5. Equilibrium and Efficiency

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A. The Robinson Crusoe Economy

In the previous chapters we have examined the behavior of individuals whose influence over market prices is sufficiently small that price-taking rather than price seeking is a natural assumption. In this chapter we examine the allocation that results if all economic agents are price takers and prices adjust until markets clear. One way to think about this adjustment process is to imagine that an auctioneer calls out a price vector. Consumers and firms respond with the demands that they would make at these prices. The auctioneer lowers prices where there is excess supply and raises them where there is excess demand.

As a first example we consider trading in a one person economy. It may seem odd that there could be room for prices in such an economy. However Robinson Crusoe, our single agent, does have two roles. As the manager of the firm (or farm,) he must choose a production plan. And as the consumer he must choose between leisure and the goods that he can purchase if he works harder. We will suppose that at any point in time, Robinson Crusoe wears only one hat, either his "manager hat" or his "consumer hat." Market equilibrium prices then guide his decision.

An alternative interpretation is that there are many agents similar to Robinson Crusoe in the economy. Thus price-taking behavior becomes a natural assumption. And if these agents happen to be identical, we can analyze the economy as if there were only one agent. In this interpretation, Robinson Crusoe is the representative agent in the economy - - representing all his fellow agents.

The Firm

Up to this point we have maintained a clear distinction between inputs and outputs of a production process, writing a production plan as an \( n+m \) vector \((z,q)\). Given an input price vector \(r\) and output price vector \(p\), the profit of the plan is \( p \cdot q - r \cdot z \). However this distinction is somewhat restrictive. Many firms produce intermediate goods so that an output of one firm is an input of another. This problem is easily surmounted if we allow quantities in a production plan to be either negative or positive. If a quantity is negative (so is to be subtracted from the aggregate supply) it is an input. If the quantity is positive it is an output. Then a production plan is a vector \( y \in Y \subseteq \mathbb{R}^N \). Given the price vector \(p\) the profit of the firm is then \( p \cdot y \).

Let us now consider what Robinson would do as a price-taking manager of the firm. All profit is distributed as dividends so shareholder's interests are served if the manager maximizes the firm's profit. Robinson then chooses a production plan \( y^f \) such that

\[
\Pi^f = p \cdot y^f = \text{Max}_y \{ p \cdot y \mid y \in Y \}
\]

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1 This section is based on T. Koopmans "Three Essays on the State of Economic Science"
In this section we will focus on the following very simple example. Suppose labor $L$ is used to produce coconuts $C$ according to the production function $C = F(L)$. Converting into the formal notation, a production plan $(y_1, y_2) = (-L, C)$ must lie in the production set

$$Y = \{ (y_1, y_2) \mid y_2 \leq F(-y_1), y_1 \leq 0 \}.$$ 

Let $p = (W, P)$ be the price vector. That is $W$ is the wage and $P$ is the price of coconuts. Robinson the manager then chooses a production plan to solve the following problem.

$$\max_{L,C} (PC - WL \mid C = F(L)).$$  \hspace{1cm} \text{(A.1)}$$

Substituting for $C$ and differentiating, we obtain the first order condition

$$PF'(L) = W.$$ 

Robinson’s optimal choice is depicted in Figure A.1 below. Each line in the figure is an iso-profit line. That is a line of the form $PC - WL = \Pi$. Robinson then maximizes profit where the iso-profit line just touches the boundary of the production set.

Fig. A.1: Profit Maximization
Robinson the manager goes home and becomes Crusoe the consumer. A consumption vector \( x \) is a list of commodities that Crusoe receives and supplies. Positive elements of this vector are items received and negative elements are items delivered (such as labor inputs.) Since Crusoe is the only shareholder, his dividend income is the profit of the firm. If his preferences are represented by the utility function \( U(\cdot) \) a price-taking Crusoe chooses a consumption plan \( x^h \) such that

\[
U(x^h) = \max_x \{ U(x) \mid p \cdot x \leq \Pi' \}
\]

Reverting to the simple example, if Crusoe works \( L \) hours he earns a wage income of \( WL \). In addition he receives his daily dividend. Crusoe the consumer then seeks to solve the following maximization problem.

\[
\max_{L,C} \{ U(L, C) \mid PC \leq WL + \Pi' \}
\]

The budget constraint and indifference curves are depicted below.

