Can Utilitarianism be Operationalized?

William R. Zame*

Department of Economics
University of California, Los Angeles

Abstract

The Utilitarian ideal is equal regard for all individuals — those now living and those not yet born. This paper shows that operationalizing this ideal is problematic: the existence of preferences that regard all individuals equally and respect the Pareto ordering is undecidable on the basis of the axioms used in almost all of formal economics; in particular, such preferences cannot be "constructed."

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1 Introduction

The Utilitarian ideal is equal regard for all individuals — those now living and those not yet born. As Sidgwick (1907, p. 424) argues: "the time at which a man exists cannot affect the value of his happiness from a universal point of view ... the interests of posterity must concern a Utilitarian as much as those of his contemporaries." Similar arguments are made by Ramsey (1928) and Rawls (1971). Operationalizing this ideal would seem to entail instructions for a Utilitarian social planner — or rules for a Utilitarian constitution — for choosing among social outcomes in a way that is invariant to permuting any finite set of individuals. Assuming (as does all of the literature) that inter-personal comparisons of utility make sense, that social choices should reflect the Pareto ranking, and that the time horizon is infinite, this amounts to constructing a complete transitive preference relation on the space of infinite utility streams that is invariant under finite permutations and respects the Pareto ordering; an *equitable, Paretian preference*, for short. The main results (Theorems 1 and 2) of this paper assert that in fact equitable, Paretian preferences *cannot be* "constructed."

I do *not* assert that equitable, Paretian preferences do not exist; indeed, Svensson (1980) shows that they do. However, Svensson's proof relies on the Axiom of Choice (via Szpilrajn's (1930) extension lemma), and is not "constructive" in any sense.[1] Because it is not constructive, Svensson's existence proof would be of little use to a utilitarian social planner who must choose an optimal growth path for an economy, or distribute a scarce resource among generations — or, for that matter, make any of the constrained optimization decisions that seem the essence of economics.

[1]Bossert, Sprumont & Suzumura (2005) expands Svensson's argument to show the existence of ethical preferences having additional desirable properties. On the other hand, Diamond (1965) shows that ethical preferences cannot be continuous in the topology induce by the supremum norm and Basu & Mitra (2003) shows that ethical preferences cannot be represented by a utility function. See also Fleurbaey & Michel (2003), Hara, Shinotsuka, Suzumura & Xu (2005) and Basu & Mitra (2005) for additional negative results. As the revision of this paper was being completed, I learned of work of Lauwers (2006), who appears to have proved a result similar to Theorem 2 of the present paper, by rather different methods.

The formal expression of the assertion that equitable, Paretian preferences cannot be "constructed" is that the existence of such preferences is independent of the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice, the axioms used in almost all of formal economics — and for that matter in all of classical analysis.[2,3] (The Zermelo–Fraenkel Axioms formalize naive set theory in a consistent way, avoiding such paradoxes as the set of sets that are not elements of themselves; the Axiom of Dependent Choice asserts the possibility of making a sequence of choices, with each choice depending on the previous choices. See Section 3 for further discussion.) This would seem to rule out any useful way of operationalizing Utilitarianism, and surely to contradict Ramsey's (1928) dictum that any argument for preferring one generation over another "arises merely from the weakness of the imagination."

To understand what independence means it may be useful to recall a little mathematical logic.[4] A set of axioms $\mathcal{A}$ is *consistent* if no contradiction can be derived from them. The Completeness Theorem asserts that the set of axioms $\mathcal{A}$ is consistent if and only if they are true in *some* model. Similarly, a proposition P is provable from $\mathcal{A}$ if and only if P is true in every model, and the negation of P is provable from $\mathcal{A}$ if and only if P is false in every model. By definition, P is *independent of* $\mathcal{A}$ (or *undecidable from* $\mathcal{A}$) if and only neither P nor its negation can be proved from $\mathcal{A}$. Hence, P is independent of $\mathcal{A}$ if and only if there is a model of in which the axioms in $\mathcal{A}$ are true and P is true and a model in which the axioms of $\mathcal{A}$ are true and P is false.

