Scale and the origins of structural change

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Abstract

We consider broad patterns of structural change: (i) sectoral reallocations, (ii) rich movements of productive activities between home and market, and (iii) an increase in establishment size, especially in manufacturing. We extend these facts and develop a unified model explaining them. The crucial distinction across manufacturing, services and home production is the scale of the productive unit. In manufacturing, scale technologies lead to industrialization and marketization. In services, they lead to marketization and later demarketization of services. A later increase in the scale of services could yield a decline in industry and a rise in services, consistent with the data.

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1. Introduction

A broad view of the process of development, dating back to Kuznets [21], includes not only the changes in the relative importance of broadly defined sectors, (e.g., agriculture, manufacturing and services), but also the marketization of home production and the introduction of modern technologies into the household. An open question is whether these structural changes should be understood as independent, perhaps merely coincident phenomena, or whether they have a common origin.

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The central contribution of this paper is to emphasize that scale technologies are the driving force of all three phenomena. Toward this end, we develop a model with multiple sectors, including home production, which vary in the efficient scale of their production. Production of modern services and, especially, manufacturing have large efficient scales, consistent with the empirical pattern that we establish. The theory shows that scale technologies inevitably lead to a marketization of production and growth in both the manufacturing and service sectors. The later spread of modern scale technologies into the home lead to a further increase in manufacturing relative to services, and a reverse product cycle in which market services become home produced. Finally, the model has implications for the role of large-scale technologies in the recent growth of service sector.

On the consumption side, agents hold a continuum of satiable wants. Wants are symmetric in that they offer the same potential utility. For each of these wants, there is a utility advantage if they are home-produced, as opposed to being procured from the market. All wants are satisfied ultimately by services; for simplicity, we model manufacturing only as an intermediate into either home or market production of services.

The asymmetry in the continuum of wants comes on the production side, where wants are ordered from simple to complex. More complex wants require relatively more labor to be produced. We model two technologies – a stagnant, traditional, subsistence technology and a high-growth, modern, scale technology. The modern sector is relatively efficient at producing more complex output. In addition, the modern technology for producing services can be used at home or on the market, but, because the technology requires an indivisible amount of manufacturing intermediates, economies of scale make market production less costly. Modern manufactured goods can only be produced on the market because their larger efficient scale makes home production prohibitive.

The model yields three key implications. First, neutral productivity growth in the modern, scale technologies drives a transition from a stagnant, traditional, home production economy to a growing market economy. The most complex want that is satisfied is the first to become marketized. These modern scale technologies involve both manufacturing and services, and this marketization force leads to an expansion of both and a decline in traditional home production. Second, the same productivity growth leads to later “mass consumption” of modern manufactured goods, as households adopt modern technologies in home production. This force increases manufacturing relative to services, as consumer demand shifts away from services toward manufacturing inputs, and home production generally requires more manufacturing inputs per unit of output (e.g. commuting separately in cars rather than riding together in a bus). Finally, we show that larger scale services tend to be produced on the market, where production can better take advantage of scale economies. We conjecture that growth in the efficient scale of service, consistent with the recent trends for service establishments, may contribute to the recent growth in the service sector in the US.

Our paper relates to several literatures. Empirically, we make several contributions. First, we establish that average scale differs systematically between manufacturing and services industries. We construct panel data patterns on the allocation of current output across agriculture, manufacturing and services for 31 countries. These patterns extend the work of Kuznets [20], Chenery and Syrquin [7], and Kravis et al. [19], and relate to the work of Duarte and Restuccia [10], who focus on real quantities, and the labor patterns emphasized by Maddison [25]. We complement
the historical work on marketization (e.g., De Vries [9], Katouzian [17], Reid [33]) by providing suggestive evidence of later reversals, where services moved from the market to home.

Our model builds on the work of several others. We adopt the non-homothetic preferences of Matsuyama [26,27], whose work also looks at industry growth patterns over development. The preferences in Foellmi and Zweimueller’s [11] work on developmental patterns and R&D are similar, but since theirs are not symmetric, they reproduce sectoral patterns mechanically. Neither paper considers the home production margin. Our work is most closely related Buera and Kaboski [4], which uses identical preferences but exclusively examines the role of increasing skill-intensity in the recent growth of services. This paper introduces scale as a potentially complementary explanation and addresses longer run phenomena as well. Finally, our assumption that market production is less customized than home production complements the work of Locay [24].

Our results fill a gap in a large literature on structural change. Traditional theories have lacked a strong quantitative explanation for the hump shape developmental pattern of manufacturing (see, for example, Kongsamut, Rebelo, and Xie [18], and Ngai and Pissarides [31]).\(^1\) We present a potential candidate.\(^2\) Specifically, we model scale as an explanation for the initial growth and conjecture its role in the later decline. Our analysis of home production patterns in development complements work by Gollin, Parente and Rogerson [13], Rogerson [34], and Ngai and Pissarides [32]. Finally, our results on the determinants and impacts of mass consumption of manufactured goods complement the earlier work by Katona [16], Matsuyama [27], and Murphy, Shleifer and Vishny [30].

The remainder of the paper is organized as follows. Section 2 develops and extends the broad facts of structural change. We present the model in Section 3 and the theoretical results of the model in Section 4. Section 5 concludes.

2. Facts of structural change

“The rate of structural transformation of the economy is high. Major aspects of structural change include the shift away from agriculture to non-agriculture pursuits, and, recently, away from industry to services; a change of the scale of productive units, and a related shift from personal enterprise to impersonal organization of economic firms, with a corresponding change in the occupational status of labor.” (Kuznets [21, 2])

This section documents empirical evidence on broad structural change that motivates our model in Section 3. Specifically, we document: (1) larger scale establishments in manufacturing relative to services, (2) sectoral reallocations of production, closely linked to consumption, and (3) the movement of some services into the home with the spread of manufacturing goods. We view the first two as stylized facts, documented for multiple countries, while the third is suggestive evidence in support of the demarketization result of our model.

\(^1\) In related work, Acemoglu and Guerrieri [1] look at the relative growth patterns of capital- and labor-intensive industries, but not manufacturing and services explicitly.

\(^2\) Recent work by Duarte and Restuccia [10] suggests an alternative for the hump-shape based on a combination of traditional theories.
2.1. Larger average scale in manufacturing than in services

In the next section, we model industrialization as marked by the advent of large scale technologies that require large scale investments. The emergence of these large scale technologies for both manufacturing (e.g., Mokyr [29], Scranton [35]) and services (e.g., Chandler [6]) has been well-documented in the historical literature. Their importance can be seen in the average scale (workers per establishment) in 19th century US manufacturing data from Atack and Bateman [2]. Our focus here is to establish the fact that manufacturing technologies are nonetheless much larger scale than services.

Given the dearth of historical data on service establishments, we instead look across countries at different stages of development: the United States, Mexico and India. We use economic data that cover both manufacturing and services, and we adopt a broad classification of manufacturing as industry (mining, manufacturing, construction, and utilities) and broad services (transportation, retail, wholesale, other services, and administration). For India and Mexico, these come from economic censuses of 2005 and 1997, respectively, while for the US, we use 1998 County Business Patterns (CBP) data. In each country, the average scale is larger in manufacturing than in services. Specifically, in the US data, (broad) manufacturing establishments averaged 47 workers, while service establishments average 14. In Mexico, a country whose output per worker was about 1/4 that of the US in 1998, the average scales in manufacturing and services were 15 and 4, respectively. In India, where output per worker was roughly 1/12 that of the US in 1998, average scales were 6 and 4, respectively. Thus, in all three countries average scale is larger in manufacturing, but the difference is larger in the two more developed economies.

