1. Introduction

The general equilibrium model of an economy is the product of nearly two centuries of conceptual innovation and continued intellectual refinement. Its roots may be found in Adam Smith's description of the behavior of capitalists motivated by considerations of profitability in the collection of economic activities. The elements of demand theory appear in John Stuart Mill's treatment of international trade and in his analysis of the responses of economic agents to changes in taxes and import duties. The model reaches its mature form later in the nineteenth century in the work of Leon Walras, who provided a general description of the functioning of a complex economic system based on the interaction of a number of independent economic units. Today, the general equilibrium model is the centerpiece of microeconomic theory.

The fundamental themes of the general equilibrium model are extremely simple and lie at the heart of economic theory. The production side of the economy, engaged in the transformation of certain commodities into other commodities, is distinguished from the consumption side, whose goals are the acquisition and eventual consumption of goods and services. Stocks of commodities, which may be consumed directly or offered as factors of production, are owned by households in their physical form or by means of a variety of financial instruments. Each consumer's income, or wealth, is determined by evaluating his stock of commodities in terms of the prices at which the commodities can be sold. Income and a knowledge of relative prices permit the consumer to express his demands for goods and services and his offerings of labor and other stocks that are made available for the productive side of the economy.

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In the general equilibrium model, producers are assumed to be
informed of the prices of all inputs and the prices at which outputs
can be sold. These prices are assumed to be independent of the scale
and composition of productive activity each producer then selects
from among the choices that are technologically available to him the
production plan which maximizes profit. Production technology is
assumed to exhibit constant or decreasing returns to scale.

The decisions of the production and consumption sides of the
economy need not be consistent with each other if they are based on an
arbitrary list of prices. If the price of a desired commodity is too
low, consumers may be motivated to demand large quantities of this
commodity, and producers may be forced to supplying a commodity whose
sales generate insufficient revenue to cover the costs of manufacture.
Equilibrium prices are those which equate demand and supply in all
markets. Once they are known, in the context of a particular model of
an economy, the entire range of economic decisions based on them is
determined.

Despite the appeal of this general model of all markets simulta-
neously clearing, progress in this field of economic research has
proved to be slow. A proof of the existence of a market-clearing set
of prices remained unsolved for more than half a century after Walras
had formalized the model. It was not until the work of Arrow, Debreu,
Gale, Kuhn, McKenzie, and McNaught in the 1950s that such a proof was
formulated by using fixed-point methods from mathematical analysis.
The next hurdle, which also proved formidable, was the computation
of the equilibrium set of prices. The proofs of existence seem to offer
little guidance since they were fundamentally nonconstructive. However
in 1967 both Harold Kuhn and Hersh Scarf developed ingenious computer-
based algorithms (based on almost identical logic) for the numerical
determination of the equilibrium set of prices for Walrasian models.
It could be shown that the methods always found an equilibrium if a
fixed-point proof of existence was available. In fact, since the
algorithms could be used to constructively establish the fundamental
fixed-point theorems on which the original proofs of existence were
based, they were just as general as the analytic proofs of existence.

The ability to compute equilibria of relatively complicated general
equilibrium models opened the door to what may be referred to as applied
equilibrium modeling. It seemed natural to be able to add to
systems models some features of the real world, such as governments,
taxes, tariffs, and transfer payments, specifying them so that they resembled
actual economic and policy realities with them. Before the
development of the algorithms, general equilibrium analysis was limited
to the two by two analytic or graphic models associated with Johansen,
Heas, and Nerger. Now, larger and more realistic models were
feasible. 1 The purpose of this paper is to describe the models and
techniques that have been developed for using applied general equilibrium
analysis for the analysis of tax policy. The paper also mentions some
of the results to date, the shortcomings of current models, and the
expected directions of further developments.

2. Fundamental structure of applied general equilibrium models

General equilibrium models have four essential ingredients. There
must be a specification of (1) the endowments of consumers, (2) their
preferences, (3) the production technology, and (4) the conditions of
equilibrium.

In general, consumers may possess endowments of any or all of
the commodities in the economy. Often, in practice, consumers are endowed
only with factors of production (capital and labor). The preferences
of consumers are specified with the demand function for each commodity.
Commodity demands are nonnegative and depend on all prices in a continuous
manner. They are homogeneous of degree zero in prices, meaning that
only relative prices matter. Market demands are the sum of individual
household demands, and they satisfy Walras's law. If some notation is
introduced, the consumer side of the model can be specified. Let \( \Pi \) be the
number of commodities (including factors), \( \Pi \) be the total endowments
of commodity \( i \) and \( \Pi (P) \), \( i = 1, \ldots, \Pi \) be the market demand functions.

With this notation, Walras's law now states

\[
\Pi (P) = \Pi (W) + \Pi (W) = 0
\]

The value of market demands must equal the value of market endowments
at all prices. This condition automatically holds if market demands are
simply the sum of individual demands when the individuals are subject to
their budget constraints.

On the production side of a general equilibrium model, technology
is usually described by a set of constant returns to scale activities
or by production functions that exhibit nonincreasing returns to scale.