The most familiar example of an independent/undecidable proposition is surely Euclid's Fifth (Parallel) Postulate: through every point not on a line there is exactly one line parallel to the given line. Until the early part of the 19$^{\text{th}}$ century, many mathematicians believed that this statement could be derived from the first four postulates. That this is not so was demonstrated

[2]Perhaps the most sgnificant exceptions in economics are in the use of non-standard analysis to model large economies (Anderson, 1991) and to analyze continuous trading (Anderson & Raimondo, 2006), and in proofs of existence of Walrasian equilibrium in economies with a non-separable commodity space (Bewley, 1972).

[3]This result confirms a conjecture of Fleurbaey & Michel (2003).

[4]Chang & Keisler (1992) and Jech (1978) are good references.

by Bolyai and Lobachevsky who constructed "non-Euclidean geometries" (spherical and hyperbolic, respectively) that satisfy the first four postulates but not the fifth. Plane geometry is a model in which the first four postulates and the Parallel Postulate are true; and spherical and hyperbolic geometries are models in which the first four postulates are true and Parallel Postulate is false, so the Parallel Postulate is independent of the first four postulates.

A more relevant example is the existence of non-measurable sets. A familiar construction (due to Vitali) shows that the Zermelo–Fraenkel Axioms together with the Axiom of Choice imply the existence of sets of real numbers that are not Lebesgue measurable. On the other hand, Solovay (1970) constructs a model in which the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice are true and in which all sets of real numbers are Lebesgue measurable.[5] Hence the existence of a non-measurable set of real numbers is independent of the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice. Informally: non-measurable sets of reals cannot be "constructed."[6]

The main result of this paper (Theorem 1) is established by first showing (Proposition 1) that no equitable, Paretian preference can be a measurable subset of the cartesian product of the space of utility streams with itself; equivalently, the existence of an equitable, Paretian preference implies the existence of a non-measurable subset of the cartesian product of the space of utility streams with itself. It is then shown that the existence of such a non-measurable set implies the existence of a non-measurable set of real numbers. In Solovay's model, the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice are true and there are no non-measurable sets of real numbers, so in this same model there are no equitable, Paretian preferences. Since non-measurable sets cannot be "constructed," equitable, Paretian pref-

[5] Solovay's construction, like most constructions in model theory, is relative, in the sense that it assumes a model of one kind and constructs a model of another kind. In the case at hand, Solovay assumes the existence of a model in which the Zermelo–Fraenkel Axioms, the Axiom of Choice, and a "large cardinal axiom" are true and deduces the existence of a model in which the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice are true and in which all sets of real numbers are Lebesgue measurable.

[6] For an interesting discussion of non-constructibility, see Schechter (1997) and the website http://www.math.vanderbilt.edu/ schectex/.

erences cannot be "constructed" either. Proposition 2 and Theorem 2 provide parallel results for equitable, Paretian preferences with a restricted domain.

2 Utility Streams and Measurable Preferences

I assume that the range of possibilities for each individual's utility is an interval, which may be taken without loss to be $[0,1]$. (The case in which the range of possibilities is a finite set is considered in Section 4.) If $\mathbb{N}$ is the set of natural numbers (positive integers), then $X = [0,1]^{\mathbb{N}}$ is the space of utility streams; for $x \in X$, x_n is the utility of individual n. (The literature frequently assumes a single individual in each generation, or a representative individual for each generation, in which case x_n may be identified with the utility of generation n.) Write $x \geq y$ if $x_n \geq y_n$ for all n, $x > y$ if $x \geq y$ but $x \neq y$, and $x \gg y$ if $x_n > y_n$ for all n.

By a *finite permutation* I mean a permutation $\sigma : \mathbb{N} \to \mathbb{N}$ that differs from the identity only on a finite set; write $\mathbb{F}$ for the group of finite permutations. For $\sigma \in \mathbb{F}$, define $T_\sigma : X \to X$ by $T_\sigma(x)_n = x_{\sigma(n)}$.

A complete, transitive preference relation $\succeq$ on X is *equitable* if

$$x, y \in X, \sigma \in \mathbb{F} \Rightarrow x \sim T_\sigma(x)$$

In view of transitivity, this is equivalent to saying that $\succeq$ is invariant under permuting the utility of any finite number of individuals. (Equivalently, $\succeq$ is equitable if and only if it is invariant under permuting the utility of any pair of individuals.) Say that $\succeq$ is *weakly Paretian* if

$$x, y \in X, x \gg y \Rightarrow x \succ y$$

Write λ for Lebesgue measure on $[0,1]$. Let $\lambda^{\mathbb{N}}$ be the product measure on $X = [0,1]^{\mathbb{N}}$, and let Λ be the completion of $\lambda^{\mathbb{N}}$. (Recall that the completion of any measure is obtained by adjoining all subsets of sets of measure zero.) Let Λ^2 be the completion of the product measure $\Lambda \times \Lambda$ on $X \times X$.

