Supplemental material for “Pricing to market, trade costs, and international relative prices,” by Andrew Atkeson and Ariel Burstein.

Notes on perfect, Bertrand, and Cournot competition

Relative to the general model described in the paper, we now analyze in detail the case when \( \rho = \infty \), that is when all varieties within a sector are perfect substitutes, and we also abstract from the fixed cost of exporting \((F = 0)\). We consider three types of competition: Perfect competition, Bertrand competition, and Cournot competition.

**Perfect competition** Under perfect competition, the final goods producer in each country \( i \) purchases each variety from the lowest cost supplier of that good to that country, and the price charged for that variety is the marginal cost of that lowest cost supplier.\(^1\) So, the price of sector \( j \) in country \( i \) is given by:

\[
P_{ij} = c_{ij}^{1},
\]

where \( c_{ij}^{1} \) is the marginal cost of the lowest cost producer. For imported varieties, this marginal cost is the marginal cost of production scaled up by the international trade cost \( D \). Clearly, for the non-tradeable varieties, with \( D = \infty \), the lowest cost provider is always domestic and the price is equal to its marginal cost. The extent to which tradeable varieties are traded depends on the balancing of the trading cost \( D \geq 1 \) and the dispersion of idiosyncratic productivities \( z \).

**Bertrand Competition**

Under Bertrand competition, the final goods producer in each country \( i \) purchases each intermediate good from the lowest cost supplier of that good to that country, just as under perfect competition, but the price charged is the marginal cost of production of the second lowest cost supplier of that good to that country. (More precisely, the price charged is the minimum of the monopoly price for the lowest cost supplier and the marginal cost of the second lowest cost supplier). By charging this price, the lowest cost supplier discourages all other potential suppliers from entering. This is the key distinction between Bertrand and perfect competition — under Bertrand competition there is no fixed relationship between the price of each intermediate good and the marginal cost of the supplier of that good.

\(^1\)This is a pricing assumption, since in our setup there is a finite number of heterogeneous producers within a sector and that wouldn’t necessarily lead to marginal cost pricing.
To model Bertrand competition, let $c^k_{ij}$ denote the marginal cost of the $k^{th}$ lowest cost supplier ($k = 1$ or $2$) of sector $j$ to country $i$. Then, the price is given by:

$$P_{ij} = \min \left\{ c^2_{ij} \cdot \frac{\eta}{\eta - 1} c^1_{ij} \right\}$$

(0.2)

Since, in each country $i$, each intermediate good is supplied by the lowest cost supplier, the accounting for labor is the same as in the case of perfect competition.

**Cournot Competition** Under Cournot competition, the number of suppliers of each intermediate good to country $i$ is endogenous and depends on the full distribution of productivities of the $2 \times K$ potential suppliers. Suppliers compete in terms of quantities and the price of each intermediate is the price resulting from the CES demand function for each variety at the quantities being supplied. Given prices for final goods $P^T_i$, $P^N_i$, and wage rates in each country $W_i$, together with the vector of productivities in sector $j$, we compute equilibrium price and quantity for that good in country $i$ with an iterative procedure as follows. We first compute the monopoly price and quantity that would arise if country $i$ were supplied only by the lowest marginal cost supplier. We then compute the price and quantity that would arise as we add each next-lowest-cost supplier to the market and continue to do so until the marginal entrant no longer earns positive profits. We assume that there are no fixed costs of production.

Let $c^k_{ij}$ denote the marginal cost of the $k^{th}$ lowest cost supplier of sector $j$ to country $i$. The resulting equilibrium price is:

$$P_{ij} = \frac{\eta}{K_{ij} \eta - 1} \sum_{k=1}^{K_{ij}} c^k_{ij}$$

(0.3)

where $K_{ij}$ is the equilibrium number of suppliers of good $j$ to country $i$. Note here that this price depends on the marginal cost of all firms that are actively supplying a given market.\(^2\)

**Aggregate Prices**

In measuring movements in aggregate prices, we use expenditure shares from the symmetric equilibrium to weight the changes in prices of the individual varieties that comprise

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\(^2\)Note that if it is optimal for the $k^{th}$ lowest cost supplier in the market to produce, then it will also be optimal for any producer with a lower marginal cost. These producers can always produce a quantity close to zero, so the sectorial price will be unchanged, and the price will be at least equal to their marginal cost (this is the case for the $k^{th}$ lowest cost producer for him to decide to produce).
the aggregate price index. We follow this procedure in the model to mimic the procedure used to construct the data.