1 It is perhaps ironic that, once the applied general equilibrium
models were developed it was found that they could usually be solved with
Newton-type methods that have long been available. Despite this, the
expanding interest in computational general equilibrium models is clearly
due to the work of Kuhn and Scarf. Improved versions of these algorithms
are now competitive with Newton methods in terms of computational speed,
even in cases where the Newton algorithms converge.
The advantage of the activity analysis approach is that the conditions for equilibrium are very simple when production is modeled in this way. On the other hand, production functions are more convenient to use in applied work. They are easily given parameters since most of the relevant econometric literature involves their estimation.

With the activity analysis approach, the \( J \) activities available to the economy can be listed in an \((N \times J)\) matrix \( A \), where the \( a_{ij} \) elements are negative for inputs and positive for outputs. The first \( N \) columns of this matrix are disposal activities. Joint products are possible; however, activities are restricted to satisfy the boundedness condition that \( Ax + \bar{w} \) is bounded that at any nonnegative set of \( J \) activity levels \( x \). The interpretation of this condition is that the production possibility is finite in all dimensions. \( I \)

In the activity analysis modeling of production, equilibrium is characterized by a nonnegative vector of \( N \) prices and \( J \) activity levels \((p^{e}, x^{e})\) so that

\[ D_{k}(p^{e}) = \sum_{j=1}^{J} a_{kj} x_{j}^{e} + w_{k} \quad \text{for} \quad k = 1, \ldots, N \]

and

\[ \sum_{j=1}^{J} a_{kj} x_{j}^{e} + w_{k} = 0 \quad \text{if} \quad x_{j}^{e} > 0 \]

A simplified numerical example may illustrate the general equilibrium structure. For expositional purposes, let us consider a model with two final goods (manufacturing and nonmanufacturing), two factors of production (capital and labor), and two classes of consumers. Consumers have initial endowments of factors but no initial endowments of goods. The "rich" consumer group owns capital, while the "poor" group owns labor. Production of each good takes place according to a constant elasticity of substitution (CES) production function, and each consumer class has demands derived from maximizing a CES utility function subject to its budget constraint.

The production functions are given by

\[
Q_{i} = \frac{1}{\delta_{i}} \left[ \left( \frac{L_{i}}{L} \right)^{\delta_{i}} + (1-\delta_{i}) K_{i} \right]^{1/(\delta_{i}-1)}
\]

where \( Q_{i} \) denotes output of the \( i \)th industry, \( \delta_{i} \) is the scale or unit parameter, \( \delta_{i} \) is the distribution parameter, \( X_{i} \) and \( L_{i} \) are the factor inputs, and \( \sigma_{i} \) is the elasticity of factor substitution.

The CES utility functions are given by

\[
U^{e} = \frac{1}{2} \left( \begin{array}{c} \sigma_{1} \\ \sigma_{2} \\ \vdots \\ \sigma_{q} \end{array} \right) \left( \begin{array}{c} x_{1}^{e} \\ x_{2}^{e} \\ \vdots \\ x_{q}^{e} \end{array} \right) \left( \begin{array}{c} \sigma_{1} \\ \sigma_{2} \\ \vdots \\ \sigma_{q} \end{array} \right) \frac{1}{q}
\]

where \( x_{j}^{e} \) is the quantity of good \( j \) demanded by the \( q \)th consumer, \( \sigma_{q} \) are share parameters, and \( \sigma_{q} \) is the substitution elasticity in consumer class \( q \)'s CES utility function.
If consumers maximize these utility functions subject to the constraint that expenditures do not exceed income derived from the sale of endowments, the resulting demand functions are:

\[ X_i^q = \frac{q_i^2}{1 - \nu} \]

\[ X_i^q = \frac{p_i^q}{1 - \nu} (a_1 q_1 + a_2 q_2) \]

where \( q_i \) is individual \( i \)'s income level.

With this structure, a "zip" model can be specified, with the following values of the parameters:

<table>
<thead>
<tr>
<th>Production</th>
<th>( q )</th>
<th>( \delta )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing (1)</td>
<td>1.5</td>
<td>0.6</td>
<td>2.9</td>
</tr>
<tr>
<td>Nonmanufacturing (2)</td>
<td>2.0</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Preference Parameters</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich households</td>
<td>25</td>
<td>0</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>Poor households</td>
<td>0</td>
<td>60</td>
<td>0.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

This model has been solved using Merrill's algorithm, which is an advanced variant of Scarf's method. The results are shown in Table 1.

At the prices computed, total demand for each output exactly matches the amount produced. It follows that producer revenues equal consumer expenditures. It also is true, to a high degree of approximation, that the labor and capital endowments are fully employed and that consumer factor incomes equal producer factor costs. The cost per unit output in each sector matches the price, which means that economic profits are zero. The expenditure of each household exhausts its income. Thus, the solution closely approximates all of the properties of an equilibrium for this economy. The closeness of the approximation can be enhanced by increasing the amount of computation time allowed for the algorithm used in the solution.