A preference relation $\succeq$ on X is by definition a subset of $X \times X$. Say $\succeq$ is *measurable* if its graph

$$\mathcal{G} = \{(x, y) \in X \times X \colon y \succeq x\}$$

is Λ^2-measurable. The key result is the following.

Proposition 1 *No equitable weakly Paretian preference relation on $X = [0,1]^{\mathbb{N}}$ is measurable.*

Proof Suppose to the contrary that $\succeq$ is equitable, Paretian and measurable. Set

$$\begin{aligned} A &= \{(x, y) \in X \times X : y \succ x\} \\ B &= \{(x, y) \in X \times X : x \succ y\} \\ I &= \{(x, y) \in X \times X : y \sim x\} \end{aligned}$$

Define an inversion

$$\tau : X \times X \to X \times X$$

by $\tau(x, y) = (y, x)$. Note that τ is one-to-one, measurable and measure-preserving, and that $\tau \circ \tau$ is the identity. Note that A, B, I are disjoint and their union is $X \times X$. Moreover,

$$\begin{aligned} I &= \mathcal{G} \cap \tau[\mathcal{G}] \\ A &= \mathcal{G} \setminus I \\ B &= X \times X \setminus \mathcal{G} \end{aligned}$$

By assumption, $\mathcal{G}$ is measurable, so each of A, B, I is measurable.

The next step is to prove the following **Claim:** $\Lambda^2(A) = \Lambda^2(B) = 0$. To this end, note first that, because τ is measure preserving and $\tau(A) = B$, it follows that $\Lambda^2(A) = \Lambda^2(B)$. Suppose this common value is $\alpha > 0$; because A, B are disjoint and Λ^2 is a probability measure, $\alpha \leq \frac{1}{2}$.

For each measurable set $E \subset X \times X$ and each $x, y \in X$ define the sections

$$\begin{aligned} E_x &= \{y \in X : (x, y) \in E\} \\ E^y &= \{x \in X : (x, y) \in E\} \end{aligned}$$

Fubini's theorem guarantees that each section is measurable and that

$$\Lambda^2(E) = \int_X \Lambda(E_x)\, d\Lambda(x) = \int_X \Lambda(E^y)\, d\Lambda(y)$$

Hence

$$\begin{aligned} \Lambda^2(E) = 0 \quad &\Leftrightarrow \quad \Lambda(E_x) = 0 \text{ for almost all } x \\ &\Leftrightarrow \quad \Lambda(E^y) = 0 \text{ for almost all } y \end{aligned}$$

It follows immediately that, for subsets $E, F \subset X \times X$ the following are equivalent:

(i) E, F differ by a set of measure 0

(ii) for almost all $x \in X$, the sections E_x, F_x differ by a set of measure 0

(iii) for almost all $y \in X$, the sections E^y, F^y differ by a set of measure 0

Now consider the sections A_x, A^y. By assumption, $\succeq$ is equitable, so each such section is invariant under each T_σ; that is,

$$\begin{aligned} y \in A_x, \sigma \in \mathbb{F} \Rightarrow T_\sigma(y) \in A_x \\ x \in A^y, \sigma \in \mathbb{F} \Rightarrow T_\sigma(x) \in A^y \end{aligned}$$

The Hewitt–Savage 0–1 law (Billingsley, 1995, p. 496) asserts that every measurable subset of X that is invariant under each T_σ has measure either 0 or 1, so each A_x and each A^y has measure either 0 or 1.

