Section I emphasizes that the output market measure can enable an analyst to perform precise measurements of social costs even when data on the directly affected input market are unavailable. Section II discusses how information from other observable input and output markets can be used to measure costs and benefits that arise in an unobservable market.

We wish to show that an input market distortion can be completely measured in the output market. Without loss of generality, we examine the case where we place a tax on only one factor, and assume inputs used to produce the output can be used to produce only this output. An obvious generalization of the proof establishes it for the case where all inputs are taxed and many outputs can be produced by the inputs. We assume competition in all markets. We do not require that factor supply curves be infinitely elastic and instead allow for the case where factor supply curves slope upwards. This means that factors earn rents. Two of the standard reasons why factor supply curves can slope upwards are that the opportunity cost of the factor may not be constant as in the labor supply decision and that the marginal cost of producing the factor may be rising as occurs, for example, in the case of mineral deposits of different extraction costs that must be mined or in the case of agricultural land that requires different amounts of fertilizer.

Let \( r_1(t) \) be the equilibrium wage received by factor 1 when the tax on factor 1 is \( t \). Therefore, a firm pays \( r_1(t) + t \) for factor 1, \( r_2(t) \) for factor 2, etc. Let \( x_1(t) \) be the amount of factor 1 demanded in equilibrium when the tax on factor 1 is \( t \). The deadweight loss \( S^1 \) of an increase in the tax on factor 1 from \( t_0 \) to \( t_1 \) is measured in the input market by...


\[ I_1(x(t)) = \int_{x_1(t)}^{x_1(t_0)} (u^d_1(x_1) - u^s_1(x_1)) dx_1 \]

where \( u^d_1(x_1) \) = the demand wage for \( x_1 \) units of factor 1
\( u^s_1(x_1) \) = the supply wage for \( x_1 \) units of factor 1.

Notice that

\[ \frac{dx}{dt_1} = \left[ u^d_1(x_1(t_1)) - u^s_1(x_1(t_1)) \right] \frac{dt}{dx} = -t \frac{dx}{dt}. \]  

Let us now show how the measurement of deadweight loss in the output market will lead to exactly the same result as (1), provided we measure costs appropriately. Let the equilibrium output when taxes on factor 1 are \( t \) be denoted by \( Q(t) \). The loss from raising the tax on factor 1 from \( t_0 \) to \( t \) can be measured as the change in the area under the output demand curve minus the change in "true" costs (i.e., excluding rent and tax revenue changes) of producing the output.

Let the costs (after excluding tax revenue) of producing output \( Q(t) \) be denoted by

\[ C(t) = \sum_{i=1}^{n} c_i x_i(t) \]  

The change in \( C(t) \) as the tax rate increases is

\[ C'(t) = \sum_{i=1}^{n} c_i \frac{dx_i}{dt} x_i(t) + \sum_{i=1}^{n} \frac{dc_i}{dt} x_i(t). \]  

The change in cost relevant for calculations of welfare loss is not \( C'(t) \) since \( C'(t) \) includes the term \( \sum_{i=1}^{n} \frac{dc_i}{dt} x_i(t) \) which is the change in rents that accrue to factor owners as taxes are altered. The cost change relevant for the cost-benefit calculation excludes rent and equals

\[ \tilde{C}'(t) = C'(t) - \sum_{i=1}^{n} \frac{dc_i}{dt} x_i(t) = C(t) - \frac{dc}{dt} x_i(t). \]

Let the area under the (inverse) output demand curve, \( P(Q) \), between the quantities corresponding to the tax rates \( t_0 \) and \( t \) be denoted by

\[ C(t) = \int_{Q(t_0)}^{Q(t)} P(Q) dQ. \]  

hence

\[ \frac{dC}{dt} = -Q'(t) P(Q(t)). \]
The measure of the input market distortion of raising the tax from \( t^* \) to \( t^\prime \) when measured in the output market is the sum of the changes in cost and consumer surplus and equals

\[
s^0 = \int_{t^*}^{t^\prime} \left( \frac{dc}{dt} + \tilde{C}(t) \right) dt.
\]

Since the output and factor markets are competitive, we have that

\[
Q'(t) = \sum_{i} \frac{dx_i(t)}{dt} = \left( \sum_{i} \frac{dx_i(t)}{dt} + t \frac{dx_i(t)}{dt} \right) \frac{1}{P(Q(t))}
\]

where \( \frac{dx_i(t)}{dt} \) is marginal physical of factor \( i \) and where we have used the relation that in competitive markets, the wage equals the value of the marginal physical product. Plugging \( Q'(t) \) into the expression for \( ds^0/dt \), and using the expressions for \( s^0 \) and \( \tilde{C} \), we obtain

\[
\frac{ds^0}{dt} = -\sum_{i} \frac{dx_i(t)}{dt} - t \frac{dx_i(t)}{dt} + \sum_{i} \frac{dx_i(t)}{dt} \frac{1}{P(Q(t))}
\]

or

\[
\frac{ds^0}{dt} = -t \frac{dx_i(t)}{dt},
\]

which is the same as \( ds^0/dt \) in (1). We therefore conclude that \( s^0 \) and \( s^1 \) yield identical measures of the distortion.