<table>
<thead>
<tr>
<th>Table 1. Equilibrium Solutions General Equilibrium for Illustrative Simple Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equilibrium prices</strong></td>
</tr>
<tr>
<td>Manufacturing output</td>
</tr>
<tr>
<td>Nonmanufacturing output</td>
</tr>
<tr>
<td>Capital</td>
</tr>
<tr>
<td>Labor</td>
</tr>
<tr>
<td><strong>Production</strong></td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Nonmanufacturing</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Nonmanufacturing</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td><strong>Expenditure</strong></td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Nonmanufacturing</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td><strong>Labor Income</strong></td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Nonmanufacturing</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>
This illustrative example shows the kind of models that can be solved with the relatively new computer-based algorithms. However, it does not indicate how data are collected and incorporated into these models and other policy variables are introduced. Also, it is necessary to develop welfare economics techniques to compare the equilibria that result from alternative policies. Let us now turn to these issues.

3. Specification of policy models

a. The inclusion of taxes

The first modification that is desirable in the simple model outlined above is the inclusion of a system of taxes and government expenditure. Taxes may be imposed on the purchase of goods and services by consumers, the use of factors and intermediate inputs by producers, the receipt of income by consumers, and the final output of the various production sectors. The tax rate may differ for each good, consumable good, and producer. The government uses the tax proceeds to finance transfer payments to consumers and to purchase final goods and services. Most of the models developed to date assume a balanced government budget, but recent work by Felstein (1973) incorporates a bound and money market into models of this type.

The method for including taxes and governments into the general equilibrium framework was first shown in Shoven and Whalley (1975) and Shoven (1974). Conditions for equilibrium become demand equal supply for each commodity, firms in operation based on after-tax revenue and government receipts (including bond sales and money issuance in Felstein's formulation) equal government expenditure. Walton's law of unproductive tax value of demand equals the value of endogenous less personal taxes plus transfer payments. It conditions known as the sum of the individual household after-tax budget constraints.

b. Equal yield tax comparisons

Often, in consideration of replacement of one system of taxes with an alternative system, the relevant policy constraint is that the replacement set of taxes should generate the same real government revenue as the original set. When economic behavior in itself as a function of tax rates, the rates required for matching the yields cannot be easily calculated. In fact, a full general equilibrium analysis is required to determine such rates correctly. The computational algorithms used can easily be extended to calculate not only an equilibrium for a new tax but also a scaler that determines the level of tax rates. The system but also a scaler that determines the level of tax rates. The system but also a scaler that determines the level of tax rates. The system but also a scaler that determines the level of tax rates.

$$Q = \min \left[ \begin{array}{c} E \cdot V(A, E) \cdot E_1 \ldots E_n \end{array} \right]$$

Most empirical models distinguish between industrial sales and consumer sales for the simple reason that the data are classified differently. Industrial sectors involve such categories as forestry and fisheries, metal mining, and publishing and printing, while consumer goods, automobiles, and books. This fact is recognized in the model by incorporating a second stage of production, which converts industrial sales into consumer sales. This technology is usually modeled as a fixed coefficient.

With some exceptions (for example, Fullerton (1982) and Dorris, de Nola, and Robinson (1983)), capital is modeled as fully mobile between production sectors and hence occurs the same after-tax rate of return from each sector. Fullerton's model allows full mobility to new investment, which earns the same rate of return in all sectors engaged in new investment, but in the industrial sector capital once it has been acquired. Dorris, de Nola, and Robinson have a similar "putty-clay" model of capital, although the allocation of investment may be set by arbitrary policy rules rather than by competitive rent seeking.

d. Consumption and saving

Computational or applied general equilibrium models were initially almost always static in nature, possibly including rather artificial saving and investment behavior. In recent years several of the models have been made dynamic, although they remain an area of active model development. The U.S. model with which the author is associated (along...
with his co-investigators John Whalley, Don Fullerton, Charles Ballard, and Larry Cudler) now computes a sequence of essentially static equilibria connected by saving and capital formation.

The 11 consumer classes in the U.S. model act as if they were maximizing the nested utility function

\[ U = u(H, \tau_1, \tau_2, \ldots, \tau_{11}, e, c_p) \]

or some monotonic transformation of it, subject to their income constraint. The \( H_1 \) are consumer goods (15 in number in the U.S. model), \( e \) is leisure, and \( c_p \) is a composite commodity of future consumption.

Both \( H \) and \( U \) are CES functions. The parameters of these functions determine the share of income devoted to each commodity, to savings (the provision for \( c_p \)), and to the "purchase" of leisure. They also determine two key elasticities in the models—the elasticity of labor supply with respect to the real after-tax wage rate and the elasticity of saving with the real after-tax rate of return to capital.

In the U.S. model, consumers have myopic expectations regarding future prices and, in particular, regarding the future rate of return to capital. Future consumption is "acquired" by buying a fixed composition portfolio of real investments that offer an infinite annuity of returns. There has been some work on incorporating both perfect foresight and limited foresight into this model (Ballard and Cudler (1983)). Work is also being done to incorporate life-cycle behavior where a utility function such as

\[ U = \int_0^T H(K, e) e^{-at} dt \]

is maximized, subject to a lifetime wealth constraint.

e. Foreign trade

Applied general equilibrium modeling is used in the evaluation of customs unions, tariffs, and trade restrictions, and several models focusing on these issues have been developed (see, for example, Miller and Spencer (1977), Volltenstein (1980) and (1982), and Whalley (1982)).