Set

$$\begin{aligned} X_0 = \{x \in X : \Lambda(A_x) = 0\} \\ X_1 = \{x \in X : \Lambda(A_x) = 1\} \end{aligned}$$

Note that the sets X_0, X_1 are disjoint and that $X_0 \cup X_1 = X$ (because each A_x has measure 0 or 1). Hence

$$\Lambda(X_0 \cup X_1) = \Lambda(X_0) + \Lambda(X_1) = 1$$

We can write

$$A = [A \cap (X_0 \times X)] \cup [A \cap (X_1 \times X)]$$

The definition of X_0 implies that $\lambda(A \cap (X_0 \times X)) = 0$, so $[A \cap (X_1 \times X)]$ and A differ by a set of measure 0. Moreover, for every $x \in X$, the sections $[A \cap (X_1 \times X)]_x$ and $(X_1 \times X)_x$ differ by a set of measure 0, so $[A \cap (X_1 \times X)]$ and $(X_1 \times X)$ differ by a set of measure 0. Hence A and $X_1 \times X$ differ by a set of measure 0, so for almost every $y \in X$, A^y and $(X_1 \times X)^y$ differ by a set of measure 0. However, $(X_1 \times X)^y = X_1$, so Fubini's theorem guarantees

$$\Lambda^2(A) = \int_X \Lambda(A^y)\, d\Lambda(y) = \int_X \Lambda(X_1)\, d\Lambda(y) = \Lambda(X_1)$$

Hence $\Lambda^2(A) = \Lambda(X^1) = \Lambda(A^y)$ for almost every $y \in X$. On the other hand, we have already seen that $\Lambda^2(A) = \alpha \leq \frac{1}{2}$ and $\Lambda(A^y)$ is equal to 0 or 1 for almost every $y \in X$, so this is a contradiction. This completes the proof of the **Claim**.

Because $\Lambda(A) = \Lambda(B) = 0$ it follows that $\Lambda(I) = 1$. For each $x \in X$, write

$$I_x = \{y \in X : (x, y) \in I\}$$

Fubini's theorem guarantees that

$$\Lambda^2(I) = \int_X \Lambda(I_x)\, dx$$

Because $\Lambda^2(I) = 1$, $\Lambda(I_x) = 1$ for almost every $x \in X$; fix any $x_0 \in X$ for which $\Lambda(I_{x_0}) = 1$. Choose and fix an increasing sequence $\{a_n\}$ of real numbers and a decreasing sequence $\{b_n\}$ of real numbers such that

$$0 < a_n < b_n < 1 \quad \text{and} \quad \prod_{n=1}^{\infty}(b_n - a_n) > \frac{1}{2}$$

Set

$$D = \prod_{n=1}^{\infty}[a_n, b_n]$$

Define a map $f : D \to X$ by

$$f(x)_n = x_n + \frac{1 - b_n}{2}$$

Note that f is one-to-one, measurable and measure-preserving. Our construction guarantees that $\Lambda(I_{x_0}) = 1$ and $\Lambda(D) > \frac{1}{2}$, whence

$$\Lambda(D) = \Lambda(D \cap I_{x_0}) = \Lambda(f(D \cap I_{x_0})) > \frac{1}{2}$$

However, if $y \in (D \cap I_{x_0})$ then $f(y) \gg y$, so $f(y) \succ y$, whence $f(y) \succ x_0$. Hence $(D \cap I_{x_0})$ and $f(D \cap I_{x_0})$ are disjoint. Since f is measure-preserving, we have $\Lambda(f(D \cap I_{x_0})) = \Lambda(D \cap I_{x_0}) > \frac{1}{2}$, which is impossible because Λ is a probability measure. We have reached a contradiction, so the proof is complete. ■

3 Independence

The Axiom of Dependent Choice is the following statement:

> **Axiom of Dependent Choice** Let X be a non-empty set and let R be an entire relation on X (that is, a binary relation such that for each $x \in X$ there is a $y \in X$ with xRy). Then there is a function $f : \mathbb{N} \to X$ such that $f(n)Rf(n+1)$ for each $n \in \mathbb{N}$.

Note that, given a set X and an entire relation R on X, the Zermelo–Fraenkel Axioms by themselves guarantee that for every natural number N there is a function $f : \{1, \ldots, N\} \to X$ for which $f(n)Rf(n+1)$ for $n = 1, \ldots, N-1$. That is, the Zermelo–Fraenkel Axioms — without any additional choice axiom whatsoever — already guarantee the existence of *arbitrarily long finite sequences* for which $f(n)Rf(n+1)$; the Axiom of Dependent Choice guarantees the existence of an *infinite* sequence for which $f(n)Rf(n+1)$. It is not hard to derive the Axiom of Dependent Choice from the Axiom of Choice itself (the easiest argument uses Zorn's Lemma), but the Axiom of Choice is not derivable from the Axiom of Dependent Choice — because the Axiom of Dependent Choice only asserts the existence of *countably many choices*.