The two figures are superimposed in Figure A.3. Note that there is excess demand for coconuts and excess supply of labor. Thus the Walrasian auctioneer raises the price of coconuts and lowers the price of labor. The Walrasian equilibrium is depicted in Figure A-4.
Note that the equilibrium point in Figure A.4 is the utility maximizing production plan. Thus the example provides a simple illustration of Adam Smith's "invisible hand" at work. Even though Robinson Crusoe is schizophrenic, his price taking behavior results in an equilibrium outcome which is optimal.

![Figure A.3: Supply and demand](image1)

![Figure A.4: Walrasian Equilibrium](image2)
Example:

Suppose that the production function $F(L) = (L + a)^{1/2}$ and preferences are represented by the utility function $U = 18 \ln C - L$.

Assuming that the firm does produce, profit $\Pi = PF(L) - WL$ is maximized where

$$\Pi'(C) = PF'(L) - W = 0,$$

that is $6P(L + a)^{-1/2} - W = 0$.

Then $L^f = (6P/W)^2 - a$ and so output is

$$C^f = 72 \frac{P}{W}.$$ (A.2)

Maximized profit is then

$$\Pi^f = PC^f - WL^f = W(a + \frac{6P}{W})^2.$$

Note that if $a < -(\frac{6P}{W})^2$, profit is negative so the firm is better off going out of business. Initially we will assume that $a > 0$ so the constraint holds for all prices.

From the budget constraint, $L$ must satisfy $L = \frac{PC - \Pi^f}{W}$. Substituting for $L$ in the utility function,

$$U = 18 \ln C - (PC - \Pi^f)/W.$$

Differentiating by $C$ we have the first order condition

$$\frac{dU}{dC} = \frac{18}{C} - \frac{P}{W} = 0.$$

Then the demand for coconuts is

$$C^h = \frac{18W}{P}.$$ (A.3)

From (A.2) and (A.3), the excess demand for coconuts is

$$C^h - C^f = 18 \frac{W}{P} - 72 \frac{P}{W}.$$

The Walrasian equilibrium "real wage" is therefore $W/P = 2$

This example has been constructed so that the equilibrium price vector is independent of the parameter $a$ which appears in the production function. Figure A.5 depicts two possible situations.
when this parameter is negative. That is, there is some minimal fixed labor cost before any output can be produced. In each case the social optimum is the point S. In the left figure the profit line intersects the vertical axis at a point $\bar{\Pi}/P > 0$. Thus the profit associated with the social optimum is positive. However, in the right-hand figure, this profit is negative. Thus the profit-maximizing firm is better off choosing to produce nothing. The reader is left to confirm that for any real wage sufficiently low to make production possible, supply exceeds demand. Thus there is no Walrasian equilibrium.

Fig. A.5: Fixed costs

From Fig. A.5, we can map out the profit maximizing output for different output prices. Only if the output price is sufficiently high relative to the wage is it optimal to produce a positive output. The firm's supply curve thus has a discontinuity.

Fig. A.6: General equilibrium supply and demand

Whenever such discontinuities occur, there may be no Walrasian equilibrium. Conversely, as long as Walrasian demand and supply curves do not exhibit such discontinuities, there is always
a Walrasian equilibrium. We shall not study the issue of existence here. Intuitively, as long as technology and preferences are convex, the troubling discontinuities cannot occur. Thus a convex economy has a Walrasian equilibrium.

**Exercise A.1:**
Robinson has a utility function \( U(L, C) = (24 - L)C \). He can produce coconuts according to the production function \( C = \sqrt{L} \).

(a) Solve for his optimum.
(b) If the price of coconuts is 1, what wage will induce Robinson, acting as a profit maximizing manager, to demand the optimal labor input.
(c) Depict this in a neat figure showing the production set, preferences and profit maximization.

**Exercise A.2:**
Suppose that Robinson can produce coconuts using production function \( C = 2L + b \). Preferences are represented by the utility function \( U(C, H) = \ln(a + C) + \ln H \), where \( H \) is leisure. Robinson divides the total time in the day \( \bar{T} \) between leisure and work.

(a) Solve for the optimum and the price-taking equilibrium.
(b) If \( a > 0 \) and \( b < 0 \) explain why there is no competitive equilibrium.
(c) If \( a > b = 0 \), under what conditions will the equilibrium supply of labor be zero?
(You might interpret \( a \) as the number of coconuts which simply fall from the palm trees without any harvesting effort.)