Although average scale is larger in manufacturing, there is a considerable amount of heterogeneity within each sector. Fig. 1 shows histograms of the distribution of employment in each sector across different establishment sizes. In the US, the distribution of manufacturing clearly lies to the left of services, but there is considerable amount of overlap. Large scale establishments are important shares of both manufacturing and services in the US. Mexico shows less overlap, with the majority of service employment at establishments with less than ten workers, while nearly two-thirds of manufacturing employment is in establishments of 100 workers or more. In India, again there is considerable overlap, and although large establishments are more important in manufacturing, in both sectors the majority of workers are in small establishments. Holmes and Stevens [15] discuss this heterogeneity in manufacturing, arguing that the nature of production is very different for large and small-scale establishments, with small-scale establishments providing more customized, service-like output. In any case, this heterogeneity is not essential to our argument, and the model we present in Section 3 will abstract from it, focusing on average scale.
instead on the technological distinction driving the difference in average efficient scale across sectors.\(^6\)

The difference in average scale across manufacturing and service sectors holds at a much more disaggregate level as shown in Fig. 2. Here we plot a histogram of 4-digit SIC manufacturing and service industries by their average scale using the 1998 CBP data.\(^7\) Despite the wide variance of scale in industry, the distributions overlap very little with about 80 percent of

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\(^6\) See our concurrent paper, Buera, Kaboski, and Shin [5] for an example where technological differences that drive differences in average scale across sector can be reconciled with within-sector heterogeneity.

\(^7\) The earliest establishment level data, 1974, shows even starker differences in scale between manufacturing and services. We discuss recent converging patterns of scale in the US at the end of Section 3.
the mass of service industries being below twenty workers (over 45 percent of the mass is less than 10 workers), and over 90 percent of the mass of manufacturer industries being greater than twenty workers (with twenty-five percent of the mass greater than 100 workers). The difference in scale is true across each broad industry in the goods sector (including agriculture, mining, utilities, and manufacturing) and services sector (transportation, services, public administration) with the exception of construction, which is typically in the manufacturing sector, but has many service-like characteristics.  

2.2. Sectoral reallocations

Here we extend Kuznets’ stylized development patterns for reallocations across industry, services and manufacturing with longer time series and a wider set of countries. We highlight that value-added in the manufacturing sector initially grows relative to the service sector but later declines. For the case of the U.S., we provide some evidence linking these value-added patterns to the consumption of manufacturing goods relative to services.


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8 For example, construction is non-tradable, and much of construction consists of small-scale contract work for which home production is a viable alternative.
Thailand (1940–2000), United Kingdom (1820–2000), and the United States (1870–2000). Based on Maddison (2006), our data covers: 68 percent of world population and 80 percent of world GDP in 2000; 72 percent and 76 percent, respectively, in 1950; and 40 percent and 60 percent, respectively in 1900. Although the numbers are lower for 1900, since the longer time series include Western Europe and its offshoots, we cover a much larger share of the population and economic activity undergoing large structural change at the time. We provide an abbreviated appendix of our sources, but the data, as well as complete documentation of sources and methodology, are available at http://jet.arts.cornell.edu/Supplementary_Materials.html.

To summarize the data, we start by filtering out any differences in the levels of the series across countries. That is, we run regressions of (log) income shares on quadratic functions of log income per capita, and we then subtract out estimated country-fixed effects from the raw data. These adjusted data series are then plotted below in Fig. 3, which shows value-added shares vs. real income per capita for industry (top panel), services (middle panel), and agriculture (bottom panel), with each dot representing a country-decade observation. The patterns are strong, and beyond the well-known decline in agriculture in the lower panel, two important features are discernible.
First, the share of manufacturing is hump shaped over development.\(^9\) Second, services constitute a substantial share of output even early on, but exhibit a late acceleration with the decline of manufacturing.\(^10\) The patterns are quite salient features of development.\(^11\)

\(^9\) Of the 30 countries, 21 – including all high income countries – have experienced an increase and then decline in industry, while the remaining lower income countries have only (yet) experienced the increase in industry. For these 21 countries, the peak share averages 0.40 (std. dev: 0.05) and occurs at an average per capita income of $7100 (st. dev.: $1800). Using this $7100 threshold to divide the country-year observations in the sample, regressions of industry’s share of country \(j\) on its log real income per capita (\(\ln y_j\)) that include country-specific fixed-effects (\(\alpha_j\)) yields the following results (standard errors in parentheses):

\[
\begin{align*}
\text{\$7100 sample:} & \quad \text{Ind. Share}_j = \alpha_j + 0.11 \ln y_j, \\
\text{\$7100 sample:} & \quad \text{Ind. Share}_j = \alpha_j - 0.13 \ln y_j.
\end{align*}
\]

\(^10\) The 25 countries for which we have data at levels of per capita income below $2000 have services shares averaging 0.39 (std. dev: 0.07), which is comparable to the average share of agriculture in that income level, 0.40. The analogous split sample regression using service shares demonstrates the late acceleration:

\[
\begin{align*}
\text{\$7100 sample:} & \quad \text{Serv. Share}_j = \alpha_j + 0.07 \ln y_j, \\
\text{\$7100 sample:} & \quad \text{Serv. Share}_j = \alpha_j + 0.20 \ln y_j.
\end{align*}
\]

\(^11\) The UN National Accounts Main Aggregates Database, which includes sector specific numbers for a much larger cross-section of 161 countries but over a shorter time period (1970-2000), yields very similar results with low- and high-income sample coefficients of 0.07 and −0.12 for industry, respectively, and 0.04 and 0.18 for services.
Together, the hump shape in manufacturing and late acceleration of services leads to an initial growth of manufacturing relative to services and a later decline. We plot this ratio explicitly in Fig. 4, which clearly shows the hump-shaped pattern in the relative size of the two sectors.

Our model will address both the increasing and decreasing portions of Fig. 4 through the consumption channel. We only have a long series of detailed consumption data for a single country, the United States, but Fig. 5 below shows that a strong connection between the value-added and consumption series in these US data.

2.3. Rich dynamics home vs. market movements

At the early stages of development, many productive activities move from being household produced to being market produced. This marketization phenomenon has been well-established in the literature.\(^{12}\) We emphasize that the data suggest richer patterns. In particular, we show evidence suggesting an important reverse process, in which market services move to home production Fig. 6 shows some important examples of this phenomenon in which services rise initially but then fall with the diffusion of the relevant manufactured goods to households.\(^{13}\)

The top panel graphs the rise and fall of important transportation services (i.e., railways, trolleys, and buses) on the left axis, and the spread of the stock of registered automobiles (i.e., all vehicles and just cars) on the right axis. Because the three services are substitutes for one another, the dynamics of individual services are complicated, but clearly the spread of the automobile has played an important role in the decline of all three.

The middle panel similarly displays the dynamics of market services for “domestic work” (i.e., laundering and dry cleaning, domestic servants) together with the diffusion of vacuums, light bulbs, and other durables.\(^{13}\) The model we present gives complicated dynamics for the value of output because of price effects, so instead we count real services and establishments. Related, in the graphs the commodities that diffuse are durables, so we choose measures related to the stocks of these items instead of the flows.

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\(^{12}\) For example, Reid [33] documents a long list of former home production activities: “spinning, weaving, sewing, tailoring, baking, butchering, soap-making, candle-making, brewing, preserving, laundering, dyeing, gardening, care of poultry, ... child care, education, and the care of the sick.” Deaton [8] discusses the importance of marketization for the measurement of changes in national income and poverty in developing countries.

\(^{13}\) The model we present gives complicated dynamics for the value of output because of price effects, so instead we count real services and establishments. Related, in the graphs the commodities that diffuse are durables, so we choose measures related to the stocks of these items instead of the flows.