Export and import price indices are defined as aggregates of price changes of traded varieties. Let \( \hat{\text{EPI}} \) be the change in country 1’s export price index (import price index for country 2). Here a hat on a variable indicates the log deviation of the variable from its symmetric equilibrium value. \( \hat{\text{EPI}} \) is given by:

\[
\hat{\text{EPI}} = \frac{1}{s_M} \int s_{2j}^1 s_{2j} \hat{P}_{2j} dj,
\]

where \( s_{ij} \) denotes the symmetric equilibrium expenditure share of variety \( j \) over total tradable consumption in country \( i \), and \( s_{ij}^1 \) is the symmetric equilibrium share of expenditure on variety \( j \) in country \( i \) supplied by producers located in country 1. \( s_M \) is the share of tradeables expenditure on imports, which is equal in both countries with symmetry and balanced trade. It is given by:

\[
s_M = \int s_{2j}^1 s_{2j} dj.
\]

The definition of the import price index for country 1, \( \hat{\text{IPI}} \) is symmetric. Note that, under perfect and Bertrand competition there is only one active producer per variety, so \( s_{2j}^1 = 1 \) for imported varieties and 0 otherwise. Under Cournot competition, \( s_{2j}^1 \in [0, 1] \).

Our log-linearization strategy assumes that the set of exporting producers and exported varieties remains unchanged after the shock. This is not the case when we compute the exact equilibrium (which we can easily do under no fixed cost of exporting). In this case we follow the methodology employed by the BLS in computing its export and import price indices as described in the Handbook of Methods, Chapter 15.3

The producer price index is defined as an aggregate of price changes of domestically produced and sold tradeable goods. For simplicity in the algebra, we abstract for now from the presence of prices of exported goods in the producer price index. Let \( \hat{\text{PPI}}_1 \) denote the change in the logarithm in the producer price index in country 1. It is given by:

\[
\hat{\text{PPI}}_1 = \frac{1}{1 - s_M} \int s_{1j}^1 s_{1j} \hat{P}_{1j} dj
\]

3 The main points that we consider are: (1) use of symmetric equilibrium expenditure shares (our base year), (2) for newly imported varieties, impute price change as the change in the import price index, and use their new actual expenditure share (reducing the total weight of the continued imported varieties), and (3) when there are discontinued imported varieties, re-balance the base-year weights of continued imported varieties so that they add up to 1.
Symmetrically, $\hat{P}PI^T_2$ is given by:

$$\hat{P}PI^T_2 = \frac{1}{1 - s_M} \int (1 - s_{2j}^1) s_{2j} \hat{P}_2 dj$$

The change in the consumer price index for tradeable goods in country 1 is given by

$$\hat{P}_1^T = (1 - s_M)\hat{P}PI^T_1 + s_M \hat{P}I,$$

and that for country 2 by

$$\hat{P}_2^T = (1 - s_M)\hat{P}PI^T_2 + s_M \hat{E}PI,$$

Hence, the change in the relative consumer price of tradeable goods is given by

$$RER^T = (1 - s_M) \left( \hat{P}PI^T_1 - \hat{P}PI^T_2 \right) + s_M \left( \hat{I}PI - \hat{E}PI \right). \quad (0.5)$$

The change in the price of the final consumption good in country $i = 1, 2$ is a weighted average of the change in the price index for tradeable goods and the price index of non-tradeable goods given by

$$\hat{P}_i = \gamma \hat{P}^T_i + (1 - \gamma) \hat{P}^N_i. \quad (0.6)$$

The change in the real exchange rate is given by $RER = \hat{P}_1 - \hat{P}_2$.

