatively without a formal "manager." Formally:

$$g_1 = (\pi_1, \gamma_1, y_1) \qquad \pi_1 = (3, 0) \qquad y_1 = (0, 3)$$

Firms of type g_2 are owned and managed by an entrepreneur who supplies all 3 units of the capital and also serves as a manager of two workers who contribute nothing but their labor. Formally:

$$g_2 = (\pi_2, \gamma_2, y_2) \qquad \pi_2 = (2, 1) \qquad y_1 = (0, \alpha)$$

Preferences are described by the utility function:

$$u(x, \ell) = \begin{cases} 8\sqrt{x_1 x_2} & \text{if the agent chooses no membership} \\ 6\sqrt{(x_1 - 1)x_2} & \text{if the agent chooses } m = (W, g_1) \\ 6\sqrt{(x_1 - 3)x_2} & \text{if the agent chooses } m = (M, g_2) \\ 4\sqrt{x_1 x_2} & \text{if the agent chooses } m = (W, g_2) \end{cases}$$

Workers prefer the working conditions of a worker-managed firm, other things being equal, but self-monitoring and joint supply of capital may be less efficient than having a single owner who controls his workers. The parameter α measures the relative efficiency of the entrepreneurial firm.

Notice that, in contrast to the preceding example, firms acquire no inputs from the market. Instead capital is provided by the workers (in firms of type g_1) or by an entrepreneur (in firms of type g_2), an obligation we build into consumption sets. An agent choosing a membership (W, g_1) is required to choose a nonnegative consumption vector (x_1, x_2) such that $x_1 \geq 1$, and the first unit consumed of commodity 1 contributes nothing to utility. Similarly,

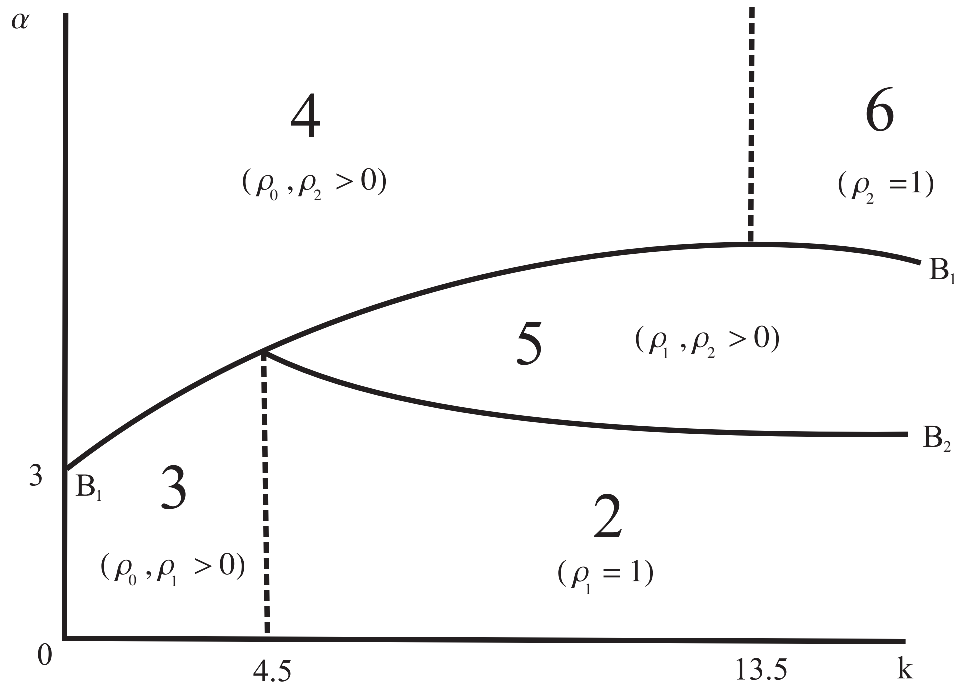

Fig. 3. Phase diagram: capitalists

an agent choosing a membership (M, g_2) must consume at least 3 units of commodity 1 and the first three units contribute nothing to utility.

As before, let ρ_0 denote the fraction of agents choosing no membership and ρ_j the fraction choosing a membership in a firm of type j $(j = 1, 2)$, and normalize so that $p_1 = 1$. We omit the computation of equilibrium, which parallels that of Example 5.2. Figure 3, which subdivides the parameter space

$$\{(k, \alpha) \in \mathbf{R}^2 \mid (k, \alpha) > 0\}$$

into a number of regions, captures the most important features of the equilibrium correspondence. The curve labeled B_1 is the graph of the equation

$$\alpha = B_1(k) := \begin{cases} \frac{9+7k}{3+k} & \text{if } k \leq 27/2 \\ \frac{12k-24}{2k-5} & \text{if } k > 27/2 \end{cases} \tag{8}$$

The curve labeled B_2, which intersects the graph of B_1 at the point $(k, \alpha) = (9/2, 27/5)$, is the graph of the equation

$$\alpha = B_2(k) := \frac{5k-9}{k-2} \qquad k \geq 9/2 \tag{9}$$

(Note that B_2 is defined only for $k \geq 9/2$.)