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laundry washers, dryers, and dishwashers. Laundering and dry cleaning services show a rise and then decline that corresponds to the spread of the washer and dryer. Domestic servants shows a continuous decline, but seems to also be related to the spread of these household items. Indeed
Greenwood et al. [14], from whom we take the data on household items, also draw a link between these trends, and Francis and Ramey [12] cite historical evidence that the spread of many household appliances were associated with increases in household production labor because activities (e.g., bread baking, laundry) moved from market to home production.

Finally, the third panel plots the rise and fall of retail food, drinking, ice and fuel service establishments, and the corresponding expansion of important household amenities: private refrigerators, freezers, electricity and central heating. We argue that the expansion of refrigerators and freezers has moved these services from market production to home production, eliminating the need for frequent shopping, neighborhood grocery stores, and milk, fuel and ice delivery services.14

Important product cycles include the decline of transportation services, such as railroads, rail lines, and buses with the spread of the private automobile. The automobile was also related to the decline in neighborhood retail services (food, apparel, ice, fuel, dairy, “five and dime stores”), as was the spread of refrigerators and freezers. Similarly, the spread of washers, dryers, vacuums, microwaves, and other home appliances (see Greenwood et al. [14]) was accompanied by declines in domestic servants, launders, and dry cleaners. Many newer activities that have started in the market have also moved toward home production. Examples include the relative decline of movie theaters (spread of televisions, VCRs, and DVD players), mail services (computers, fax machines), and recently internet cafes (computers, cable internet connections).

These examples of demarketization are quantitatively important. The examples in Fig. 6 constitute 75 percent of all declining service industries between 1950 to 2000 (see the industry-level data in Buera and Kaboski [4]). Also, returning back to Fig. 5 we see an actual increase in home production time between 1900 and 1950 in the US, which accompanies an increase in the share of manufactured (non-food) goods in consumption and the increase in the value-added share of manufacturing.15 Similarly, all three trends show a decline after 1950, which at least suggests some link between the three. Unfortunately, while we know the pattern in value-added is robust to many countries, the patterns in Figs. 5 and 6 remain suggestive since we lack comparable data over long periods for other countries.

2.4. Summary

We have established two important facts using data from multiple countries. First, while modern scale technologies characterize both manufacturing and services, the average scale of manufacturing establishments greatly exceeds that of service establishments. Second, both modern industry and services play an important role early in development, but industry follows an extended hump-shape, even relative to services. Finally, using available data for the United States, we have presented evidence suggesting demarketization of services that accompanies the spread of manufacturing goods and is linked to patterns in manufacturing consumption and value-added.

In the next section we present a model consistent with the first fact, which in turn yields the second fact, through a mechanism consistent with the suggestive evidence on marketization and demarketization.

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14 Lagakos [22] examines the relationship between automobiles, retailing consolidation, and productivity in the context of developing countries.

15 These data are only available for the US. Home production time is taken from Francis and Ramey [12], while consumption data pre-1929 is from Lebergott [23].
3. A theory of structural change

We model the consumption decision over a continuum of discrete wants. Individuals choose whether to home produce or to procure these wants from the market. Production can be done using a traditional or a modern technology. Production using the modern technology requires the use of fixed amount of intermediate manufactured goods in combination with labor to produce up to a maximum scale. To satiate each want requires the use of both manufactured goods and services. In the model economy, as in the data, manufacturing differs from services by requiring a larger fixed cost and operating at a larger scale.

3.1. Preferences

The household holds preferences over a continuum of discrete, satiable wants indexed by the service that satisfies them, \( z \in \mathbb{R}^+ \). Thus, all final consumption takes the form of services. Let the function \( C(z) : \mathbb{R}^+ \rightarrow \{0, 1\} \) indicate whether a particular want is being satisfied. Wants can be satisfied via market production or home production. Define the function \( H(z) : \mathbb{R}^+ \rightarrow \{0, 1\} \) to take the value 1 if want \( z \) is satisfied by home production and 0 otherwise. Together the set of indicator functions mapping \( \mathbb{R}^+ \) into \( \{0, 1\} \) defines the consumption set. The utility function is therefore:

\[
    u(C, H) = \int_0^\infty [H(z) + \nu(1 - H(z))] C(z) \, dz.
\]

where \( H(z) \leq C(z) \). The parameter \( \nu \) represents the relative utility of having a want satisfied via market production instead of home production. We assume \( 0 < \nu < 1 \) to capture the fact that home production of a service will be more customized to the specific wants of a household (e.g., driving precisely when and where you want rather than riding the bus on fixed schedules).

3.2. Technologies

We model three alternative technologies for producing services, (i) a traditional technology which we identify with agriculture, (ii) a modern home technology and (iii) a modern service technology. The production functions are as follows:

Traditional: \( S_0(z) = z^{-1} L_0 \).

Modern Market Service: \( S_M(z, t) = \begin{cases} 0 & \text{if } K < q, \\ \min\{n, e^{\nu t} z^{-\lambda} L_M\} & \text{if } K = q, \end{cases} \)

Modern Home Service: \( S_H(z, t) = \begin{cases} 0 & \text{if } K < q, \\ \min\{1, e^{\nu t} z^{-\lambda} L_H\} & \text{if } K = q. \end{cases} \)

The three production functions differ in several important ways. First, while all use labor \( L \), the two modern technologies require at least \( q \) units of intermediate goods \( K \) (e.g., a laundry machine required to produce laundry services), while the traditional does not. Second, labor productivity is lower for high \( z \) goods, but we assume that \( 1 > \lambda \), so this is especially true in

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16 One could easily introduce a second continuum of wants that are directly satisfied by manufactured goods, but it would contribute little to the analysis.
the traditional sector. Equivalently, since $z$ indexes complexity, the modern technologies are relatively more productive in complex goods. Third, the two modern technologies experience productivity growth at a rate $\gamma$, while the traditional sector is stagnant. Finally, the traditional sector exhibits constant returns to scale, while the min functions in the modern technologies capture a maximum scale. The maximum scale is driven by a capacity limit of the intermediate goods $K$ (e.g., the maximum number of loads of a laundry machine operated at full capacity). We denote this maximum as 1 for modern home and $n > 1$ for modern market services to capture the scale advantages available on the market.$^{17}$

Finally, we choose a simple analogous specification for the manufacturing technology to produce type $z$ intermediate goods:

\[
\text{(Market) Manufactured Goods: } G(z, t) = e^{\gamma t} z^{-\lambda} L_G.
\]

Note that we abstract from any efficient scale considerations in the manufacturing technology. The fact that manufactured goods are only market produced implicitly appeals to the larger scale of manufactured goods, i.e., for manufacturing $q, n \to \infty$, with $q/n \to 0$.

We should note four further simplifications in the environment, none of which are critical. First, although manufactured goods will be “final goods” from the national accounts perspective (i.e., purchased by consumers as intermediates into home production), we abstract from manufacturing goods that may provide direct utility. This would be straightforward to incorporate but would add little to the analysis. Second, we abstract from services being used as intermediates into production. This helps emphasize our focus on home-market consumption patterns with regards to services, and it is consistent with the evidence that recent growth in service value-added is explained by consumption (see Buera and Kaboski [4]). Third, we do not model any home production of manufactured goods. This direct assumption is motivated by the evidence that modern manufacturing technologies are highly productive and large-scale, and that home production of goods is not quantitatively large. Finally, we do not model any manufactured goods in the traditional sector, identifying agriculture with the services produced using the traditional technology. Implicitly we are assuming that the value-added of modern agriculture is zero, and that all of the costs associated with the production of modern agricultural goods, e.g., food, correspond to manufactured inputs and modern services. Each of these assumptions is clearly an abstraction, but they help to simplify the analysis.