**Log-linearization**

To understand the logic of pricing in our model, we take a log-linear approximation to express the change in the price of individual varieties and various price aggregates of interest as linear combinations of the change in marginal costs in each country. We then study how the coefficients in these expression differ according to the nature of competition.

Let $\hat{P}_{ij}$ be the percentage change in the price of variety $j$ in country $i$. Under all three forms of competition that we consider, this price change can be approximated as a weighted average of the change in marginal cost in each country

$$\hat{P}_{ij} = a_{ij} \hat{W}_1 + (1 - a_{ij}) \hat{W}_2,$$  \quad (0.7)

where $0 \leq a_{ij} \leq 1$, differs according to the nature of competition. Here, $\hat{W}_i$ is the change in marginal cost for producers in country $i$.

In defining the term *pricing-to-market* for an individual firm, we focus on the prices charged by a single producer of a traded variety $j$ in different locations. Consider a variety
that is exported by (at least) one producer from country 1 to country 2. Our pricing-to-market measure is \( PTM_j = \hat{P}_1 - \hat{P}_2 \). It reflects the extent to which deviations from the law of one price arise from the pricing decision of a single producer supplying different locations as opposed to the pricing decisions of different producers supplying different locations. In terms of (0.7), there is pricing-to-market for a traded variety when \( a_{1j} \neq a_{2j} \). Note that the type of pricing-to-market that has been generally documented in the data is \( a_{1j} > a_{2j} \) for country 1 exporters and \( a_{1j} < a_{2j} \) for country 2 exporters. In these cases, after an increase in their marginal cost, exporters increase their domestic price relative to the export price.

The change in the producer price index is given by:

\[
\hat{PPIT}_1 = v\hat{W}_1 + (1 - v)\hat{W}_2
\]

where \( 0 \leq v \leq 1 \) differs according to the nature of competition and is given by

\[
v = \frac{1}{1 - s_M} \int a_{1j}s_{1j}^1s_{1j}dj
\]

Given the symmetry in our model,

\[
\hat{PPIT}_2 = v\hat{W}_2 + (1 - v)\hat{W}_1,
\]

and \( v \) does not vary across countries.

Similarly, the change in the export price index in country 1 is given by:

\[
\hat{EPI} = b\hat{W}_1 + (1 - b)\hat{W}_2,
\]

where

\[
b = \frac{1}{s_M} \int a_{2j}s_{2j}^1s_{2j}dj.
\]

The coefficient \( b \) varies according to the nature of competition. We interpret \( b \) as a measure of the extent to which changes in marginal costs in country 1 are passed-through to the import price index in country 2. Given that the 2 countries are symmetric, it follows that:

\[
\hat{IPI} = b\hat{W}_2 + (1 - b)\hat{W}_1.
\]

Note that without trade costs (\( D = 1 \)), the law of one price holds, so \( a_{1j} = a_{2j} \). Moreover, given that the two countries are symmetric, \( s_{1j}^1 = s_{2j}^1, s_{1j} = s_{2j}, s_M = 0.5 \), and thus \( v = b \).

We can decompose the relative change in producer prices across countries as:

\[
\hat{PPIT}_1 - \hat{PPIT}_2 = \left( \hat{PPIT}_1 - \hat{EPI} \right) + \left( \hat{IPI} - \hat{PPIT}_2 \right) + \left( \hat{EPI} - \hat{IPI} \right)
\]

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We can re-write each of the three terms as:

\[ \hat{PPI}_T^1 - \hat{EPI} = (\nu - b) \left( \hat{W}_1 - \hat{W}_2 \right) \]

\[ \hat{IPI} - \hat{PPI}_T^2 = (\nu - b) \left( \hat{W}_1 - \hat{W}_2 \right) \]

\[ \hat{EPI} - \hat{IPI} = (2b - 1) \left( \hat{W}_1 - \hat{W}_2 \right) \]

In the case without trade costs \((D = 1)\) \(v = b\), so the first two terms are zero. So, there is no pricing-to-market and \(\hat{PPI}_T^1 - \hat{PPI}_T^2 = \hat{EPI} - \hat{IPI}\). Note that in this case there might still be incomplete pass-through (i.e.: \(b < 1\)) even though there is no pricing-to-market. In order for the terms of trade to change by less than the PPI-based RER we need \(b < v\).