Below the curve $\min\{B_1, B_2\}$ (Regions 2 and 3), where the productivity of entrepreneurial firms is low, there are no entrepreneurial firms: agents either belong to a worker-managed firms or choose not to work. Above the curve B_1 (Regions 4 and 6), where the productivity of entrepreneurial firms is high, there are no worker-managed firms: agents either are members of entrepreneurial firms or they choose not to work. On the curve B_1 and, for wealth $k > 9/2$, in the area bounded by the curves B_1 and B_2, worker-managed and entrepreneurial firms coexist. Region 5, along with the curve B_1, is where worker-managed firms and entrepreneurial firms coexist.

For equilibria in this region, it would, at least in principle, be possible to test for the presence of a trade-off between "economic democracy" and the efficiencies of a more hierarchically organized firm. We are unaware of any study of that sort comparable to Mokyr's comparison of domestic and factory production.

This application has ruled out by fiat the possibility of acquiring capital from the market. This seems reasonable since otherwise capitalists are providing capital without insisting on control. How then would we propose capturing the publicly-owned corporation? By recognizing that shareholders are also members of the group, supplying capital but delegating control to a board of directors. The advantage to such an investor is the opportunity to diversify risk by holding relatively small stakes in many different firms; the disadvantage is the introduction of an agency problem. We leave such an extension for another time.

5.4 Contracts

Here we give a simple example illustrating the interpretation of groups as contracts and the effect of competition on the choice of contracts.

We consider an economy in two dates $0, 1$. There is a single good (grain), which can be consumed at either date but can also be planted at date 0. Two methods of planting are possible:

a) Two agents can work side-by-side, planting 2 units of grain at date 0 and harvesting 2α units of grain. Because the agents work side-by-side, they *must* share the harvest equally, obtaining α units of grain each. We refer to this arrangement as *partnership*.
b) Two agents can work in sequence, the first planting 2 units of grain at date 0, the second harvesting β units. Because the agents work in sequence, the second agent cannot be prevented from eating the entire harvest. We refer to this arrangement as *ownership*, to the second agent as the *entrepreneur* and to the first agent as the *worker*.

We view each of these choices as indivisible and full-time, so each agent can choose to participate in only one (or neither of course).[17] We take output levels α, β as parameters.

Agents are *ex ante* identical. Agents are endowed with 4 units of grain at each date; their utility for consumption patterns over time is:

$$U(c_0, c_1) = \sqrt{c_0 c_1}$$

Aside from the contractual arrangements for planting grain, *no intertemporal contracts are enforceable*; in particular, there is no market at date 0 for consumption at date 1.

We model this story as an atemporal economy with one good, embedding consumption in the second date into utility functions. Formally, we distinguish three membership characteristics P, E, W (partner, entrepreneur, worker) and two group types

- partnership $\mathcal{P} = (\pi_{\mathcal{P}}, \gamma_{\mathcal{P}}, -2)$ where

$$\pi_{\mathcal{P}}(\beta) = \begin{cases} 2 \text{ if } & \beta = P \\ 0 \text{ if } & \beta = E \\ 0 \text{ if } & \beta = W \end{cases}$$

- ownership $\mathcal{O} = (\pi_{\mathcal{O}}, \gamma_{\mathcal{O}}, -2)$ where

$$\pi_{\mathcal{O}}(\beta) = \begin{cases} 0 \text{ if } & \beta = P \\ 1 \text{ if } & \beta = E \\ 1 \text{ if } & \beta = W \end{cases}$$

Consumption sets permit consumption of any non-negative quantity of the (date 0) private good and choice of at most one membership. In this formulation, utility for date 0 consumption and membership choice reflects the *actual* consumption at date 0 and *implicit consumption* at date 1:

$$\begin{aligned} u(c, 0) &= \sqrt{c \cdot 4} \\ u(c, (P, \mathcal{P})) &= \sqrt{c \cdot (4 + \alpha)} \\ u(c, (E, \mathcal{O})) &= \sqrt{c \cdot 4} \\ u(c, (W, \mathcal{O})) &= \sqrt{c \cdot (4 + \beta)} \end{aligned}$$

To solve for the equilibrium (which will depend on the productivity parameters α, β) we take the private good as numeraire. Budget balance entails that membership prices sums to the cost of the input, so equilibrium prices satisfy:

[17] We might allow for part-time participation, or for smaller-scale planting, but this would add substantial complication without adding much interest.