The main result is the following.

Theorem 1 *The existence of an equitable, weakly Paretian preference on $[0,1]^{\mathbb{N}}$ is independent of the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice.*

Proof We need to exhibit a model in which the Zermel–Fraenkel Axioms and the Axiom of Dependent Choice are true and in which there is no equitable, weakly Paretian preference on $[0,1]^{\mathbb{N}}$. A suitable example is provided by Solovay's (1970) model $\mathbb{M}$, in which the Zermel–Fraenkel Axioms and the Axiom of Dependent Choice are true and in which every set of real numbers is Lebesgue measurable.

In view of Proposition 1, no equitable, weakly Paretian preference on $[0,1]^{\mathbb{N}}$ can be measurable with respect to Λ^2. Put differently, the existence of an equitable, weakly Paretian preference on $X = [0,1]^{\mathbb{N}}$ implies the existence of a non-measurable subset of $X \times X$. I show that this implies the existence of a non-measurable set of real numbers. This is an immediate consequence of a famous theorem of Maharam (1942) characterizing complete non-atomic probability spaces, but a simple argument is available for the case at hand.

For each $s \in [0,1]$, write $s = .s_1 s_2 \ldots$ for its decimal expansion. Define a map $\Phi : [0.1]^{\mathbb{N}} \times [0.1]^{\mathbb{N}} \to [0,1]$ by

$$\Phi(s^1, t^1) = .s_1^1 t_1^1 s_2^1 t_2^1 s_1^2 t_1^2 \ldots$$

Let F be the set of elements of $[0,1]$ whose decimal expansions end in an infinite string of 0's or an infinite string of 9's; note that F is a countable set. Set $X^* = ([0,1] \setminus F)^{\mathbb{N}}$, $Y^* = \Phi(X^* \times X^*)$ and $G = [0,1] \setminus Y^*$; let $\Phi^* : X^* \times X^* \to Y^*$ be the restriction of Φ. Note that G consists of those elements $s \in [0,1]$ for which one of countable number of particular subsequences of the decimal expansion of s ends in an infinite string of 0's or an infinite string of 9's; hence $\lambda(G) = 0$ and $\lambda(Y^*) = 1$. Because F is countable, $\Lambda(F) = 0$ and $\Lambda^2(X \times X \setminus X^* \times X^*) = 0$.

It is easily seen that Φ^* is one-to-one, onto, measurable with a measurable inverse, and measure-preserving. If $\mathcal{G} \subset X \times X$ is not measurable then neither is $\mathcal{G}^* = \mathcal{G} \cap (X \times X \setminus X^* \times X^*)$. Hence $\Phi^*(\mathcal{G}^*)$ is a non-measurable subset of Y^*, and thus of $[0,1]$.

To summarize: the existence of an equitable, weakly Paretian preference on $X \times X$ entails the existence of a non-measurable subset of $[0,1]$. Hence, in Solovay's model $\mathbb{M}$, no equitable, weakly Paretian preferences do not exist, which is what was necessary to prove. ■

4 Restricted Domain

A common response to impossibility results in social choice is to look for restricted domains on which possibility is restored. In the present instance, one might require that each individual's utility lies in a finite set, say $\{0,1\}$; see Basu & Mitra (2003) and Basu & Mitra (2005) for instance. However, even this very strong restriction makes little difference.

To see this, write $Z = \{0,1\}^{\mathbb{N}}$. Let γ be normalized counting measure on $\{0,1\}$. Write $\gamma^{\mathbb{N}}$ for the infinite product measure on $Z = \{0,1\}^{\mathbb{N}}$ and let Γ be its completion. Let Γ^2 be the completion of $\Gamma \times \Gamma$ on $Z \times Z$. As before, say that a preference relation $\succeq$ on Z is *measurable* if its graph

$$\mathcal{G} = \{(z,w) \in Z \times Z : z' \succeq z\}$$

is measurable with respect to Γ^2. Say that $\succeq$ is *strongly Paretian* if

$$z, w \in X, w > z \Rightarrow w \succ z$$

Proposition 2 *No equitable, strongly Paretian preference relation on $Z = \{0,1\}^{\mathbb{N}}$ is measurable.*

Proof The proof is almost the same as the proof of Proposition 1. Suppose to the contrary that $\succeq$ is an equitable, strongly Paretian preference on Z. Write

$$I \quad = \quad \{(z,w) \in Z \times Z : w \sim z\}$$

Arguing exactly as in the proof of Proposition 1, we see that $\Gamma^2(I) = 1$, and hence that there is some $z_0 \in Z$ for which $\Gamma(I_{z_0}) = 1$.