Here, let us concentrate on the foreign trade specification of models basically designed for evaluating domestic tax policies.

International trade is usually modeled extremely simply. In the U.S. model, the standard specification is one which has a constant elasticity of export demand and has import supply equations. Trade balance is imposed, and there is no international mobility of capital. A richer specification of the foreign sector, including international capital markets, was investigated by Cudler, Shoven, and Whalley (1982). The impact of domestic tax policies was shown to be quite sensitive to international capital mobility and to the share granted in the United States for foreign taxes paid. Other tax models (Keller (1980), Ballantine and Thirl (1979)) include capital flows, while some (Simson (1981) and Auerbach, Katzko, and Shimmin (1981)) have no foreign sector at all.

f. Financial sectors

Current general equilibrium tax models clearly owe a great deal to the pathfinding work of Arnold Harberger (1959, 1962, 1976). He introduced the two-sector general equilibrium framework to public finance and was one of the first to investigate the issue of tax incidence so it is known today. In many ways, the proper approach to thinking about the current models is to super-Harberger models.

One serious shortcoming of these models is the total absence of financial markets. They are "real" models solving for relative prices, but there are no debt instruments, money, financial intermediation, or deficits. Integrating financial and real markets in these models is perhaps the current area of greatest research activity. Volltenstein (1983) has added to this general model money and government bonds as well as foreign exchange markets. Shimmon (1981) has attempted to incorporate modern portfolio behavior on the part of consumers, while Fullerton and Gordon (1981) have begun to deal with issues of corporate financial policy and behavior toward risk.

Given that countries experience large government deficits, current account trade imbalances, and sizable accumulated foreign debt, the inclusion of these features in policy models is clearly important. The general issue of the "crowding out" of private sector investment through government borrowing can also be addressed in this framework.

8. Data requirements and parameter specification

In applying general equilibrium models, a complete equilibrium data set must be assembled. This includes factor wages by industry, an input-output table, consumer expenditures by commodity and income by source, government expenditure and tax collections, and information on foreign trade and investment. The normal practice is to gather this data from available sources for a particular year. In general, such data are inconsistent. For example, total labor payments by employers
do not match total labor income. To be useful, the data must be adjusted for consistency. This requires some judgment as to which data are most reliable and which should be changed so as to be consistent.

The consistent data represent what is often referred to as the "benchmark" equilibrium. The strong assumption is made that the data represent an equilibrium of the economy. The construction of data sets of this type is described in St. Hilaire and Whalley (1980), Pigott and Whalley (forthcoming), and Ballard, Fullersten, Shoven and Whalley (forthcoming). Since the benchmark data are usually presented in value terms, units must be chosen for goods and factors in order to obtain separate price and quantity observations. A common used type of unit conversion, originally adopted by Barberger, is to choose units for both goods and factors that have a price of unity in the benchmark equilibrium.

With the benchmark observation at hand, parameters are then chosen so that the solution to the model explicitly replicates the benchmark data. This procedure is termed "model calibration." Values of the parameters thus generated can then be used to solve for a different equilibrium under alternative policy regimes. This is usually termed a "counterfactual" or "policy replacement" equilibrium.

The typical calibration procedure involves only one year's data or one single observation, which may be an average over a number of years. Depending on the complexity of functional forms used, the data may not uniquely identify the parameters. With Cobb-Douglas functions, a single benchmark observation serves to uniquely identify the values of the parameters, since expenditure share by sector are known. With other functions, it is typically the case that an infinite number of combinations of parameters can replicate the data in the required manner. In such cases, extra information is needed for identifying restrictions. Once specified, these allow the other parameters to be determined uniquely from the equilibrium observation.

The autonomous specification of elasticities can be thought of as determining the curvature and position of indifference curves. This is sufficient to uniquely determine the parameters of the function if CES functions are used. Additional assumptions of substitution elasticities are required, since the curvature of indifference curves, described by the single elasticity parameter, is not given by benchmark data. If the elasticity parameter is known, the demand functions are determined, once the origin coordinates for utility measurement are known. The current procedure in setting the additional parameters is to scan empirical literature to select appropriate values of substitution elasticities for the underlying utility and production functions.

The primary role of calibration is thus to determine the shares and unit parameters in these functions, once elasticities are known. No statistical test of the chosen model specifications is involved, since a deterministic procedure is employed for calculating the values of the parameters from the equilibrium observation. This entire procedure is clearly dependent on the accuracy of the assembled data and the model that is chosen to represent an equilibrium. Also, the key role played by elasticities used in these models becomes immediately apparent.

Once the calibration procedure is completed, a full model is available and can be used for policy analysis. As indicated in Figure 1, any policy change can be specified and a counterfactual equilibrium for a new policy regime can be computed. Policy appraisal then proceeds on the basis of pair-wise comparisons of counterfactual and benchmark equilibria. If further policy changes are to be evaluated, the specification of policy changes is repeated.