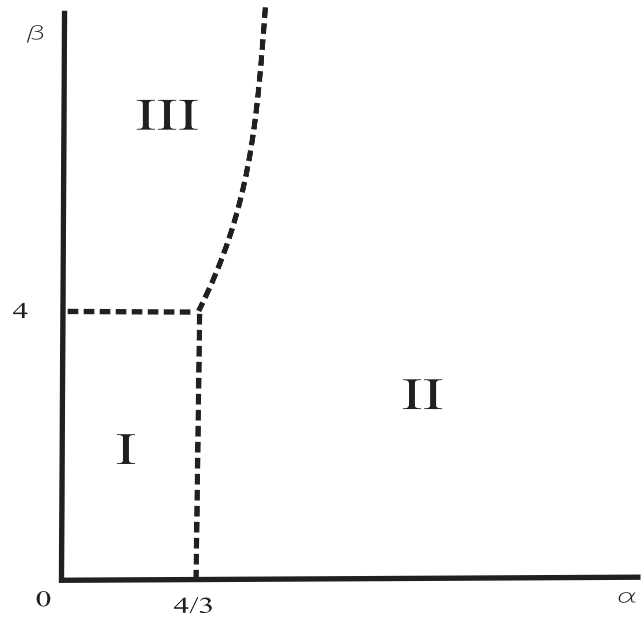

Fig. 4. Competitive contracting.

$$p = 1$$
$$q(P, \mathcal{P}) = 1$$
$$q(E, \mathcal{O}) + q(W, \mathcal{O}) = 2$$

If membership prices are q, then the (indirect) utility of agents choosing various memberships is

$$\hat{u}(0) = 4$$
$$\hat{u}(P, \mathcal{P}) = \sqrt{[4 - q(P, \mathcal{P})][4 + \alpha]}$$
$$\hat{u}(W, \mathcal{O}) = \sqrt{[4 - q(W, \mathcal{O})][4]}$$
$$\hat{u}(E, \mathcal{O}) = \sqrt{[4 - q(E, \mathcal{O})][4 + \beta]}$$

(An agent choosing a partnership will pay the price $q(P, \mathcal{P})$, hence consume $4 - q(P, \mathcal{P})$ units of grain at date 0 and $4 + \alpha$ units of grain at date 1, and so forth.)

Keep in mind that some of these memberships may not be chosen at equilibrium. Indeed, the space of parameter values can be decomposed into three (open) regions and their boundaries; within each region, only a single kind of membership is chosen; see Figure 4.

I In this region, neither partnership nor ownership are chosen. In order that this be the case, the indirect utility $\hat{u}(0)$ must be at least as large as every other indirect utility. Simple algebra gives:

$$\hat{u}(0) \geq \hat{u}(P, \mathcal{P}) \Leftrightarrow \alpha \leq \frac{4}{3}$$
$$\hat{u}(0) \geq \hat{u}(W, \mathcal{O}) \Leftrightarrow q(W, \mathcal{O}) \geq 0$$
$$\hat{u}(0) \geq \hat{u}(E, \mathcal{O}) \Leftrightarrow q(E, \mathcal{O}) \geq \frac{4\beta}{4+\beta}$$

Keeping in mind that $q(W, \mathcal{O}) + q(E, \mathcal{O}) = 2$ and solving, we conclude that this region is the rectangle bounded by

$$0 \leq \alpha \leq \tfrac{4}{3}$$
$$0 \leq \beta \leq 4$$

II In this region only partnership is chosen. In order that this must be the case, the indirect utility $\hat{u}(P, \mathcal{P})$ must be at least as large as every other indirect utility. Simple algebra gives:

$$\hat{u}(0) \leq \hat{u}(P, \mathcal{P}) \Leftrightarrow \alpha \geq \frac{4}{3}$$
$$\hat{u}(P, \mathcal{P}) \geq \hat{u}(W, \mathcal{O}) \Leftrightarrow q(W, \mathcal{O}) \geq 1 - \frac{3}{4}\alpha$$
$$\hat{u}(P, \mathcal{P}) \geq \hat{u}(E, \mathcal{O}) \Leftrightarrow q(E, \mathcal{O}) \geq \frac{\alpha + 4 + 4(\beta - \alpha)}{4+\beta}$$

Solving, we conclude that this region is infinite above and to the right:

$$0 \leq \beta \leq \frac{8\alpha}{4-\alpha} \quad \text{for} \quad \frac{4}{3} \leq \alpha \leq 4$$
$$\beta \text{ arbitrary} \quad \text{for} \quad 4 < \alpha$$

(If $\alpha > 4$ then there is no wage rate the owner can afford to pay from endowment that will make the worker prefer working to being in a partnership.)