**Exercise A.3: Robinson meets Friday**
Suppose that Robinson owns the firm but is unable to work. Friday has a utility function \( U(C, L) = \ln C + \ln(24 - L) \). The technology is the same as in the previous question.

(a) Solve for Friday’s labor supply curve.
(b) Hence, or otherwise, solve for the Walrasian equilibrium.
(c) Depict the equilibrium in a neat figure, showing Friday’s budget constraint and the firm’s profit maximizing production plan.

**B. Equilibrium and Efficiency in an exchange Economy**

Before turning to general results, we can develop further insights by focussing on a simple exchange economy. In this economy each consumer has an endowment of commodities and can trade freely with others. Consumer \( h, \ h = 1, ..., H \) has a strictly increasing utility function \( U^h(x^h) \) over his consumption set \( X^h = \mathbb{R}^N_+ \). Utility is strictly increasing in consumption of all commodities. His endowment is \( \omega^h \).
Let $\bar{x}^h$ be the utility maximizing consumption vector of consumer $h$, given that the price vector is $p$. That is

$$\bar{x}^h \text{ solves } \max_x U^h(x) \mid p \cdot x \leq p \cdot \omega^h$$

The resulting allocation is a Walrasian equilibrium allocation if all markets clear, that is,

$$\sum_{h=1}^H \bar{x}^h = \sum_{h=1}^H \omega^h.$$

Suppose that there are just two consumers and two commodities. (Equivalently there are two groups of consumers, equal in numbers. Within each group everyone has the same endowments and preferences.) Then we can illustrate the Walrasian equilibrium allocation in the familiar Edgeworth-box diagram.

![Edgeworth-box diagram](image)

Fig. B.1: Excess demand for commodity 1

As depicted, in Figure B.1, Alex wants to trade from the endowment point $N$ to $C^A$, while Bev wishes to trade from $N$ to $C^B$. Thus, there is excess demand for commodity 2 and excess supply of commodity 1. The Walrasian auctioneer thus raises the price of commodity 2 relative to commodity 1 and so flattens the budget line until supply equals demand. The Walrasian equilibrium $E$ is depicted in Fig. B.2.

For the point $E$ to be optimal for Alex, it must be the case that all strictly preferred allocations are above the budget line $NE$. Moreover, since utility is strictly increasing, any weakly preferred bundle must be on or above this line. Similarly, for the point $E$ to be optimal for Bev, it must be the case that all strictly preferred allocations are below the budget line $NE$. Thus any allocation
which strictly preferred by Bev over her equilibrium allocation $\bar{x}^B$ is an allocation which is strictly worse for Alex, and vice versa. This is the sense in which the Walrasian equilibrium is "efficient."

Generalizing to many consumers and many commodities, we have the following definition of efficiency.

**Pareto Efficiency**

A feasible allocation $\{\hat{x}^h\}_{h=1}^H$ in an exchange economy with endowments $\omega^h$, $h = 1, \ldots, H$ is Pareto efficient if there is no other allocation which is strictly preferred by at least one consumer and is weakly preferred by all consumers.

As we shall see, the simple diagrammatic argument readily generalizes. That is, a Walrasian equilibrium of an exchange economy is Pareto efficient.

Suppose we now consider the allocation rule from the perspective of an altruistic, omniscient and omnipotent central planner. In deciding on the socially optimal allocation, the logical first step is to first ask what allocations are non-wasteful in the sense of Pareto efficiency. The next step is to choose from among these allocations according to some notion of social justice. The question then arises as to whether, for each Pareto-efficient allocation, there is a Walrasian equilibrium price vector. If so, it follows that the planner can achieve any social objective by introducing an appropriate set of taxes and transfers. There is no further gain to introducing direct centralized allocation rules or other market regulation.
For the exchange economy, this will be the case if preferences are convex. Again we illustrate using an Edgeworth box diagram. Consider all allocations which provide Bev with a utility level of at least $\hat{U}^B$. This is the shaded region in Figure B.3. Choose from this set the allocation $(\hat{x}^A, \hat{x}^B)$ which is best for Alex. In the figure, this is the point $\hat{C}$. By construction, any allocation which is better for Alex must be worse for Bev. Thus $(\hat{x}^A, \hat{x}^B)$ is Pareto efficient.

In Figure B.3, the Pareto efficient allocations are shown as a heavy curve. This is reproduced in Figure B.4. This time the allocations preferred by each consumer over $\hat{C}$ are shaded. By assumption, preferred sets are convex. Then, by the separating hyperplane theorem there is a supporting hyperplane through $\hat{C}$.