There are a number of reasons why this calibration approach is so widely used, rather than a more direct econometric approach, in setting the parameters for applied models. First, in some of the models, many thousands of parameters are involved; to estimate all of these parameters of the model simultaneously by using time series methods would require an unrealistically large number of observations. Second, the way in which benchmark data sets are used to generate the values of the parameters under calibration involves taking an observation in value terms and then decomposing it into separate price and quantity observations. Benchmark equilibrium prices, by construction, represent unity in each benchmark equilibrium. This makes it difficult to sequence equilibrium observations with consistent units through time, as would be required for time series estimation. These problems, combined with the difficulty of incorporating equilibrium restrictions into a satisfactory estimation procedure, have thus far largely excluded complete econometric estimation of general equilibrium systems, although some progress in this direction has been made in recent work by Meur (1981). Meur, for instance, notes the difficulties in simply writing down a likelihood function for a maximum likelihood procedure incorporating full-equilibrium restrictions. He equates equilibrium prices, by construction, with changes in production and demand systems, with a third step incorporating their equilibrium interdependence. Other attempts at econometric estimation of complete equilibrium systems occur in the work of Allingham (1972) and Jorgenson (1983).

b. Welfare evaluations

A counterfactual equilibrium is computed and compared with the observed economy (which is assumed to represent an equilibrium). In the case of a dynamic model, a dynamic path of prices and endowments is computed (the capital endowments being exogenous). This is compared
with the path of the economy when there is no policy change. For the U.S. model, the investigators have assumed that the base year's data (1971) represent not only a static equilibrium, but one which lies on a steady-state growth path. Thus, without any policy change, relative prices remain constant and the economy simply gets larger in a completely balanced manner. When an unanticipated policy change is announced, the economy goes through a transition period but eventually resettles into a new steady-state growth path. The model thus computes both the transition path and the long-run comparative steady states.

Without a social welfare function, it is impossible to unambiguously state that one equilibrium or a path of equilibria is better than an alternative, unless the improvement follows Pareto's law—that is, everyone is better off. This is, unfortunately, rarely the case. What the investigators do in this model in measuring the change in economic efficiency or the welfare of a policy change is analogous to the measurement of costs and benefits in cost-benefit analysis—that is, apply the Kalder criterion: a situation is superior if the winners could compensate the losers, even if this compensation does not take place. The criterion has well-known theoretical shortcomings (Buchanan showed, for instance, that it need not be transitive), but it is widely used for policy evaluation. In the U.S. model, the investigators calculate the dynamic or static compensating variation for each household and sum these for an overall welfare measure. The government's expenditures do not enter into this calculation—an omission that is less serious owing to the equal revenue-equal expenditure constraint described above; that is, the government has the same real resources available to it under both the old and new policy regimes.

It should be noted that the general equilibrium approach offers such a complete description of the economy for alternative policy scenarios. Substantial information is lost in the endeavor to compare the equilibria with a single number. The changes in welfare for each commodity can be computed, and the changes in factor usage, expenditure patterns, and industrial output levels can be examined. It should also be said that, for large policy changes, such as the institution of a new tax regime or the construction of a project such as the Aswan Dam, only a general equilibrium analysis can capture the interactive effects.

4. Applications

In this section, let us review some of the applications that have been completed using the U.S. model. It should be stated at the outset that there are a large number of other models that have been used for policy evaluation in other countries. These include Miller and Spencer's (1977) assessment of the United Kingdom's entry into the European Community, Whalley's (1975) evaluation of the major 1973 U.K. tax reform package, Feldstein's (1980) analysis of trade restrictions in Argentina,
Serra-Puche's (1981) policy model for Mexico, Shleifer's (1982a) examination of the effects of the Tokyo Round trade agreement, and John Pigott's (1979) evaluation of Australian tax policy. Similar models are being used for development policy (see Barrig, de Mello, and Robinson (1982), energy economics (Rudebusch and Jorgenson (1978) and Borger and Coulter (1982)), and even economic history (Jones (1981)).

The U.S. model consists of a production sector of 19 products, 16 consumer goods, and 12 consumer groups. It is a dynamic model incorporating the complete tax system (federal, state, and local personal income taxes, corporate taxes, sales and excise taxes, social security taxes, etc.). The benchmark data set represents the 1975 economy. The model's development was financed by the U.S. Treasury Department, and it is currently in use there, most recently in evaluating flat-tax proposals.

The policy that has received the most attention in the United States from general equilibrium models is the integration of the U.S. corporate and personal income tax probably because Barberger originally examined the incidence and efficiency consequences of the corporate income tax with his two-sector model. Corporate equity capital is taxed twice in the United States in that the earnings of firms are subject to the 46 percent corporate income tax. After-tax earnings are either distributed as dividends and taxed at the personal level or retained. If retained, the earnings may lead to partially taxable capital gains. Capital income from other sectors, particularly real estate and some extent agriculture, is lightly taxed. The result is an insufficient allocation of capital across sectors and, quite possibly, a distortion of the consumption/saving decision.

Another policy that has been evaluated with the U.S. general equilibrium model is the possibility of taxing consumption rather than income at the personal level. This could be accomplished by first establishing the household's income and then allowing a deduction for all saving. As the tax would be direct, it could have special tax advantages for the blind, the elderly, those with large families, etc., and could have increasing marginal rates. The advocates of a consumption tax argue that it does not distort the consumption decision as does the income tax, and that it is better to base taxes on a household's withdrawal from the social product (consumption) than on a rough approximation of their contribution to it (income).