3.3. Equilibrium

We can now state the household’s problem and the competitive equilibrium. For each want $z$, the household makes three linked binary decisions: whether to consume or not, $C(z)$; if so whether to home produce or not, $H(z)$; and again if so, whether to use the modern technology in home production, $M(z)$.

$^{17}$ Essentially, since home demand for any service is at most one, a household can only utilize more than one unit by selling it and therefore becoming a market service. For this implication, it is important that the intermediates are indivisible (one cannot be half as productive with half a laundry machine) and specialized (a car cannot substitute for a laundry machine in doing laundry). Thus, $K(z) = q$ would be a more precise constraint capturing this specialization of capital.
Normalizing labor as the numeraire, the household takes the prices of each good \( p_G(z) \) and market service \( p_S(z) \) as given, and solves the following static problem at each point in time:

\[
\max_{M(z) \leq H(z) \leq C(z)} \int_0^\infty \left[ H(z) + \nu(1 - H(z)) \right] C(z) \, dz
\]

s.t.

\[
\int_0^\infty C(z) \left[ H(z) M(z) q p_G(z) + \left[ 1 - H(z) \right] p_S(z) \right] \, dz = 1 - \int_0^\infty C(z) \left[ e^{-\gamma t} M(z) z^\lambda + \left[ 1 - M(z) \right] z \right] \, dz. \tag{2}
\]

The left-hand side of the budget constraint is total market expenditures, while the right-hand side is income/labor supply.

At each point in time, the competitive equilibrium is a static equilibrium given by: price functions \( p_G(z) \), \( p_S(z) \), household decisions \( C(z), H(z) \) and \( M(z) \); and labor allocations \( L_G(z), L_M(z), L_H(z), \) and \( L_0(z) \) such that:

- given prices, household decisions solve (2);
- firms maximize profits taking prices as given;
- labor and output markets clear:

\[
\int_0^\infty \left[ L_G(z) + L_M(z) + \right] \, dz = 1 - \int_0^\infty \left[ L_H(z) + L_0(z) \right] \, dz,
\]

\[
C(z) \left[ H(z) M(z) + \left[ 1 - H(z) \right] / n \right] q = G(z),
\]

\[
C(z) \left[ 1 - H(z) \right] = S_M(z),
\]

and home production decision satisfy feasibility:

\[
C(z) H(z) M(z) = S_H(z),
\]

\[
C(z) H(z) \left[ 1 - M(z) \right] = S_0(z).
\]

3.3.1. Simplified characterization

Much of the equilibrium can be characterized quite simply. Free entry leads to a zero-profit condition for firms that immediately yields market prices:

\[
p_G(z, t) = e^{-\gamma t} z^\lambda,
\]

\[
p_S(z, t) = \left( 1 + \frac{q}{n} \right) e^{-\gamma t} z^\lambda.
\]

Returning to the household problem (2), decisions are characterized by simple thresholds, since costs are ordered in \( z \), and wants enter utility symmetrically but costs increase in complexity \( z \). Hence, consumers will satisfy and home produce the least complex wants first. In particular, the consumption decision takes the form \( C(z) = 1 \) iff \( z \leq \bar{z} \), with \( \bar{z} \) therefore denoting the most
complex want that is satisfied. $\mathcal{M}(z)$ shows up only in the budget constraint, so the modern technology is chosen for a home-produced service only if it is cheaper. The assumption that $\lambda < 1$ makes the modern technology relatively more productive for high $z$ goods, which defines a single crossing for the costs. Hence, $\mathcal{M}(z) = 1$ iff $z_0 \leq z \leq \bar{z}$, where $z_0$ is the upper threshold for wants satisfied using the traditional technology. Finally, the $\mathcal{H}(z)$ decision of whether to home produce or market purchase is either characterized by $z_0$ (all home produced services use the traditional technology) or by $z > z_0$ (the modern technology is used for home production in the range $z_0 \leq z \leq \bar{z}$).

The market technology is therefore used for any $z$, $z_0 < z \leq \bar{z}$. We can highlight the role of scale by writing the condition for a service to be produced in the market:

$$\mu \left[ p_G(z)q \left( 1 - \frac{1}{n} \right) \right] > 1 - \gamma$$

where $\mu$ is the multiplier on the budget constraint (the marginal utility of income).

The bracketed term, the goods cost-savings of market production, stems from the efficient scale of market production, which requires paying only a fraction $(1/n)$ of the intermediate goods cost as opposed to the full goods cost from purchasing the input. This cost savings is higher for output that requires large or expensive intermediates (high $q$ or $p_G$) or has a large efficient scale $n$.

### 4. Evolution of structural change

This section presents the results of the paper, which tie in closely with the facts presented in Section 2, given our assumption of large scale modern technologies, and a scale so large in manufacturing that it ensures market production. We provide four main propositions. The first characterizes the behavior of the thresholds, which lead immediately to the result of rich product cycles. The second characterizes the evolution of shares, which reproduce both growth in manufacturing share of value-added and consumption, and even growth relative services. The third proposition shows how time spent in traditional activities declines, but later home production time increases in conjunction with the spread of manufactured goods to consumer. Finally, we show the effect of average scale of services on the share of manufacturing, and conjecture its implications for the decline of manufacturing at high levels of income.

In order to ensure that the scale economies in services do indeed produce market services, we need to restrict parameter values. Specifically, we assume that the cost savings of market production is sufficiently large relative to the disutility of market consumption as follows:

**Condition 1.** $\frac{q(1 - \frac{1}{n})}{1 + \frac{q}{n}} > \frac{1 - \gamma}{\gamma}.$

Given the above condition, the following proposition characterizes the behavior of the thresholds defining allocations in three periods: a period of a stagnant “traditional economy,” a period of “industrialization” involving growth and marketization, and a later period of “mass consumption.”

---

18 This result is consistent with our assumptions that manufacturing is always market produced, since it is generally done requires large intermediates/capital inputs and is done on a large scale.
Proposition 2. Assume Condition 1. There exist time intervals characterized by \( t_0 \) and \( t_1 \), \( t_0 < t_1 \), such that:

i) For \( t \in [0, t_0) \), \( z_0(t) = z( t ) = z(t) = 2^{1/2} \); 

ii) For \( t \in [t_0, t_1) \), \( \partial z_0(t)/\partial t = \partial z(t)/\partial t < 0 \) and \( \partial z(t)/\partial t > 0 \); 

iii) For \( t \in [t_1, \infty) \), \( \partial z_0(t)/\partial t < 0 \), \( \partial \ln z(t)/\partial t = \partial \ln z(t)/\partial t > 0 \).

The first interval is trivial and occurs at low \( t \) when the modern technology is insufficiently productive relative to the traditional technology. In the second interval, \( z_0 = \bar{z} < \bar{z} \). In this case, some modern market services are purchased, but no modern technologies are used in the home, since for this range of \( z \), the traditional technology is still more productive giving the additional goods cost of modern production in the home. Technological progress reduces the cost of the modern technology, and so the set of activities performed in traditional technology declines, the range of wants satisfied defined by \( \bar{z} \) expands, and hence the range of wants satisfied on the market expands. Finally, in the third interval, \( z_0 < \bar{z} < \bar{z} \), so that the traditional technology, modern home technology, and modern market technology are all utilized. At this point, the modern technology has become so productive that the modern home technology is preferable despite the additional goods required for modern home production.

Proposition 1 has the following corollary for the product cycles.