Suppose that the endowment allocation is $(\omega^A, \omega^B)$. Thus, to achieve the outcome $(\hat{x}^A, \hat{x}^B)$ the central planner taxes consumer $h$ an amount $T^h = p \cdot (\omega^h - \hat{x}^h)$, $h = A, B$. Then, given the supporting price vector $p$, Alex has a budget constraint,

$$p \cdot x \leq p \cdot \omega^A - T^A = p \cdot \hat{x}^A.$$  

A symmetric argument for Bev establishes that her budget constraint is

$$p \cdot x \leq p \cdot \omega^B - T^B = p \cdot \hat{x}^B.$$
Note finally that the total tax revenue \( \sum_{h=A}^{B} T^h = p \cdot (\sum_{h=A}^{B} \omega^h - \hat{x}^h) \) is zero.

The Lagrange method provides a convenient way to solve for these prices. The following optimization problem formalizes the discussion summarized in Figure B.3.

\[
\text{Max} \{ U^A(x^A) \mid U^B(x^B) \geq \hat{U}^B, \ x^A + x^B \leq \omega \}.
\]

Forming the Lagrangian,

\[
L = U^A(x^A) + V^B(U^B(x^B) - \hat{U}^B) + \sum_{j=1}^{2} \mu_j (\omega_j - x^A_j - x^B_j).
\]

If we define \( V^A = 1 \), the necessary conditions for Pareto efficiency are as follows.

\[
\frac{\partial L}{\partial x^A_j} = \nu^h \frac{\partial U^h}{\partial x^A_j} - \mu_j \leq 0, \text{ with equality if } x^A_j > 0. \quad (B.1)
\]

It is a straightforward exercise to confirm that for any consumer whose allocation of both goods is strictly positive,
\[ MRS^h = \frac{\partial U^h}{\partial x_1} / \frac{\partial U^h}{\partial x_2} = \frac{\mu_1}{\mu_2}. \]  

(B.2)

Thus, for an interior solution (that is, a Pareto efficient allocation in the interior of the Edgeworth-box) it must be the case that

\[ MRS^A = \frac{\partial U^A}{\partial x_1} / \frac{\partial U^A}{\partial x_2} = \frac{\mu_1}{\mu_2} = \frac{\partial U^B}{\partial x_1} / \frac{\partial U^B}{\partial x_2} = MRS^B \]  

(B.3)

Thus the optimum is supported by the shadow price vector \((\mu_1, \mu_2)\)

**Example: Quasi-linear preferences**

Alex has a utility function \(U^A = x_1^A + \ln x_2^A\) while Bev has a utility function \(U^B = x_1^B + 2\ln x_2^B\). In this case the Pareto efficient allocations are as depicted below.

If \(U^B(x^B) \geq U^B\), Alex’s utility is maximized by choosing the allocation where the slopes of the two indifference curves (marginal rates of substitution) are equal. Formally, condition (B.3) must
hold. In this special case $MRS^A = x_2^A$ and $MRS^B = \frac{1}{2} x_2^B$. Thus, for an interior allocation in the Edgeworth Box to be Pareto Efficient, it must be the case that $x_2^A = \frac{1}{2} x_2^B$.

Since total consumption of commodity 2 is $\omega_2$, it follows that for an allocation in the interior of the Edgeworth Box to be Pareto Efficient, $x_2^A = \frac{1}{3} \omega_2$ and $x_2^B = \frac{2}{3} \omega_2$. The Pareto efficient allocations are shown as heavy line segments in Fig. B.5.

For any efficient allocation in the interior of the box, the marginal rate of substitution $MRS^A = x_2^A = \frac{1}{3} \omega_2$. Thus the supporting prices are the same for all these allocations.

Consider the Pareto efficient allocation $(\hat{x}_1^A, \hat{x}_2^A)$ on the boundary of the Edgeworth box. The separating hyperplane must separate the two shaded areas. Thus it must be tangential to the indifference curve for Alex, whose allocation is in the interior of his consumption set.

The two welfare theorems offer a useful indirect way to analyze equilibrium. Often is it easier to characterize the Pareto Efficient allocations of an economy than to compute Walrasian equilibria. To illustrate this we return to the Edgeworth Box world and examine the effects on Walrasian equilibrium prices of changes in the distribution of wealth.