Before evaluating the consumption tax, it is important to recognize that the United States already has a partial consumption tax since roughly half of saving is not subject to tax. Thirty percent is saved through retirement plans and life insurance, where the tax is deferred until withdrawal (as with a consumption tax). Another 20 percent of saving is in the form of new housing construction. Housing must be purchased with after-tax dollars (i.e., the saving/investment is not deductible), but the return on it, implied or otherwise, is very lightly taxed. Thus, it is not taxed twice, as with an income tax; its treatment is more nearly analogous to a consumption tax.

Table 2 presents the dynamic efficiency gains for a consumption tax and corporate tax integration. The figures are in 1973 dollars. The key parameters of the model are set at 0.4 for the saving elasticity and 0.15 for the labor supply elasticity. The elasticity for factor substitution is value added varies by industry, but is generally slightly less than unity. The gain in efficiency depends on how the lost revenue is compensated for. For example, if a consumption tax is instituted by raising 80 percent of saving deductable (over and above the 20 percent currently saved through new housing acquisition), the first row of Table 2 shows that the gain would be $456 billion if the revenue shortfall was made up with lump-sum tax increases. However, if marginal tax rates are increased in a multiplicative manner (everyone's rate is multiplied by a common $>1.0), the gain is $621 billion, while if they increased in an additive manner ($+t*$), the welfare measure increases by $636 billion. These numbers are about 1.25 percent of the present value of future national income, expanded to include the value of leisure. The discount rate used is each consumer's after-tax rate of return to capital before the tax change, which averaged a real rate of return of 4 percent.

The second row of Table 1 shows the welfare gains of integrating the two income tax systems. The results are more sensitive to the replacement tax used for maintaining government revenues, both because integration involves the loss of more tax receipts and because it does not stimulate saving, capital formation, and growth.

The third row combines the policies of the first two systems and shows that the efficiency improvement is approximately additive. Since 80 percent of total savings are deductible under the plans of rows 1 and 3 and 20 percent of total savings flow into tax-favored housing, these plans capture the intertemporal effects of a partial consumption tax. However, since any savings can be used for housing, these plans leave an intersectoral distortion in favor of owner occupancy. The plan of row 4 allows full deductibility of savings and eliminates the preference for housing. Gains are larger, as expected. The efficiency gain of the plan in row 4 relative to the current tax system is roughly $1.5 trillion with lump-sum revenue replacement, $1.15 billion with multiplicative marginal rate surcharges, and $1.39 billion with additive marginal rate surcharges. Row 5 examines a partial move toward a consumption tax (halfway between the current 30 percent sheltering for retirement plans and the 50 percent of row 4), while row 6 examines all
Table 2. Dynamic Welfare Effects in Present Value of Compensating Variations Over Time
(In billions of 1973 dollars) 1/

<table>
<thead>
<tr>
<th>Tax Replacement</th>
<th>Types of Scaling to Preserve Tax Yield</th>
<th>Lump sum</th>
<th>Multiplicative</th>
<th>Additive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Consumption tax</td>
<td></td>
<td>686.167</td>
<td>(1.375)</td>
<td>(1.245)</td>
</tr>
<tr>
<td>(80 per cent savings deduction)</td>
<td></td>
<td>630.652</td>
<td>(1.124)</td>
<td>(1.135)</td>
</tr>
<tr>
<td>2. Corporate tax integration with indemnity of capital gain</td>
<td></td>
<td>731.350</td>
<td>(1.667)</td>
<td>(0.606)</td>
</tr>
<tr>
<td>3. Consumption tax with integration</td>
<td></td>
<td>339.503</td>
<td>(2.001)</td>
<td>(2.007)</td>
</tr>
<tr>
<td>4. Pure consumption tax with integration</td>
<td></td>
<td>1300.881</td>
<td>(3.024)</td>
<td>(2.601)</td>
</tr>
<tr>
<td>5. Partial consumption tax (55 per cent savings deduction)</td>
<td></td>
<td>320.360</td>
<td>(0.658)</td>
<td>(0.654)</td>
</tr>
<tr>
<td>6. Full savings deduction with housing preference</td>
<td></td>
<td>991.706</td>
<td>(1.999)</td>
<td>(1.991)</td>
</tr>
<tr>
<td>7. Pure income tax without integration</td>
<td></td>
<td>-179.177</td>
<td>(-1.182)</td>
<td>(-0.964)</td>
</tr>
<tr>
<td>8. Pure income tax with integration</td>
<td></td>
<td>120.398</td>
<td>(-0.230)</td>
<td>(0.045)</td>
</tr>
</tbody>
</table>

1/ The numbers in parentheses represent the gain as a percentage of the present discounted value of consumption plus leisure in the base sequence. This number is $49.963 trillion for all comparisons and accounts for only the initial population.