It is useful to illustrate diagramatically how one could calculate deadweight loss in an output market. Suppose that the tax on factor \( i \) is increased from \( 0 \) to some small \( t^\prime \). Let us first calculate what the relevant cost change \( (\tilde{C}) \) equals. By definition, see (2), \( (\tilde{C}) \) will approximately equal \( \tilde{C}(t) \) or the difference in total costs after taxes and rents have been excluded. Mathematically,

\[
(\tilde{C}) = (TC_{t^\prime}(Q_{t^\prime}) - RENT_{t^\prime}(Q_{t^\prime})) - (TC_{0}(Q_{0}) - RENT_{0}(Q_{0})),
\]

where \( Q_{t^\prime} = \) output when tax is \( 0 \)

\( Q_{t^\prime} = \) output when tax is \( t \)

\( TC_{0}(Q_{t^\prime}) = \) total cost of producing \( Q \) when tax is zero

\( TC_{t^\prime}(Q_{t^\prime}) = \) total cost of producing \( Q \) when tax is \( t \) (does not include tax payments)

\( RENT_{t^\prime}(Q_{t^\prime}) = \) rent earned when \( Q \) is produced and tax is \( 0 \)

\( RENT_{t^\prime}(Q_{t^\prime}) = \) rent earned when \( Q \) is produced and tax is \( t \).

Rewrite the above expression for \( (\tilde{C}) \) as

\[
(\tilde{C}) = (TC_{t^\prime}(Q_{t^\prime}) - RENT_{t^\prime}(Q_{t^\prime})) - (TC_{0}(Q_{0}) - RENT_{0}(Q_{0})).
\]

Let \( C_{t^\prime}(Q_{t^\prime}) \) be the supply (unit cost) curve when the tax is zero and \( C_{t^\prime}(Q_{t^\prime}) \) be the supply (unit cost) curve when the tax on factor \( i \) is \( t^\prime \) and when tax revenues have been subtracted from costs. \( C_{t^\prime}(Q_{t^\prime}) \) lies above \( C_{0}(Q_{0}) \) because the tax causes output to be produced with inefficient factor proportions. Notice that the first term in (3), \( (TC_{t^\prime}(Q_{t^\prime}) - RENT_{t^\prime}(Q_{t^\prime})) \), is given exactly by the area beneath the \( C_{t^\prime}(Q_{t^\prime}) \) curve between 0 and \( Q_{t^\prime} \).

Similarly the second term in (3), \( (TC_{0}(Q_{0}) - RENT_{0}(Q_{0})) \), is given by the area beneath the \( C_{0}(Q_{0}) \) curve between 0 and \( Q_{0} \). The difference between these first two terms is thus the area between the \( C_{t^\prime}(Q_{t^\prime}) \) and \( C_{0}(Q_{0}) \) curves from 0 to \( Q_{t^\prime} \). The last expression in (3) is nothing more than the incremental social cost of producing \( Q_{t^\prime} - Q_{0} \), which approximately equals \( (Q_{t^\prime} - Q_{0})C'(0) \).

Using the above analysis we can decompose \( (\tilde{C}) \) into the difference between two areas in Figure 1 below. The first is a "banana" shaped area representing the difference between the first two expressions in (3). The area of production "banana" is the increased cost caused by input distortions that arise from the tax on the factor. The second area is a "box" that represents the last term in (3). The box represents the decrease in cost that comes from a
reduction in output. By construction, the difference between the "banana" and the "box" is precisely what \( C'(q) \delta t \) measures for \( \delta t \rightarrow 0 \) in (2).

