Corrigendum to "Communication and Equilibrium in Discontinuous Games of Incomplete Information"

(*Econometrica*, Vol. 70, No. 5, September 2002, 1711–1740)

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We thank Andreas Blume for pointing out an error in a claim that we made in Example 1, and Eric Balder for pointing out that the proof of a lemma contains an assertion that is far from obvious and that two steps of the main proof contain typos and misstatements that make the steps difficult to follow. We offer corrections here.

In Example 1 we claim that no equilibrium exists for any type-independent tie-breaking rule. Andreas Blume has pointed out the following counter-example to that claim. Consider a tie-breaking rule where all ties are broken in favor of bidder 1.[1] Bidder 2 bids 3.5 for all types. Bidder 1 bids 3.5 if his type is .5 or higher, and bids $3.5 - \varepsilon t_1$ when of type $t_1 < .5$. For small $\varepsilon > 0$, this is an equilibrium. The error in our proof of the non-existence claim comes in the assertion that if there exists an equilibrium then there exists one in weakly increasing strategies. To justify that assertion, we appealed to Proposition 1 of Maskin and Riley (2000), to which the above is also a counter-example.[2]

A slight modification of our Example 1 resurrects the claim that there does not exist an equilibrium for any type-independent tie-breaking rule. Consider exactly the same setting except for the following: with probability $1-\varepsilon$ there is one item available that is awarded to the high bidder at the high price; and with probability $0 < \varepsilon < 1/200$ there are two items available, in which case the auctioneer randomly draws a number x from a uniform distribution on $[0, 6]$ and gives an item to each bidder whose bid exceeds x at a price of 0.[3] With this modification, whenever there exists an equilibrium there also exists one in weakly increasing strategies. This follows from the observation that if one type of a bidder weakly prefers a high bid to a low one, then a higher type of the

[1] The claim that there are no equilibria is true if the tie-breaking rule gives both bidders a positive probability of winning an object.

[2] The statement of Proposition 1 in earlier working paper versions of Maskin and Riley (2000) asserts monotonicity only when there is a strictly positive probability of winning, and is correct.

[3] This has some flavor of an Amsterdam auction (see Goeree and Offerman, (2004)) in that its effect is to make bidders care about their bids even in the presence of asymmetries (here, in the tie-breaking).

bidder will strictly prefer the higher bid to the lower one, as now any increase in bids increases the probability of obtaining an object. The remaining proof shows that in any equilibrium the bottoms of the supports of the bidding strategies must be identical and must be bid by both bidders with positive probability. A contradiction is then reached by showing that for any choice of this lowest bid which results in a positive probability of ties, at least one bidder would deviate. The (non-trivial) details can be found at www.econ.ucla.edu/zame/CorrigendumProof.pdf. There are other modified versions of the auction that also have no equilibria with type-independent tie-breaking rules, but the details of formulating such examples are very delicate.

In the proof of Lemma 2 part (i) we state that "we may implicitly define a unique element $\mu(E) = \Omega$ by requiring that $\phi \cdot \mu(E) = \lambda_\phi(E)$ for every $\phi \in \mathcal{E}^*$." As it is not obvious why this is true, we provide a fuller proof of part (i) of Lemma 2 here.

Proof of Lemma 2 (i) To show that $M(X, \Omega)$ is weak-* compact, we must show that every net in $M(X, \Omega)$ has a weak-* convergent subnet. To this end, let $\{\mu_\alpha\} \subset M(X, \Omega)$ be a net. For each integer $k \geq 1$ and each k-tuple $\Phi = (\varphi_1, \ldots, \varphi_k) \in (\mathcal{E}^*)^k$, consider the net $\{\Phi \cdot \mu_\alpha\}$ of $\mathbb{R}^k$-valued measures (equivalently, of k-tuples of scalar measures). Since the range of each μ_α lies in Ω, the range of each $\Phi \cdot \mu_\alpha$ lies in $\Phi(\Omega)$, which is a compact convex subset of $\mathbb{R}^k$. Hence we can find a subnet $\{\mu_\beta\}$ of $\{\mu_\alpha\}$ such that, for each Φ the net $\{\Phi \cdot \mu_\beta\}$ converges weak-* to some $\mathbb{R}^k$-valued measure λ_Φ. Because each of the measures $\Phi \cdot \mu_\beta$ has its range in $\Phi(\Omega)$, which is a compact convex set, so does the limit measure λ_Φ. That is, for each Borel set $E \subset X$ we have $\lambda_\Phi(E) \in \Phi(\Omega)$. Hence, for each Borel set $E \subset X$ the set

$$E_\Phi = \Phi^{-1}(\lambda_\Phi(E)) \cap \Omega = \{\omega \in \Omega : \Phi(\omega) = \lambda_\Phi(E)\}$$

is not empty. Since Φ is continuous and Ω is compact, E_Φ is also compact. Note that if $\Phi \in (\mathcal{E}^*)^k, \Psi \in (\mathcal{E}^*)^\ell$ then $(\Phi, \Psi) \in (\mathcal{E}^*)^{k+\ell}$ and

$$E_\Phi \cap E_\Psi = \Phi^{-1}(\lambda_\Phi(E)) \cap \Psi^{-1}(\lambda_\Psi(E)) = (\Phi, \Psi)^{-1}(\lambda_{(\Phi,\Psi)}(E)) = E_{(\Phi,\Psi)}$$