The results shown in rows 7 and 8 indicate that the United States could move to a pure income tax and integrate the corporate tax with no loss in efficiency, but that a pure income tax alone would lose efficiency. For row 7, the tax base is increased, since imputed income from housing is included and existing savings deductions are eliminated. Thus, the tax rate can be lowered, rather than raised, in order to maintain government revenues. Results in row 7 show that moving to a pure income tax alone involves an efficiency loss of $379 billion if marginal tax rates are not lowered—primarily because the intertemporal distortions of the current system are worsened. However, if the marginal rates are reduced, the efficiency loss to the economy is lowered to roughly $470 billion. The improvement in the interindustry allocation of capital (resulting primarily from the taxation of the return to owner-occupied housing) tends to offset the deterioration in the intertemporal efficiency (now reduced by the marginal rate adjustments). Row 8 shows the results from a comprehensive single level income tax plan involving corporate tax integration as well. Such a tax system lowers revenues and thus necessitates a rate increase to maintain the yield. When the rates are adjusted either multiplicatively or additively, the net efficiency impact of the package is negligible.

The results in Table 2 are sensitive to the elasticities incorporated in the model. For example, the $471 billion from row 1, with a multiplicative scaling of the marginal tax rates, becomes $41 billion if the uncompensated saving elasticity is zero and $1,379 billion if this elasticity is 2.0. A more thorough evaluation of these results appears in Fullerton, Shoven, and Shleifer (1982).

Table 3 provides information on how long the economy takes to reconcile into a steady-state growth path after a tax change occurs. Once the economy has completely adjusted to the new policy regime, all relative prices will again remain constant. In the case of consumption tax proposals, the new steady state is characterized by a higher capital intensity and a lower relative return to capital. The results of Table 3 indicate that, for the cases with a 0.6 savings elasticity, roughly 40 percent of the adjustment is completed after 10 years and 80 percent is completed after 30 years. The economy then asymptotically approaches the new steady-state growth path. The transition is accomplished much more rapidly with a savings elasticity of 2.0, despite the fact that the total adjustment is larger. Adjustments in capital/labour ratios proceed in patterns similar to the adjustments of the price ratios in Table 3.
Interestingly, Ballard and Goulder (1982) find that the adjustment to a new steady state is slightly slower with perfect foresight and the institution of a consumption tax, since consumers are deterred from additional saving by the recognition that future capital deepening will depress the rate of return to capital.

In previous literature, estimates of the length of the long run vary widely. Riefler (1963) finds the adjustment to be extremely long (more than 100 years), while Summers (1981) and Hall (1968) find it to be surprisingly short (about 5 years). It is difficult to reconcile these various findings, but it is clear that a prime determinant in the strength of substitution effects in the model used for the analysis.

Charles Ballard, John Whalley, and the author (1982) made another set of computer runs, asking a question of more theoretical interest: What are the efficiency costs of the entire U.S. tax system? This question is of interest because efficiency losses are often treated as minor ones relative to those of economic stability. Our aim was also to estimate the marginal cost of a government dollar raised by increasing taxes. In the past, efficiency costs have frequently been quoted as fractions of gross national product or as the deadweight loss relative to the revenue raised. The former measure is ridiculous if the question is whether a tax on automobile tires or restrictions on steel imports are inefficient. The latter measure—the average distortionary per dollar raised—does not often give the right answer either (average figures seldom do in economics). What has been computed, therefore, is the marginal distortionary cost per marginal dollar raised for each of the major tax systems in the United States.

Our estimate for the hypothetical experiment of removing the entire tax system and replacing it with a set of lump-sum levies (proportional to income and with sales taxes actually paid so as to minimize income and wealth transfers) is that the present value of welfare would increase by $3.3 trillion, which is roughly 6.7 per cent of national income plus transfers or 10 per cent of national income. The primary result of removing all marginal taxes is tremendous capital deepening. The net of tax capital-output ratio immediately climbs by 117 per cent and gradually climbs from there to become 30 per cent higher than its present value in the new steady-state growth path. The capital-labor ratio in 30 per cent higher after 30 years. The labor supply also grows, being 19 per cent higher in the first period, because leisure is no longer the ultimate tax shelter that it is under the current system.

These results are sensitive to the values of the key elasticity parameters, as shown in Table 4, although the general picture is preserved. The standard case is shown in the second row. The total loss represents 3.33 per cent of expanded national income, even when both the uncompensated labor supply and saving elasticities are zero.
Table 4. Sensitivity Analysis with Respect to Key Parameters of Deadweight Loss of the Total Tax System

<table>
<thead>
<tr>
<th>Labor Supply Elasticity</th>
<th>Saving Elasticity</th>
<th>Welfare Gain (trillions of 1973 dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.0</td>
<td>2.211</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.40)</td>
</tr>
<tr>
<td>0.15</td>
<td>0.4</td>
<td>3.228</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6.69)</td>
</tr>
<tr>
<td>0.15</td>
<td>2.0</td>
<td>0.236</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(16.32)</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.772</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.55)</td>
</tr>
<tr>
<td>0.0</td>
<td>0.4</td>
<td>2.709</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5.42)</td>
</tr>
<tr>
<td>0.0</td>
<td>2.0</td>
<td>7.037</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(14.07)</td>
</tr>
</tbody>
</table>

1/ The figures in parentheses express the welfare gains as a percentage of the total present value of welfare from consumption and leisure, which is $49.863 trillion.