Corollary 3. The range of wants follow one of three distinct product cycles over time:

i) services \( z \in (0, z_0(t_1)) \) transition from being home produced using the traditional technology to being home produced using the modern technology during the interval \( t \in [t_1, \infty) \); 

ii) services \( z \in [z_0(t_1), z_0(t_0)] \) transition from being home produced using the traditional technology to being market services during interval, \( t \in (t_0, t_1) \), and then transition to home produced using the modern technology after \( t_1 \); 

iii) services \( z \in (z_0(t_0), \infty) \) transition from being market purchased to being home-produced using the modern technology after \( t_2 > t_1 \), where \( t_2 \) is defined by \( z(t_2) = z_0(t_0) \).

The above corollary emphasizes that the model is consistent with the rich product cycle dynamics discussed and presented in Section 2.3. In particular, the marketization and demarketization of the second product cycle would include local transportation in the top panel of Figure 6, while the third product cycle, where activities that started in the market and moved to the home, would include the retailing services in the bottom panel of Fig. 6. Moreover, the demarketization aspects of both cycles involve the purchase of manufactured goods directly by the home, and affects the sectoral shares of manufacturing and services.  

The following proposition presents the implications of the threshold movements for the sectoral patterns. In particular, we characterize the movement of value-added shares, \( y_A(t) \), \( y_G(t) \), and \( y_S(t) \), and consumption shares, \( c_A(t) \), \( c_G(t) \), and \( c_S(t) \), in manufacturing and services, respectively.

---

19 If Condition 1 does not hold, then market services are bypassed, and services using the modern technology are produced in the home. Thus, a model with heterogeneity in the scale of services can produce still richer product cycles. Analysis of that model are available upon request.

20 Explicitly, we define the consumption shares as \( c_A(t) \equiv C_A(t)/C(t) \), \( c_S(t) \equiv C_S(t)/C(t) \), \( c_G(t) \equiv C_G(t)/C(t) \), where \( C_A(t) = \int_0^{z_0(t)} z \, dz \), \( C_S(t) = \int_{z_0(t)}^{\bar{z}} p_S(t) \, dz \), \( C_G(t) = \int_{z_0(t)}^{\bar{z}} q_P(t) \, dz \), \( C(t) = C_S(t) + C_G(t) + C_A(t) \). To ex-
Proposition 4. Assume Condition 1. The evolution of the value-added and consumption share of agriculture, manufacture and services are characterized by three phases:

i) For $t \in [0, t_0)$, $y_A(t) = c_A(t) = 1$, $y_G(t) = y_S(t) = c_G(t) = c_S(t) = 0$;

ii) For $t \in [t_0, t_1)$, $\frac{\partial y_A(t)}{\partial t} < 0$, $\frac{\partial y_G(t)}{\partial t}, \frac{\partial y_S(t)}{\partial t} > 0$, $y_G(t) = qn$, $\frac{\partial c_S(t)}{\partial t} > 0$, $c_G(t) = 0$;

iii) For $t \in [t_1, \infty)$, $\frac{\partial y_A(t)}{\partial t} < 0$, $\frac{\partial y_G(t)}{\partial t}, \frac{\partial (y_G(t) - y_S(t))}{\partial t}, \frac{\partial (c_G(t))}{\partial t}, \frac{\partial (c_G(t) - c_S(t))}{\partial t} > 0$. In addition, $\frac{\partial y_S(t)}{\partial t} > 0$ iff $q < \frac{1 + \lambda}{1 - \lambda}$.

By definition, the stagnant traditional economy is entirely agricultural. During industrialization, when the economy begins to grow, agriculture declines while the value-added of both manufacturing and services grow. The consumption of services grows, while the direct consumption of manufactured goods by households is still zero. Direct consumption of manufacturing goods characterizes the mass consumption phase and leads to growth in the consumption share of manufacturing. It also leads to extra growth of manufacturing relative to services in terms of value-added (and consumption), since market services are substituted for by modern home production which has higher requirements of manufactured goods. The growing value-added patterns in the industrialization and mass consumption phases capture the early sectoral patterns from Section 2.2; i.e., the sectoral dynamics before the late decline of manufacturing. Asymptotically, the model converges to a constant share of manufacturing and services, in terms of both value-added and consumption.

The following proposition characterizes the dynamics of home-production time, both traditional/agricultural $h_T(t)$ and modern $h_M(t)$, over these three phases.

Proposition 5. Assume Condition 1. The evolution of the time used in home production is characterized by three phases:

i) For $t < t_0$, $h_T(t) = 1$, $h_M(t) = 0$;

ii) For $t \in [t_0, t_1)$, $\frac{\partial h_T(t)}{\partial t} < 0$, $h_M(t) = 0$;

iii) For $t \geq t_1$, $\frac{\partial h_T(t)}{\partial t} < 0$ and $\frac{\partial h_M(t)}{\partial t} > 0$.

The proposition shows that after the stagnation phase traditional/agricultural labor declines. In national accounts, the output produced by agricultural labor is measured, and hence this labor is part of the labor force. In contrast, during the mass consumption phase, modern home production increases. Therefore, while Proposition 3 linked the growth of manufacturing value-added to the growth in manufacturing consumption, in the mass consumption phase, Proposition 4 links the growth of home production time to this phase. Thus, during this mass consumption phase, the model is consistent with the common trends of manufacturing value-added, non-food consumption, and home production time documented for the United States in Fig. 5.
While the dynamics of the model capture the growth of manufacturing value-added, consumption, and home production time, it does not capture the salient decline at higher incomes that we document. We believe scale has important implications for the decline of manufacturing too, however. In the mass consumption phase, the model yields a comparative static result that the larger the scale of services, parameterized by n, the smaller the relative size of the manufacturing sector, and the smaller is home production time. Proposition 3 formalizes this result.

**Proposition 6.** Assume Condition 1 and \( t \geq t_1 \), then \( \frac{\partial [yM(t)/yS(t)]}{\partial n} < 0 \), and \( \frac{\partial [cM(t)/cS(t)]}{\partial n} > 0 \).

There are two intuitive reasons for this result. First, the larger the scale, the less the goods cost per unit. That is, keeping \( q \) constant, the share of intermediate manufactured goods is decreasing in scale. Second, the larger the scale, the larger the cost savings of market production of services (which produces at this efficient scale) relative to home production. Thus, households substitute toward market services and away from home production. While the proposition refers to the shares of manufacturing relative to services, the absolute share of manufacturing grows and the absolute share of manufacturing declines as long as the share of agriculture is substantially small (i.e., small \( z_0 \)).

While Proposition 3 is a comparative static result with respect to the parameter, \( n \), we believe it is relevant for the empirical time series of development. That is, during the mass consumption phase, growth in the optimal scale of services technologies, captured as an exogenous increase in the parameter \( n \) over time, could increase the relative share of services in value-added and consumption and decrease home production time, \( h_M(t) \).

While lacking a theory for an increase in \( n \), such an increase in the scale of service establishments has been empirically observed alongside the (post-1950) decline in manufacturing in the US. Data from the County Business Patterns show a steady increase of 70 percent in the average scale of services from 1947–1998, while the scale in the goods sector has actually declined. Moreover, at a disaggregate level the growth in the service sector has been dominated by services whose scale has grown, and who are now among the largest scale services. Using scale and payroll information by 3-digit level from the 1959 and 1998 County Business Patterns, OLS regressions yield the following estimates (with standard errors in parentheses):

\[ \Delta \text{share}_i = 0.20 + 0.69 \Delta \log \text{scale}_i \]

where \( i \) represents 3-digit SIC industry (based on IPUMS 1950 coding, which allows us to link it to IPUMS data on schooling levels of workers in each industry), and \( \Delta \text{share}_i \) is the absolute change in the percentage share of industry in total payroll payments between 1959 and 1998. The positive coefficient on \( \Delta \log \text{scale}_i \), the change in log employees per establishment, is significant at the one percent level. That is, industries that have grown in share have been the industries whose scale has increased.