Suppose that the two (groups of) individuals in the economy have different convex and homothetic preferences. Suppose also, that at the aggregate endowment, $(x_1, x_2)$ Alex has a higher marginal valuation for commodity 1 than Bev. That is, Alex is willing to give up more units than of commodity 2 in exchange for one additional unit of commodity 1.

**Assumption:** Consumer A places a higher marginal value on commodity 1 at the aggregate endowment.

\[
MRS_A(\omega_1, \omega_2) = \frac{\partial U^A}{\partial x_1} / \frac{\partial U^A}{\partial x_2} > \frac{\partial U^B}{\partial x_1} / \frac{\partial U^B}{\partial x_2} = MRS_B(\omega_1, \omega_2)
\]

\[\text{(B.4)}\]

\[U^A(x) = U^A(\omega) \]

\[U^B(x) = U^B(\omega) \]

Fig. B.6: Alex places a higher marginal value on commodity 1
We now explore the implications of this assumption on the Pareto efficient allocations. Consider the Edgeworth box diagram below.

![Edgeworth Box Diagram](image)

**Fig B.7: Pareto Efficient Allocations**

First note that, along the dotted diagonal line, \( (x_1^h, x_2^h) = \theta^h \omega \). By hypothesis, preferences are homothetic. Since Alex places a higher value on commodity 1,

\[
MRS_A(\theta^4 \omega) > MRS_B(\theta^4 \omega) = MRS_B(\theta^8 \omega)
\]

It follows that the Pareto efficient allocations must lie below the diagonal. Let \( C \) be an efficient allocation and \( C' \) be a second such allocation preferred by Alex. In the Figure \( C' \) must lie to the North-east of \( C \). Let the \( MRS \) at \( C \) be \( m \). Given homothetic preferences, for any point above the line \( O_A D \),

\[
MRS_A^4 (x_1, x_2) > m.
\]

Also, for any point above the line \( O_B F \),

\[
MRS_B^8 (x_1, x_2) < m.
\]

Hence in the upper shaded region Alex has a \( MRS \) exceeding \( m \) while Bev has a \( MRS \) less than \( m \). It follows that no such point can be Pareto efficient. A symmetric argument establishes that allocations in the lower shaded region are also not Pareto efficient.
Consider the efficient allocation \( C' \) to the North-East of \( C \). Since this point lies below \( O_A D \) and above \( O_B F \), it follows that

\[
\frac{x_2^h}{x_1^h} < \frac{x_2^h}{x_1^h}, \quad h = A, B,
\] (B.5)

and

\[
MRS^h(C') > \alpha = MRS^h(C), \quad h = A, B.
\] (B.6)

It follows that, as we move along the locus of Pareto efficient allocations from \( C \) to \( C' \), the marginal rate of substitution increases. Hence the supporting prices must have the property that the relative price of commodity 1 rises. Thus the greater the wealth of Alex relative to Bev, the higher will be the relative price of commodity 1. The intuition is clear. Since Alex has a stronger taste for commodity 1, as his wealth increases, the aggregate demand for commodity 1 rises and so its price is pushed up relative to that of commodity 2. And given the higher relative price of commodity 1, his consumption ratio \( x^A_2 / x^A_1 \) also rises.

Since we shall have reason to refer to these results later, we summarize them below.

**Proposition B.1:** Pareto efficient allocations with homothetic preferences

In the \( 2 \times 2 \) exchange economy, suppose each consumer has homothetic preferences. Suppose also that at the aggregate endowment, consumer A places a higher marginal value on commodity 1. Then at any interior efficient allocation,

\[
\frac{x_2^A}{x_1^A} < \frac{x_2^B}{x_1^B}
\]

Moreover, along the locus of efficient allocations, as consumer A’s utility rises, the consumption ratio \( x_2^h / x_1^h \) and marginal rate of substitution of \( x_1 \) for \( x_2 \) of both consumers rises.

**Existence of equilibrium in an exchange economy**

In section A we noted that if there are discontinuities in Walrasian demand and supply curves, equilibrium may not exist. Parallel problems arise in an exchange economy. Figure B.6 illustrates the problem.