III In this region, only ownership is chosen. This region is the complement of the union of regions I, II and we can describe it as:

$$0 \leq \alpha \leq 4$$
$$\beta \ \geq \ \min\left\{4, \tfrac{8\alpha}{4-\alpha}\right\}$$

In this region, half the agents choose to be entrepreneurs and half choose to be workers. Because *ex ante* identical agents obtain the same utility in equilibrium, the membership prices are:

$$q(W,\mathcal{O}) = -\frac{2\beta - 8}{8+\beta}$$
$$q(E,\mathcal{O}) = +\frac{2\beta - 8}{8+\beta} + 2$$

Remember that negative membership prices are wages: the entrepreneur pays the worker the wage $\frac{2\beta-8}{8+\beta}$ *and* bears the cost of planting.

It is instructive to contrast the competitive environment discussed above to an environment in which there are only two agents. If there are only two agents, there is no reason to view equilibrium as the appropriate solution notion. Rather, it seems that we should permit as a solution any efficient, individually rational configuration.

Given a specified transfer t from the entrepreneur to the worker, straightforward calculations (as above) let us compare ownership, partnership and working alone.

(i) the worker prefers ownership to working alone if $t > 0$

(ii) the entrepreneur prefers ownership to working alone if

$$t < \frac{2\beta - 8}{4+\beta}$$

(iii) the worker prefers ownership to partnership if

$$t > \frac{3}{4}\alpha - 1$$

(iv) the entrepreneur prefers ownership to partnership if

$$t < \frac{2\beta - 3\alpha - 4}{4+\beta}$$

From this, we can identify the regions in which various arrangements are individually rational and efficient:

IV Ownership is individually rational and efficient if there is a transfer t (from the entrepreneur to the worker) so that both the entrepreneur and the worker prefer the relationship to working alone and at least one of them prefers the relationship to partnership. Thus ownership is individually rational and efficient if either: there exists a transfer t for which (i), (ii) and (iii) are satisfied, OR there exists a transfer t for which (i), (ii) and (iv) are satisfied. After a little algebra, we if there is a transfer t for which (i), (ii) and (iv) are satisfied then there is a transfer for which (i), (ii) and (iii) are satisfied, and that ownership is efficient and individually rational if

$$\beta > 4 \text{ and } \beta > \frac{3}{2}\alpha + 2$$

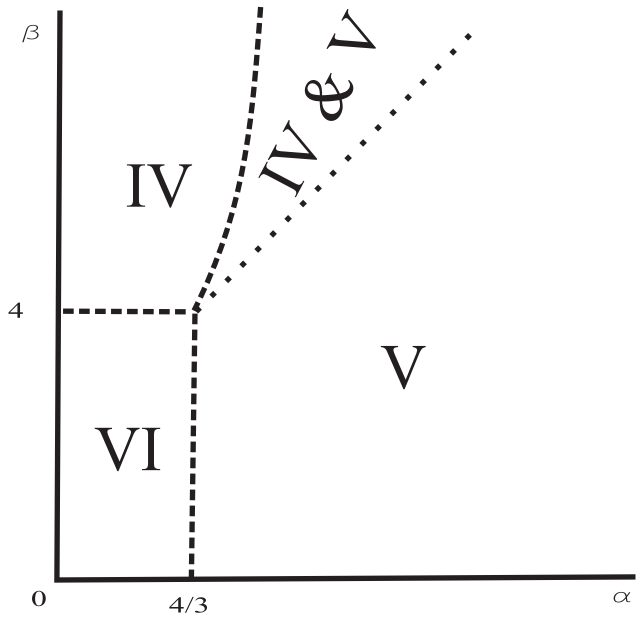

Fig. 5. Bilateral contracting.

V Partnership is individually rational if $3(4+\alpha) > 16$; equivalently, if $\alpha > \frac{4}{3}$. Partnership is efficient if there does not exist a transfer for which *both* (iii) or (iv) are satisfied. After a little algebra, we see that partnership is individually rational and efficient if

$$\alpha > \frac{4}{3} \qquad \text{and} \quad \beta < \frac{8\alpha}{4-\alpha}$$

Note that this region coincides with Region **II** above.

VI Working alone is individually rational and efficient only in the complement of the union of regions **IV, V**.

Figure 5 provides a sketch of these regions. Note that regions **IV, V** overlap. In the intersection of these regions, *both* ownership and partnership are individually rational and efficient contractual relationships — and either might be chosen in a world with only two agents. However, as we have seen above, (except for parameters in a set of measure 0) only one contractual arrangement survives in a competitive market.

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