Define a map $g : Z \to Z$ by

$$g(z)_n = \begin{cases} 0 & \text{if} \quad n = 1 \text{ and } z_1 = 1 \\ 1 & \text{if} \quad n = 1 \text{ and } z_1 = 0 \\ z_n & \quad\quad \text{otherwise} \end{cases}$$

(That is, g reverses the first component of z.) It is evident that g is one-to-one, measurable and measure-preserving. By assumption, $\succeq$ is strongly Paretian, so if $z_1 = 0$ then $g(z) > z$ and $g(z) \succ z$, while if $z_1 = 1$ then $g(z) < z$ and $z \succ g(z)$; in either case, $g(z) \not\sim z$. Hence, if $z \in I_{z_0}$ then $g(z) \not\sim z_0$, so $g(z) \notin I_{z_0}$. It follows that $g(I_{z_0}) \cap I_{z_0} = \emptyset$. Because g is measure-preserving and $\Gamma(I_{z_0}) = 1$, this is absurd, so we have reached a contradiction and the proof is complete. ■

The corresponding independence result for strongly Paretian preferences on $\{0,1\}^{\mathbb{N}}$ is:

Theorem 2 *The existence of an equitable, strongly Paretian preference on* $\{0,1\}^{\mathbb{N}}$ *is independent of the Zermelo–Fraenkel Axioms and the Axiom of Dependent Choice.*

Proof As in the proof of Theorem 1, it suffices to show that the existence of a non-measurable subset of $Z \times Z$ entails the existence of a non-measurable subset of $[0,1]$.

Note first that the map $(z,w) \mapsto (z_1, w_1, z_2, w_2, \ldots)$ is a one-to-one, onto, and measure-preserving map of $Z \times Z$ to Z. Hence the existence of a non-measurable subset of $Z \times Z$ entails the existence of a non-measurable subset of Z. Now let $F \subset Z$ be the set of sequences which are eventually constant (either eventually 0 or eventually 1) and let $G \subset [0,1]$ be the set of real numbers which have a binary expansion that ends in all 0's or all 1's. Set $Z^* = Z \setminus F$, and define a map $h : Z^* \to [0,1] \setminus G$ by

$$h(z) = \sum_{n=1}^{\infty} z_n 2^{-n}$$

That is, h maps the sequence z of 0's and 1's to the real number having z as its binary expansion. It is easily seen that h is one-to-one, measurable (in fact continuous) and measure-preserving, and that its inverse is also measurable. Because F and G are is countable, $\Gamma(F) = 0$ and $\lambda(G) = 0$. Hence the existence of a non-measurable subset of Z entails the existence of a non-measurable subset of $[0, 1]$, as required. ■

5 Conclusion

Propositions 1 and 2 make very weak use of the Pareto property, and weaker assumptions along the same lines would do. For example, the conclusion of Proposition 2 would obtain under the weaker assumption that $w \succ z$ whenever $w_n > z_n$ for any infinite set of indices n. Indeed, without any Pareto assumption at all, we can conclude from the arguments of Propositions 1 and 2 that if $\succeq$ is an equitable, measurable preference relation then $x \sim y$ for almost all pairs x, y.

Completeness is required for the conclusions here: there *do* exist equitable, Paretian, measurable incomplete preferences on $[0, 1]^{\mathbb{N}}$. (For example, we can define $x \succ y$ if and only if there is a permutation $\sigma \in \mathbb{F}$ such that $T_\sigma(x) > y$ and $y \succ x$ if and only if there is a permutation $\sigma \in \mathbb{F}$ such that $T_\sigma(y) > x$; otherwise, x, y are incomparable.) However, the argument of Proposition 1 shows that for such an incomplete preference, almost all pairs (x, y) are incomparable.

It might be argued that the universe will be of finite duration so that an infinite horizon model is inappropriate. Certainly, if the horizon is finite and known, there is no difficulty constructing equitable Paretian preferences. However, if the horizon is finite but *not known*, it is seems unclear how to model preferences, equity and the Pareto property; whether equitable Paretian preferences exist seems equally unclear.

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