Suppose that each consumer has the same endowment vector and preferences. Thus, the Walrasian equilibrium allocation must be the no trade allocation. Note that the budget constraint \( \hat{p} \cdot x = \hat{p} \cdot \omega \) touches the indifference curve at \( A \) and \( A' \) thus there are two equilibrium demands at the price vector \( p \). At any higher price ratio \( p_2 / p_1 \) there is excess demand for commodity 2 and for any lower price ratio there is excess demand for commodity 1. Thus there is no equilibrium.
Exercise B.1: Exchange economy with identical Cobb-Douglas preferences

Suppose that consumer \( h \) has a logarithmic utility function

\[
U^h = \sum_{j=1}^{N} \alpha_j \ln x_j^h, \quad h = 1, \ldots, H.
\]

and an endowment \( \omega^h = (\omega_1^h, \ldots, \omega_N^h) \).

(a) Explain carefully why these preferences are homothetic.

(b) Hence or otherwise solve for the Walrasian market demands for the \( N \) commodities if the price vector is \( p \).

(c) Solve for the Walrasian equilibrium prices.

Hint: With a single aggregate consumer, the equilibrium volume of trade is zero.

Exercise B.2: Equilibrium with CES Preferences

Consumer \( h \), \( h=1, \ldots, H \), has a utility function \( U^h(x_1, x_2) = v(x_1) + v(x_2) \), where \( v'(x) = x^{-1/\sigma} \), \( \sigma > 0 \). The aggregate endowment is \( (\omega_1, \omega_2) \).

(a) Solve for the Walrasian equilibrium price vector.

(b) Show that the higher the elasticity of substitution, the smaller is the price effect of a change in the aggregate endowment of commodity 1.

(c) Provide the intuition for this result.

Exercise B.3: Prices with quasi-linear preferences
Consider a 2 person economy in which the aggregate endowment is \((\omega_1, \omega_2) = (100, 200)\) and both have the same quasi-linear utility function \(U(x) = x_1^h + \sqrt{x_2^h}\).

(a) Solve for the Walrasian equilibrium price ratio under the assumption that the equilibrium consumption of commodity 1 is positive for both individuals.

(b) What is the range of possible equilibrium price ratios in this economy?

**Exercise B.4. Walrasian equilibrium**

Suppose half the population (the Biggs) each have an endowment (24,8) and the other half (the Littles) each have an endowment (20,10). Each Mr Bigg has a utility function

\[ U^B = \ln c_1^B + \ln c_2^B \]

Each Ms. Little has a utility function

\[ U^L = \ln(4 + c_1^L) + \ln(6 + c_2^L) \]

a) By solving for and then aggregating individual demands (or otherwise) solve for the equilibrium price ratio.

b) Solve also for the contract curve and depict it in a neat Edgeworth Box diagram showing the bottom left hand corner as the zero consumption point for a representative Ms. Little and the top right hand corner as the zero consumption point for a representative Mr. Bigg.

c) Explain carefully why the equilibrium price will not change if endowments are reallocated in favor of the Biggs.

d) What will be the equilibrium price ratio if the Littles have an endowment of (8,0) while the Biggs have an endowment of (36,18).

e) What if the Littles have an endowment of (2,0) and the Biggs have endowments of (42,18)?

f) If initial endowments are (1,1) and (43,17) will the equilibrium price ratio be the same as in part (e). Explain carefully.

**Exercise B.5. More on the Biggs and Littles**

Suppose that the Biggs and Littles have endowments as given at the beginning of question B.4. Suppose the Biggs only like commodity 1 while the Littles only like commodity 2. Depict the Pareto Efficient allocations in a neat Edgeworth Box diagram. Is there a Walrasian equilibrium? If so depict it.
Exercise B.6. Equilibrium and efficiency in an exchange economy

Consider a 2 person economy in which Alex's preferences are represented by the utility function $U^A(x^A, y^A) = x^A(5 + y^A)$ while Bev's preferences are represented by the utility function $U^B(x^B, y^B) = x^B y^B$. Alex has an endowment of (5,0) while Bev has an endowment of (0,5).

(a) In an Edgeworth box, depict the individuals indifference curves through their endowments.

(b) Obtain a condition for Pareto Efficiency in the interior of the Edgeworth box.

(c) Depict the competitive equilibrium in the figure. Can you determine the equilibrium price ratio without solving for each individual's demands and then equating aggregate demand and supply?

(d) Alex is forced to transfer $z$ units of his endowment to Bev. What will be the new price ratio if $z = 1$?

(e) What if $z = 3$? Be careful with this part. Also, depict your result in a neat Edgeworth Box diagram.