---

Table 5. Relationships Between Net Present Value of Total Tax Reform and Marginal Change in Welfare (Annotated for First Period) for Various Parts of the Tax System

<table>
<thead>
<tr>
<th>(1) Marginal Rates Net Present Value (hillion)</th>
<th>(2) Change in Wages (hillion)</th>
<th>(3) Change in Wages (hillion)</th>
<th>(4) Change in Wages (hillion)</th>
<th>(5) Change in Wages (hillion)</th>
<th>(6) Change in Wages (hillion)</th>
<th>(7) Change in Wages (hillion)</th>
<th>(8) Change in Wages (hillion)</th>
<th>(9) Change in Wages (hillion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All marginal rates</td>
<td>3.321</td>
<td>1.101</td>
<td>0.233</td>
<td>0.281</td>
<td>0.267</td>
<td>0.212</td>
<td>0.252</td>
<td>0.233</td>
</tr>
<tr>
<td>Capital taxes by industry</td>
<td>0.615</td>
<td>0.230</td>
<td>0.233</td>
<td>0.233</td>
<td>0.233</td>
<td>0.233</td>
<td>0.233</td>
<td>0.233</td>
</tr>
<tr>
<td>Labor taxes by industry</td>
<td>0.707</td>
<td>0.282</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td>Consumer purchases by output goods</td>
<td>0.131</td>
<td>0.033</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Motor vehicle taxes</td>
<td>0.011</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Marginal income</td>
<td>1.000</td>
<td>0.622</td>
<td>0.622</td>
<td>0.622</td>
<td>0.622</td>
<td>0.622</td>
<td>0.622</td>
<td>0.622</td>
</tr>
</tbody>
</table>
Table 3 contains the results of computing the marginal distorting costs of the U.S. tax system. If all marginal tax rates were multiplied by 1.01, the effect would be to raise government receipts by $28.5 billion. Transfers to consumers amount to just under a third of government revenues and the question is whether this fraction of a marginal tax increase would also be returned to households as transfers. Table 3 computes the marginal cost of funds for household consumption expenditures under the assumptions that transfers are adjusted when receipts increase and that they are held fixed.

If transfers increase, then $1.181 billion is returned to households. Further, of the $3.331 billion raised, the government itself pays $0.233 billion. Putting net the transfers and the government's own tax payments, the funds available for a public project are $3.997 billion, shown in column 5. The decrease in consumer welfare is $3.520 billion, or 1.76 times as much as the money available for the government project. Column 7 reflects this 0.76 distorting cost per dollar transferred to the public sector. If transfer payments are not increased, the government ends up with more net revenue and households have a larger decrease in utility, with the net result that consumers lose 1.52 times as much as the government raises. This is reflected in column 8.

The implications of these numbers, if accepted, are great. The $1.76 or $1.52 private cost of a marginal government dollar means that cost-benefit studies that use unity as the critical benefit-cost ratio support projects that are socially inefficient. They do this by not taking into account the resource waste caused by the distorting tax used to raise the additional revenues. The correct critical ratio would be 1.76 or 1.52, depending on how transfer payments react to the enlarged government budget. At a more theoretical level, the results indicate that the Samuelson conditions for the optimal provision of public goods should include

\[
\frac{\text{MRS}_G}{\text{MRS}_C} = \lambda \cdot \frac{\text{MRS}_G}{\text{MRS}_C}
\]

where \(\lambda\) is 1.76 or 1.52. Public goods not only use up resources in their manufacture but are a cost to economic welfare in the distorting taxes they necessitate.

Rows 2-7 of Table 3 compute the marginal cost of each major type of tax. If these tax types were used in a third-best optimal manner, they would each have the same marginal distorting cost per dollar raised. This would minimize the total deadweight loss for a given revenue using this given set of instruments. The results of columns 7 and 8 show that the U.S. tax system is far from this optimality condition. An additional flat tax on labor by industry (payroll tax) could raise a dollar for no little as $1.19 in private welfare, whereas increasing the 1973 personal

Income tax rates would result in government dollars that cost 1.57 each at the margin, even if transfer payments are frozen.

The results of this table are elaborated on in Ballard, Shaw, and Whalley (1982). They illustrate a kind of analysis that cannot appropriately be carried out with partial equilibrium techniques.

5. Conclusion

General equilibrium analysis has developed from an abstract economic theory to a computational procedure and now to a tool that can be used for policy purposes. It is not always the appropriate technique; certainly, there are many issues that are best examined at a very fine level of detail by using partial equilibrium methods. The model also is inappropriate for very short-run forecasting of business and the business cycle and does not yet have the many rigidities (unions, rent controls, monopolies, transaction costs, etc.) that characterize real economies. Nonetheless, for analysing the likely medium-run to long-run adjustments of an economy to a large policy change, the applied general equilibrium model seems to be appropriate and to be ready for application.


Koo, T. J., and John Walley, "Uniqueness of Equilibrium in a Large Scale Numerical General Equilibrium Model" (University of Western Ontario, 1982), mimeographed.