Hence the family of sets $\{E_\Phi : \Phi \in (\mathcal{E}^*)^k, \text{ some } k\}$ has the finite intersection property. Because the sets E_Φ are compact, there is an element of Ω which belongs to all of them. Because $\mathcal{E}$ is locally convex, continuous linear functionals separate points of $\mathcal{E}$, so there is a unique such element. Define $\mu(E)$ to be this unique element of Ω.

By construction, $\varphi \cdot \mu(E) = \lambda_\varphi(E)$ for each Borel set $E \subset X$, so weak countable additivity of μ follows immediately from weak countable additivity of each λ_ϕ. Weak-* convergence of $\{\mu_\beta\}$ to μ is immediate from the definition and construction. We have shown that every net in $M(X, \Omega)$ has a weak-* convergent subnet, so the proof is complete.

The last paragraph on page 1735 contains several typographical errors (t_i^*'s should be a_i^*'s) and a misstatement of (36). Delete from the beginning of the paragraph ("Fix an arbitrary ...") through equation (37) and substitute the following.

> Fix an arbitrary $(s_i^*, a_i^*) \in H$. Continuity of of u_i (in outcomes and types) and continuity of Ψ on T_i^k guarantees that there is a compact neighborhood L of $h(s_i^*, a_i^*)$ in T_i^k such that if $(s_i, a_i) \in S_i \times A_i$ and $t_i, t_i' \in T_i^k$ then
>
> $$|Eu_i(s_i, a_i|\sigma_{-i}, t_i, \theta) - Eu_i(s_i, a_i|\sigma_{-i}, t_i', \theta)| < \frac{1}{4j} \tag{36}$$
>
> Continuity of h on H means that we can choose a compact neighborhood K of (s_i^*, a_i^*) in H such that $h(K) \subset L$. Applying (36), then (35) and then (36) again yields that
>
> $$\begin{aligned} Eu_i(s_i, a_i|\sigma_{-i}, t_i, \theta) &> Eu_i(s_i, a_i|\sigma_{-i}, h(s_i, a_i), \theta) - \frac{1}{4j} \\ &> Eu_i(\sigma_i|\sigma_{-i}, h(s_i, a_i), \theta) + \frac{3}{4j} \\ &> Eu_i(\sigma_i|\sigma_{-i}, t_i, \theta) + \frac{1}{2j} \end{aligned} \tag{37}$$

The β's and B's in Step 5 are missing subscripts that make the argument hard to follow. On page 1737, delete the two sentences starting "Define B:" and "For each r, define β^r..." and substitute the following

> For each r and each type/action pair $(\bar{s}_i, \bar{a}_i) \in S_i^r \times A_i^r$, define $\beta^r_{(\bar{s}_i, \bar{a}_i)} : \Delta_{-i} \to \Omega$ by $\beta^r_{(\bar{s}_i, \bar{a}_i)}(s_{-i}, a_{-i}, s_{-i}) = \theta^r(\bar{s}_i, \bar{a}_i, s_{-i}, a_{-i})$ and $B_{(\bar{s}_i, \bar{a}_i)} : \Delta_{-i} \to \Omega$ by
>
> $$B_{(\bar{s}_i, \bar{a}_i)}(s_{-i}, a_{-i}, s_{-i}) = \Theta(\bar{s}_i, \bar{a}_i, s_{-i}, a_{-i})$$
>
> Note that $\beta^r_{(\bar{s}_i, \bar{a}_i)}$ is a selection from $B_{(\bar{s}_i, \bar{a}_i)}$.

Correspondingly, two paragraphs later replace "$\beta^{r_m}\bar{\sigma}^r_{-i} \to \xi$" with "$\beta^{r_m}_{(s_i^{r_m}, a_i^{r_m})}\bar{\sigma}^r_{-i} \to \xi$". Finally, in the last paragraph on page 1737 replace "there is a selection β from B" with "there is a selection β from $B_{(s_i,a_i)}$."

References

Goeree, J. and T. Offerman, (2004) "The Amsterdam Auction," *Econometrica*, 72, 281-294.

Maskin, E. and J. Riley, (2002) "Equilibrium in Sealed High Bid Auctions," *Review of Economic Studies*, 67, 439-452.

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