---

23 Scale is not the only force at work in the growth of services, however. Buera and Kaboski [4] focus on a related, and complementary explanation for the growth in services: their increasing skill intensity.

24 A straight forward alternative to introduce this directly into the model would be making \( n \) an increasing function of \( z \) beyond a threshold value of \( z \). One would need to increase \( q \) in a similar pattern so as to keep \( q(z)/n(z) \) constant (i.e., only change the relative cost of home and market services, without changing the relative costs across \( z \)). This model would capture the idea that services consumed at higher incomes (e.g., medicine, finance) are inherently larger scale.

25 Scale is again defined as workers per establishment or workers per firm. In 1974, there is a change from a “reporting unit” (firm) concept to establishment. The pre- and post-1974 changes are 59 and 17 percent, respectively.
This result is robust in two important ways. First, excluding the five largest and five smallest changes in shares still yields an estimate that is positive and still significant at a five percent level. Second, the relationship is not simply capturing the relationship between growth and skill intensity observed in Buera and Kaboski [4]. Controlling for \(\text{skill}_i\), the fraction of labor in an industry that was college-educated in 1940\(^{26}\), yields the following estimates:

\[
\Delta \text{share}_i = -0.31 + 0.71 \Delta \log \text{scale}_i + 5.01 \text{skill}_i.
\]  

(5)

The coefficient on \(\Delta \log \text{scale}_i\) is nearly identical and still significant at the one percent level. Thus, growth in scale appears to be independently related to the growth of disaggregate services.

5. Conclusions

We have highlighted several empirical aspects of broad structural change, including: (1) the importance of scale technologies, which exist in both manufacturing and services, but are larger in manufacturing; (2) the growth and decline of manufacturing over development, even relative to services, and the late acceleration of services; and (3) rich product cycles between home and market, including the movement of services from the market to the home. Our theory has emphasized that scale technologies are important in understanding the movement across economic sectors and the rich product cycles. In particular, a spread of manufactured goods into the home leads to the demarketization of services and a growth in manufacturing relative to services. Moreover, technological change leading to growth in the scale of services can lead to marketization and relative growth of the service sector.

Our treatment of scale has abstracted from heterogeneity within these sectors, in order to emphasize the patterns that sector-specific technological differences, such as the size of fixed costs, can produce. We conjecture that the mechanisms would be robust to within-sector heterogeneity driven by differences in the productivity of establishments, as in Buera, Kaboski, and Shin [5], as long as their remains a sector-specific scale component to technology. Alternatively, we can interpret the heterogeneity in the size of establishments within manufacturing and services as reflecting the grouping of technologies with different efficient scale of production within these broad sectors. The recent evidence by Holmes and Stevens [15] about the heterogeneity on the tradability of the goods produced by manufacturing establishments within narrowly-defined sectors is a promising avenue to explore these questions.

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26 Using the fraction that was college-educated in 2000 yields similar results for the role of scale, though the coefficient on skill is somewhat smaller given the higher education levels.
Appendix A. Data sources for country panel

The following is a simple list of the original data sources. A full data appendix is available at http://www.econ.ucla.edu/fjbuera/BK2_DataAppendix.zip.

Argentina

Australia

Belgium

Brazil

Canada

Chile

China

Columbia

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Denmark  

Egypt  


Finland  

France  


Germany  


India  

Indonesia  

Italy  


Japan


Korea


Mexico


Netherlands


Norway


Pakistan/Bangladesh


South Africa


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Appendix B. Proofs of the results in the paper

Proof of Proposition 2. The household chooses the set of wants to home produce using the traditional technology, \( z \in [0, z_0] \), the set of wants to home produce using the modern technology, \( z \in (z_0, z^\ast] \), and the set of want to market purchase, \( z \in (z^\ast, \bar{z}] \), where \( z_0 \leq z \leq \bar{z} \), to maximize

\[
\max_{0 \leq z_0 \leq z \leq \bar{z}} (1 - \nu)z + \nu \bar{z}
\]
subject to the budget constraint

\[
\int_{z_0}^{\bar{z}} q p_G(z,t) \, dz + \int_{\bar{z}}^{z_0} p_S(z,t) \, dz = 1 - \int_{0}^{\bar{z}} z \, dz - e^{-\gamma t} \int_{z_0}^{\bar{z}} z^\lambda \, dz
\]

where \( p_G(z,t) = e^{-\gamma t} z^\lambda \) and \( p_S(z,t) = (1 + \frac{q}{n}) p_M(z,t) \). The first-order conditions of the household’s problem are

\[
v + \theta_2 = \mu p_S(\bar{z},t),
\]

\[
(1 - v) + \theta_1 - \theta_2 = \mu [e^{-\gamma t} \bar{z}^\lambda + q p_G(\bar{z},t) - p_S(\bar{z},t)],
\]

and

\[
-\theta_1 = \mu [z_0 - e^{-\gamma t} \bar{z}^\lambda - q p_G(z_0,t)]
\]

where \( \mu \) is the Lagrange multiplier of the budget constraint, while \( \theta_1 \) and \( \theta_2 \) are the Lagrange multipliers of the inequality constraints, \( z_0 \leq \bar{z} \) and \( \bar{z} \leq \bar{z} \).

There are three cases to be considered: i) traditional economy, \( z_0 = \bar{z} = \bar{z} \), ii) industrialization, \( z_0 = \bar{z} < \bar{z} \), iii) mass consumption, \( z_0 < \bar{z} < \bar{z} \). Condition 1 rules out the case with \( z_0 < \bar{z} = \bar{z} \).

**Case 1: \( z_0 = \bar{z} = \bar{z} \) (traditional economy).**

In this case, all production is done at home using the traditional technology. The most complex want that is satisfied using the traditional technology solves the budget constraint:

\[
\int_{0}^{z_0} z \, dz = 1 \Rightarrow z_0 = 2^{\frac{1}{2}}.
\]

This will be the optimal solution as long as Condition 1 and the following inequality

\[ v \leq \frac{p_S(z_0,t)}{z_0}, \]

are satisfied.

Substituting in \( p_S(z_0,t) = (1 + \frac{q}{n}) p_G(z_0,t) \), \( p_G(z_0,t) = e^{-\gamma t} \bar{z}^\lambda \), and \( z_0 = 2^{\frac{1}{2}} \) into this inequality, we obtain the following condition on \( t \)

\[ 2^{\frac{1}{2}} \leq \left( 1 + \frac{q}{n} \right) e^{-\gamma t} \bar{z}^\lambda \Rightarrow t < t_0 = \frac{1}{\gamma} \log\left( 2^{\frac{1}{2}} \left( 1 + \frac{q}{n} \right) \right). \]

**Case 2: \( z_0 = \bar{z} < \bar{z} \) (industrialization).**

In this case, the first-order conditions simplify to

\[
v = \mu \left( 1 + \frac{q}{n} \right) e^{-\gamma t} \bar{z}^\lambda,
\]

\[
(1 - v) = \mu \left[ z_0 - \left( 1 + \frac{q}{n} \right) e^{-\gamma t} \bar{z}^\lambda \right],
\]

and the budget constraint equals

\[
(1 + \frac{q}{n}) e^{-\gamma t} \int_{z_0}^{\bar{z}} z^\lambda \, dz = 1 - \int_{0}^{\bar{z}} z \, dz.
\]
Combining (9) and (10) and integrating (11) yields two simple equations in $\bar{z}$ and $z_0$

\[
(1 + \frac{q}{n}) e^{-\gamma t} \bar{z}^\lambda = \frac{v}{1-v} \left[ z_0 - (1 + \frac{q}{n}) e^{-\gamma t} \bar{z}_0^\lambda \right]
\]