It is also useful to decompose \(\hat{RER}_T\) using expression (0.5) as:

\[
\hat{RER}_T = (1 - 2s_M) \left( \hat{W}_1 - \hat{W}_2 \right)
+ (1 - 2s_M) \left( \hat{PPI}_T^1 - \hat{W}_1 - \hat{PPI}_T^2 + \hat{W}_2 \right)
+ s_M \left( \hat{IPI} - \hat{PPI}_T^2 - \hat{EPI} + \hat{PPI}_T^1 \right).
\]

This equality decomposes the change in the relative consumer price of tradeable goods into three components. The first component is simply the change in relative wages, which we take to be exogenous. The second component depends on the degree to which the change in relative wages, \(\hat{W}_1 - \hat{W}_2\) is passed through to the prices paid for domestic sales in the two countries as measured by \(\hat{PPI}_T^1 - \hat{PPI}_T^2\). The third component depends on the degree of pricing to market as measured by the change in the relative price of exports and domestic sales for each country given by \(\hat{IPI} - \hat{PPI}_T^2\) and \(\hat{EPI} - \hat{PPI}_T^1\).

The last two terms can be re-written as:

\[
(1 - 2s_M) \left( \hat{PPI}_T^1 - \hat{W}_1 - \hat{PPI}_T^2 + \hat{W}_2 \right) = 2(1 - 2s_M) (\nu - 1) \left( \hat{W}_1 - \hat{W}_2 \right)
\]

\[
s_M \left( \hat{IPI} - \hat{PPI}_T^2 - \hat{EPI} + \hat{PPI}_T^1 \right) = 2s_M (\nu - b) \left( \hat{W}_1 - \hat{W}_2 \right)
\]

Under no trade costs \((D = 1)\), these two terms are equal to zero because \(s_M = 0.5\) and \(v = b\). So, \(\hat{RER}_T = 0\). In general, the first term will negatively contribute to the change in the relative price of tradeables when \(v < 1\), and the first term will contribute positively to the change in the relative price of tradeables when \(b < \nu < 1\).

In all three versions of our model, \(a_{ij} = 1\) for all non-tradeable varieties since all competitors producing these varieties are domestic. This implies that the price of non-tradeable
varieties moves one-to-one with change in the producer’s marginal costs. Thus, the change in the relative price of the final tradeable good relative to the change in the RER is given by:

$$\frac{\hat{RER}_T}{RER} = \frac{\frac{\hat{RER}_T}{\gamma \hat{RER}_T + (1 - \gamma) \left( \hat{W}_1 - \hat{W}_2 \right)}}{\gamma \hat{RER}_T + (1 - \gamma) \left( \hat{W}_1 - \hat{W}_2 \right)}.$$ (0.11)

We have calibrated the parameter $\gamma = 0.4$ to match the share of goods in the U.S. consumption. Hence, our model produces a larger movement in the relative price of tradeable goods, the larger is $\hat{RER}_T$. We now consider the determination of parameters $\nu$ and $b$ under our three forms of competition.

**Perfect Competition** In this case, prices are set at the marginal cost of the producer, so $a_{1j} = 1$ for all varieties produced in country 1 (both those that are not traded and those that are exported to country 2), and $a_{1j} = 0$ for all imported varieties. Likewise, $a_{2j} = 0$ for all varieties produced in country 2 (both those that are not traded and those that are exported to country 1), and $a_{2j} = 1$ for all imported varieties. Hence, there is no pricing-to-market since $a_{1j} = a_{2j}$ for all varieties that are actually traded.

Hence, under perfect competition

$$\nu = \frac{1}{1 - s_M} \int s_{1j}^1 s_{1j} dj = 1$$

and

$$b = \frac{1}{s_M} \int s_{2j}^1 s_{2j} dj = 1$$

So, we have:

$$\hat{PPI}_1 - \hat{PPI}_2 = \hat{EPI} - \hat{IPI} = \hat{W}_1 - \hat{W}_2$$

and

$$\hat{RER}_T = (1 - 2s_M) \left( \hat{W}_1 - \hat{W}_2 \right)$$

With a small share of goods that are actually traded in the cost of the CPI bundle that is considered tradeable, the change in $\hat{RER}_T$ can be significant.