(12) and

\[
\frac{1}{\lambda + 1} \left( 1 + \frac{q}{n} \right) e^{-\gamma t} \bar{z}^{\lambda+1} + \frac{z_0^2}{2} - \frac{1}{\lambda + 1} \left( 1 + \frac{q}{n} \right) e^{-\gamma t} \bar{z}_0^{\lambda+1} = 1. \tag{13}
\]

Eqs. (12) and (13) define upward and a downward sloping curves in the $(\bar{z}, z_0)$ space, respectively. It is straightforward to see that $d\bar{z}/dt > 0$ as both curves move upward with time (productivity). Applying the Implicit Function Theorem we obtain that the effect of technological progress on the upper bound of the set of wants that are home produced using the traditional technology $z_0$ is given by

\[
\frac{d\bar{z}_0}{dt} = -\gamma (1 + 1) e^{-\gamma t} \left[ \frac{z_0^2}{2} - \gamma t + \frac{\gamma}{\lambda + 1} \right] = 0,
\]

where the inequality follows from the condition defining case 1, $z_0 - (1 + \frac{q}{n}) e^{-\gamma t} \bar{z}_0^\lambda > 0$, and $\lambda < 1$.

This case corresponds to the optimal solution if the following set of inequalities are satisfied:

\[
z_0 \leq z_0^* e^{-\gamma t} (1 + q) \quad \text{and} \quad z_0 > z_0^* e^{-\gamma t} \left( 1 + \frac{q}{n} \right) \frac{1}{v}.
\]

Using the first order conditions, these inequalities can then be expressed in terms of an interval of time as $t_0 = \frac{1}{v} \log \left( \frac{1 + q}{1 + q/n} \right) < t < t_1$, where $t_1 = -\frac{1 - \gamma}{2\gamma} \log \{T\}$, and

\[
T = (1 + q/n)(1 + q) \left[ \frac{1}{\lambda + 1} \left( \frac{v}{1-v} \right)^{\frac{\lambda+1}{\lambda}} \left( \frac{1 + q}{1 + q/n - 1} \right)^{\frac{\lambda+1}{\lambda}} + \frac{1}{2} \frac{1 + q}{1 + q/n - 1} \right] > 0.
\]

Case 3: $z_0 < \bar{z} < \bar{z}$ (rise of mass consumption).

In this case, the first order conditions simplify to

\[
v = \mu \left( 1 + \frac{q}{n} \right) e^{-\gamma t} \bar{z}^\lambda,
\]

\[
(1 - v) = \mu \left[ e^{-\gamma t} \bar{z}^\lambda + q e^{-\gamma t} \bar{z}^\lambda - (1 + \frac{q}{n}) e^{-\gamma t} \bar{z}_0^\lambda \right],
\]

\[
z_0 - (1 + q) e^{-\gamma t} \bar{z}_0^\lambda = 0, \tag{14}
\]

and

\[
\frac{z_0^2}{2} + \frac{(1 + q)}{\lambda + 1} \left[ e^{-\gamma t} \bar{z}^{\lambda+1} - e^{-\gamma t} \bar{z}_0^{\lambda+1} \right] + \frac{(1 + q)}{\lambda + 1} \left[ e^{-\gamma t} \bar{z}_0^{\lambda+1} - e^{-\gamma t} \bar{z}_0^{\lambda+1} \right] = 1. \tag{15}
\]

Using these condition we can be solved for $z_0$

\[
z_0 = (1 + q) e^{-\frac{\lambda}{1 + q}}.
\]
and a log-linear relationship between $\bar{z}$ and $\bar{z}$
\[
\bar{z} = \left(1 - \frac{\nu}{n} \frac{1 + q/n}{q(1 - 1/n)}\right) \frac{1}{\lambda} \bar{z}.
\]

Finally, using the budget constraint it is straightforward to see that $\bar{z}$ and $\bar{z}$ increase over time. This corresponds to the optimal solution if $t \geq t_1$. □

**Proof of Proposition 4.** The evolution of the time used in traditional home production, $h_T(t) = \int_0^{z_0(t)} z \, dz$, follows straightforwardly from the behavior of $z_0(t)$. The change in modern home-production time, $h_M(t) = e^{-\gamma t} \int_{z_0(t)}^{\bar{z}(t)} z^\lambda \, dz$, equals
\[
\frac{\partial h_M(t)}{\partial t} = e^{-\gamma t} \left\{-\frac{\gamma}{\lambda + 1} z_0(t)^{\lambda + 1} + \frac{1}{\lambda + 1} \frac{\partial z_0(t)}{\partial t} \right\} - \frac{1}{\lambda + 1} \int_0^{z_0(t)} \frac{\partial z_0(t)}{\partial t} \, dz.
\]

Using $\frac{1}{\bar{z}(t)} \frac{\partial \bar{z}(t)}{\partial t} = \frac{\nu}{\lambda + 1} \left[1 - \frac{\nu}{n} \frac{(1 + q)z_0(t)^{\lambda + 1}}{q(1 - 1/n)z_0(t)^{\lambda + 1} + (1 + q/2)\bar{z}(t)^{\lambda + 1}}\right]$ and $\frac{1}{z_0(t)} \frac{\partial z_0(t)}{\partial t} = -\frac{\gamma}{1 - \lambda}$,
\[
e^{-\gamma t} \left\{-\frac{\gamma}{\lambda + 1} z_0(t)^{\lambda + 1} + \frac{1}{\lambda + 1} \frac{1 + q(z_0(t)^{\lambda + 1})}{q(1 - 1/n)z_0(t)^{\lambda + 1} + (1 + q/2)\bar{z}(t)^{\lambda + 1}}\right\}
\]
\[-\frac{\nu}{\lambda + 1} \left(\bar{z}_0(t)^{\lambda + 1} - \frac{1}{\lambda + 1} \frac{\nu}{n} \bar{z}_0(t)^{\lambda + 1}\right)\bar{z}(t).
\]

Using $\bar{z} < \bar{z}$
\[
> \frac{e^{-\gamma t} \nu z_0(t)^{\lambda + 1}}{1 - \lambda}.
\]

**Proof of Proposition 5.** For the first and second phases, $t \in [0, t_0)$ and $t \in [t_0, t_1)$, the result follows straightforwardly from the characterization of thresholds discussed in Proposition 1 and the definition of $y_A(t)$, $c_A(t)$, $y_M(t)$, $c_M(t)$, $c_S(t)$ and $c_S(t)$ (see Footnote 20). We consider in detail the characterization of the third phase, $t \in [t_1, \infty)$.