**Bertrand Competition** The logic of pricing is quite different from the competitive model because pricing is determined by the costs of the second lowest cost supplier of a good. For all non-tradeable varieties, the second lowest cost supplier is domestic, so, $a_{1j} = 1$ and $a_{2j} = 0$ as under perfect competition. For tradeable varieties, the second lowest cost supplier can be local or located abroad. To compare pricing under Bertrand and perfect competition, it is useful to group tradeable varieties purchased in each country into four
categories: (1) varieties that are produced locally and priced at the marginal cost of a local
competitor (we denote the share of this category of goods in tradeable goods consumption by
\(s^{LL}\)), (2) varieties that are produced abroad and priced at the marginal cost of a competitor
that is also located abroad (share \(s^{MM}\)), (3) varieties produced locally but priced at the cost
of a competitor abroad (share \(s^{LM}\)), and (4) varieties produced abroad but priced at the
cost of a local competitor (share \(s^{ML}\)). Locally produced products priced at the monopoly
markup of \(\eta/(\eta - 1)\) are included in \(s^{LL}\) while imported varieties priced at this markup are
included in \(s^{MM}\). Note that, given the symmetry in our model, these shares do not vary
across countries.

In country 1, for varieties in categories (1) or (4), \(a_{1j} = 1\), and for varieties in categories (2)
and (3), \(a_{2j} = 0\). Hence \(v\) is given by the expenditure share on category (1) as a fraction of the
expenditure share on both local categories (1) and (3), that is \(v = s^{LL}/(s^{LL} + s^{LM})\). Likewise,
in country 2, for varieties in categories (1) or (4), \(a_{2j} = 0\), and for varieties in categories
(2) and (3), \(a_{2j} = 1\), and again \(v = s^{LL}/(s^{LL} + s^{LM})\). Similarly, \(b = s^{MM}/(s^{MM} + s^{ML})\). In
general we will have that \(\nu > b\). Note that even under \(D = 1\) we can have \(b < 1\) (incomplete
pass-through but no pricing-to-market).

Two comments are worth noticing in terms of the decompositions discussed above. First,
when \(s^{MM} = 0\) (all exporters compete with local producers), then \(b = 0\), and \(\hat{EPI} - \hat{IPI} = -\left(\hat{W}_1 - \hat{W}_2\right)\). In this extreme case, the terms of trade would move \(-100\%\) with the change
in relative marginal costs — the exact mirror image of the competitive case. Second, in
expression (0.10) the sum of the last two terms is proportional to:

\[
(1 - 2s_M) (\nu - 1) + s_M (\nu - b) \\
= (1 - s_M) (\nu - 1) + s_M (1 - b) \\
= -(1 - s_M) \frac{s^{LM}}{1 - s^{LM} + s^M} \\
= s^{ML} - s^{LM} \\
\]

So, when \(s^{ML} = s^{LM}\), \(\hat{RER}^{T}\) is like under perfect competition. Another way of showing
this is to note that under Bertrand competition we have:

\[
\hat{P}_t^T = (s^{LL} + s^{ML}) \hat{W}_1 + (s^{LM} + s^{MM}) \hat{W}_2 \\
\]

This will be the same as under perfect competition when \(s^{LL} + s^{ML} = 1 - s^M\), or \(s^{ML} = s^{LM}\).

\[\text{It can be shown that if the } z' \text{'s are exponentially distributed, then } s^{LM} = s^{ML}.\]
Pricing-to-market under Bertrand competition can be understood as follows. Consider the pricing of a variety $j$ that is produced in country 1 and exported to country 2. For a portion of such varieties, the producer will face the same latent competitor as the second lowest cost supplier to both markets. If that latent competitor is located in country 1, then $a_{1j} = a_{2j} = 1$, while if that competitor is in country 2, then $a_{1j} = a_{2j} = 0$. In either of these cases, there is no pricing-to-market since $\hat{P}_{1j} = \hat{P}_{2j}$. For the remainder of these varieties that are produced in country 1 and actually traded, the producer will face a local latent competitor when selling in country 1 and a different local latent competitor when selling in country 2. Hence $a_{1j} = 1$ and $a_{2j} = 0$. For these varieties, there is pricing-to-market since

$$\hat{P}_{1j} - \hat{P}_{2j} = \hat{W}_1 - \hat{W}_2.$$ 

Similar arguments apply for exporters located in country 2.