The value-added and consumption shares of agriculture equal
\[
y_A(t) = c_A(t) = \frac{\int_0^{z_0(t)} z \, dz + q e^{-\gamma t} \int_{z_0(t)}^{\bar{z}(t)} z^\lambda \, dz + (1 + q/n) e^{-\gamma t} \int_{z_0(t)}^{\bar{z}(t)} z^\lambda \, dz}{1 + \frac{q}{\lambda + 1} \left[\frac{e^{-\gamma t} z_0(t)^{\lambda + 1}}{z_0(t)^2} - \frac{e^{-\gamma t} z_0(t)^{\lambda + 1}}{z_0(t)^2}\right] + (1 + q/n) \left[\frac{e^{-\gamma t} \bar{z}(t)^{\lambda + 1}}{z_0(t)^2} - \frac{e^{-\gamma t} z_0(t)^{\lambda + 1}}{z_0(t)^2}\right]}.
\]

Using $z_0(t) = (1 + q)z_0(t)^{\lambda}$ and $\frac{\nu}{\lambda + 1} = \frac{1 + q}{q(1 - 1/n)} \frac{\bar{z}_0(t)^{\lambda}}{\bar{z}_0(t)}$,
\[
= \frac{1}{1 + \frac{q}{\lambda + 1}(1 + q) \left[\bar{z}_0(t)^{\lambda + 1} - 1\right] + \frac{q}{1 + q/2} \frac{\bar{z}(t)^{\lambda + 1}}{z_0(t)^{\lambda + 1}} \left[\frac{\nu}{\lambda + 1} \frac{q(1 - 1/n)}{1 + q/2} \frac{\bar{z}_0(t)^{\lambda + 1}}{z_0(t)^{\lambda + 1}} - 1\right]}.
\]

From this expression and the fact that $\frac{\partial z(t)}{\partial t} > 0$ and $\frac{\partial z_0(t)}{\partial t} < 0$ it follows that $\frac{\partial y_A(t)}{\partial t} = \frac{\partial c_A(t)}{\partial t} < 0$. The value-added and consumption shares of services equal

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\[
c_S(t) = \left(1 + \frac{q}{n}\right)y_S(t) = \frac{(1 + \frac{q}{n})e^{-\gamma t} \int_{z_0(t)}^{z(t)} z^\lambda \, dz}{\int_0^{z_0} z \, dz + q e^{-\gamma t} \int_{z_0(t)}^{z(t)} z^\lambda \, dz + (1 + \frac{q}{n})e^{-\gamma t} \int_{z(t)}^{z(t)} z^\lambda \, dz} = \frac{\frac{\lambda + 1}{2} e^{-\gamma t} \frac{z_0(t)^2}{z(t)^{\lambda+1} - z(t)^{\lambda+1}} + q \frac{z(t)^{\lambda+1} - z_0(t)^{\lambda+1}}{z(t)^{\lambda+1} - z(t)^{\lambda+1}} + (1 + \frac{q}{n})}{(1 + \frac{q}{n})}.
\]

Using \(z_0(t) = (1 + q)z_0(t)^{\lambda}\)
\[
= \frac{(1 + \frac{q}{n})}{[\frac{\lambda + 1}{2} (1 + q) - q]} \frac{z_0(t)^{\lambda+1}}{z(t)^{\lambda+1} - z(t)^{\lambda+1}} + q \frac{z(t)^{\lambda+1}}{z(t)^{\lambda+1} - z(t)^{\lambda+1}} + (1 + \frac{q}{n})
\]

From this expression and the fact that \(\frac{\partial z(t)}{\partial t} > 0\) and \(\frac{\partial z_0(t)}{\partial t} < 0\) it follows that \(\frac{\partial y(t)}{\partial t} < 0\) iff \(q < \frac{\lambda + 1}{1 + \frac{q}{n}}\).

Finally,
\[
y_G(t) - y_S(t) = \frac{q \int_{z(t)}^{z(t)} z^\lambda \, dz + \frac{q}{n} \int_{\hat{z}(t)}^{z(t)} z^\lambda \, dz}{\int_{\hat{z}(t)}^{z(t)} z^\lambda \, dz} = \frac{q [z(t)^{\lambda+1} - z_0(t)^{\lambda+1}]}{z(t)^{\lambda+1} - z(t)^{\lambda+1}} + \frac{q}{n}.
\]

Using \(\frac{\nu}{1-\nu} = \frac{1 + \frac{q}{n}}{q (1 - \frac{1}{n}) \hat{z}}\),
\[
= q - \frac{1 - (\frac{z(t)}{z_0(t)})^{\lambda+1}}{[\frac{\nu}{1-\nu} (1 + \frac{q}{n})]^{\lambda+1} - 1} + \frac{q}{n}.
\]

From this expression and the fact that \(\frac{\partial z(t)}{\partial t} > 0\) and \(\frac{\partial z_0(t)}{\partial t} < 0\) it follows that \(\frac{\partial (y_G(t) - y_S(t))}{\partial t} > 0\). A similar argument can be used to show that \(\frac{\partial (c_G(t) - c_S(t))}{\partial t} > 0\). \(\Box\)

**Proof of Proposition 6.** The ratio of services to manufacturing output equals:
\[
y_S(y_G) = \frac{\int_{\hat{z}}^{\bar{z}} p_S(z, t) \, dz - \int_{\hat{z}}^{\bar{z}} \frac{q}{n} p_G(z, t) \, dz}{\int_{\hat{z}}^{\bar{z}} q p_G(z, t) \, dz + \int_{\hat{z}}^{\bar{z}} \frac{q}{n} p_G(z, t) \, dz} = \frac{\int_{\hat{z}}^{\bar{z}} z^\lambda \, dz}{\int_{\hat{z}}^{\bar{z}} z^\lambda \, dz} = \frac{q \int_{\hat{z}}^{\bar{z}} z^\lambda \, dz + \frac{q}{n} \int_{\hat{z}}^{\bar{z}} z^\lambda \, dz}{\int_{\hat{z}}^{\bar{z}} z^\lambda \, dz} = \frac{q \hat{z}^{\lambda+1} + q (1 - \frac{1}{n}) \bar{z}^{\lambda+1}}{\hat{z}^{\lambda+1} - \bar{z}^{\lambda+1}}
\]

using that \(z = \left(\frac{1-\nu}{\nu q (1-\frac{1}{n})}\right)z_0^{\lambda}\).
\[
y_S(y_G) = \frac{1 - (1-\nu)(1 + \frac{q}{n})^{\lambda+1}}{\nu q (1-\frac{1}{n})^{\lambda+1}}.
\]
Defining \( X = \frac{(1-\nu)(1+\frac{q}{n})}{\nu q(1-\frac{1}{n})} \cdot \frac{1+1}{\lambda} < 1 \) (by Condition 1), we get

\[
\frac{y_S}{y_G} = \frac{1 - X}{\frac{q}{n} + q(1 - \frac{1}{n})X}
\]

and

\[
\frac{\partial (\frac{y_S}{y_G})}{\partial n} = -\frac{q \frac{\partial X}{\partial n} + \frac{q}{n^2} (1 - X)^2}{(\frac{q}{n} + q(1 - \frac{1}{n})X)^2} > 0
\]

where

\[
\frac{\partial X}{\partial n} = -\frac{\lambda + 1}{\lambda} \cdot \frac{1 - \frac{1}{n}}{1 + \frac{q}{n}} \cdot \left[ 1 + \frac{q}{n^2} \right] + \frac{1 + \frac{q}{n}}{n^2 (1 - \frac{1}{n})^2}
\]

\[
= -\frac{X}{(1 + \frac{q}{n}) (1 - \frac{1}{n}) n^2} < 0.
\]

Similarly, the ratio of service to goods consumption equals,

\[
\frac{c_S}{C_G} = \frac{(1 + \frac{q}{n})(1 - X)}{q X}
\]

The effect of changes in the scale of services on the share of services in consumption is given by

\[
\frac{\partial c_s}{\partial n} = \frac{(1 + \frac{q}{n}) q \frac{\partial X}{\partial n} - \frac{q^2}{n^2} (1 - X) X}{(q X)^2}
\]

\[
= \frac{(1 + \frac{q}{n}) q X \frac{1}{(1 + \frac{q}{n}) (1 - \frac{1}{n}) n^2} - \frac{q^2}{n^2} (1 - X) X}{(q X)^2}
\]

\[
= \frac{X q(n+q)}{n^2(n-1)} + \frac{q^2 X^2}{(q X)^2} > 0.
\]

Finally, the effect of \( n \) on modern home production time follows straightforwardly from \( n \) increasing \( z \) and having no impact on \( z_0 \). □

References