Clearly, these deviations from relative PPP for those goods that are actually traded that occur under Bertrand competition contribute to the fluctuations in the relative price of tradeable goods. Bertrand competition introduces an offsetting effect, however, for the prices of tradeable varieties that are not traded in comparison to the benchmark of perfect competition. Under perfect competition, for all tradeable varieties that are produced in country 1 and not exported, $a_{1j} = 1$, while for those produced in country 2 and not exported, $a_{2j} = 0$. Hence, for all tradeable varieties that are not traded

$$\hat{P}_{1j} - \hat{P}_{2j} = \hat{W}_1 - \hat{W}_2.$$ 

Under Bertrand competition, the pricing of tradeable varieties that are not traded is still determined by the cost of the latent competitor, and this competitor can be located in either country. Hence, for those varieties produced in country 1 and not exported, a fraction face local latent competition and hence have $a_{1j} = 1$, while the remainder face foreign latent competition and have $a_{1j} = 0$. Applying the same argument to producers in country 2 who do not export, we get that, on average, the fluctuations in the relative price of those tradeable varieties that are not actually traded is smaller under Bertrand competition than it is under perfect competition. Again, to the extent that $s^{LM} \simeq s^{ML}$, the effects of Bertrand competition on pricing-to-market and on the pricing of tradeable varieties that are not actually traded essentially offset when aggregated to determine the fluctuations in the overall relative price of tradeable goods.
Cournot Competition: To analyze pricing under Cournot competition, we log-linearize (0.3) holding fixed the number of competitors in each market. Doing so, it is evident, in response to a shock, the price of each variety changes by a weighted average of the change in marginal costs of the producers of that variety, with the weights given by the symmetric equilibrium cost share of each producer. Thus, the fluctuations in the price of any particular variety will be a weighted average of the fluctuations in costs in countries 1 and 2, with the weight depending on the extent to which the variety is supplied by local producers or imported from foreign producers.

Expressing this log-linearization in terms of expenditure shares instead of cost shares gives

\[ a_{ij} = s_{ij}^1 + \mu_{ij} \left( \frac{K_{ij}^1}{K_{ij}} - s_{ij}^1 \right) \],

(0.12)

where

\[ \mu_{ij} = \frac{P_{ij}}{\sum_{k=1}^{K_{ij}} c_{ik}^k / K_{ij}}. \]

Here \( K_{ij}^1 \) is the symmetric equilibrium number of producers located in country 1 and selling in country \( i \), and \( \mu_{ij} \) is the steady-state average markup in country \( i \).

For country 1, the price of good \( j \) will move with costs in country 1 by the fraction of costs of producers in good \( j \) located in country 1 in the total costs of production. This share of costs differs from the share of tradeables expenditure purchased from producers located in country 1 (given by \( s_{ij}^1 \)), because markups are not equated across producers. The second term in (0.12) corrects for this difference between cost and expenditure shares.

Exporters will practice pricing-to-market whenever \( a_{1j} \neq a_{2j} \). In particular, for country 1 exporters \( a_{1j} > a_{2j} \) whenever their share in total costs is larger in country 1 relative to country 2. This will be more likely when their share of expenditure in the foreign market is low relative to the domestic expenditure share, and when the number of producers of country 1 exporting to country 2 is small relative to the number producers of country 1 selling in country 1.

Integrating (0.12) across tradeable varieties in country 1, we obtain \( v \) and \( b \) defined above. We can show that when \( a_{2j} < a_{1j} \) on average, we will have \( b < \nu < 1 \) (TO DO). So, the same qualitative results as under Bertrand competition will apply.