Figure 1

If we let \( \{ t_1, t_2, \ldots, t_n \} \) be a sequence whose sum converges to \( t \) and if we let \( C(q) \) be the supply (unit cost) curve when the tax on factor 1 is \( t \) and taxes are excluded from costs, then we can build up the area relevant for consideration of the cost side for a tax \( t \) on factor 1 that reduces output from \( Q_0 \) to \( Q_1 \). That area is given as the sum of all the infinitesimal "bananas" minus "boxes" as \( \int t_1 - t \). It equals \( \text{ABCD} \) minus \( \text{DEFG} \) in Figure 2. In Figure 2 the curve \( C(q) \) is the unit cost curve including tax revenues when the tax on input 1 is \( t \). The demand curve is denoted as \( D(p) \). The intersection of \( C^*(q) \) and \( D(p) \) at \( M \) determines the post tax quantity of output, \( Q_1 \). The area under the demand curve "lost" as a result of the tax is \( \text{HABC} \). The net loss equals \( \text{ABCD} \) minus \( \text{DEFG} = \text{ABCD} - \text{HABC} \) and is illustrated as the shaded area in Figure 2. The tax revenues are given by the area \( \text{HABC} \).

Since \( C^* \) was defined as the "relevant" area (i.e., a "banana" minus a "box") [see (2) and (3)], an alternative diagram to use in the benefit cost analysis is Figure 3 below. The deadweight loss is given as a "triangle" ABC. The reason this diagram is not as useful as Figure 2 is because \( C^* \) is a complicated function [see (2)] that must be derived from the information contained in the cost curves of Figure 2.

The rather complicated construction of loss in Figure 2 should be contrasted with the simple determination of loss in the directly affected factor market. From our previous proof we are guaranteed that the shaded area in Figure 2 equals the familiar triangle ABC of Figure 4, where S in Figure 4 and D refer to the supply and demand for the factor on which the tax is placed. In Figure 4 the tax revenues are given by the area of the rectangle ABCD.

Figure 4

When information about supply of all inputs, prices of all inputs, and demand for output are available, then there is no reason to prefer using Figure 4 or Figure 2. On practical grounds, Figure 4 may often be the preferred alternative since its calculation requires knowledge only of one supply and demand curve. Information on other factor markets or on the output market is not needed.

Is it ever the case that information on the affected input market is so poor that Figure 2 can be used but not Figure 4? The answer is yes and this result provides the sole justification for trying to measure input distortions (or benefits) in output markets. A simple example is the easiest way to illustrate this point.

Suppose that an undistorted output market is observed for a long time. Observable shifts in exogenous (e.g., income, weather) variables enable the analyst to identify the relevant pre-tax supply and demand curves. Data on certain input prices and quantities demanded are not available. A tax is imposed on one input and the policy remains in effect for several years. Observable shifts in exogenous variables enable the analyst to identify the post-tax supply curve. Information on taxes collected \( T(q) \) is available. If information on pre-tax factor prices and quantities is unavailable, then the distortion cannot be measured in the input market. However, using Figure 2, the distortion can readily be
measured in the output market. We let \( C(Q) \) be the pre-tax supply curve, \( C^e(Q) \) be the post-tax supply curve, and \( C_e(Q) \) be \( C^e(Q) - (TQ)/Q \). The shaded area in Figure 1 gives the correct measure of deadweight loss.

Measuring an input market distortion in an output market is more tedious than measuring the distortion in the directly affected input market. However, it may be possible to use the output market distortion measure in cases where data unavailability precludes the use of the input market distortion measure. In any practical application, one must keep in mind the usual caveats about using compensated demand curves and “reduced form general equilibrium” demand curves (Harberger 1964) and Diamond-McFadden (1974) and giving proper attention to distributional issues. Moreover, if inputs are used to produce several outputs then a tax on an input(s), when not measured in its own market(s), must be measured in all the affected output markets. The need to look at several markets when output distortion measures are used emphasizes the simplicity of using input distortion measures when data permit. However, when data are lacking, the more complicated output market distortion measures can be used to accurately reflect costs and benefits in input markets. This property is one that should make the technique of measuring input market distortions or benefits in the output market a powerful and useful one for cost benefit analysts.

II

Can the analyst ever use information on other input markets to evaluate an input market benefit or distortion? A closely related question is whether the analyst can ever use information on other output markets to evaluate an output market benefit or distortion. The answer to both questions is yes but only with special though plausible assumptions.

The intuition is that if the price of one input (output) is affected, the demand curve for other inputs (outputs) should also be affected.

If one has prior knowledge about production, then it is sometimes possible that information from only some input markets can provide complete information about all input markets. The issue is whether the prior plus observable information allows the derivation of the missing data. For example, suppose output \( Q \) is produced by a Cobb-Douglas technology using two factors labor, \( L \), and capital, \( K \), according to \( Q = f(L, K) = A L^a K^{1-a} \). Suppose that factor prices and \( L \) are observed and that \( a \) is known. Then, knowledge of \( L \) together with the ratio of factor prices is sufficient to construct the demand for \( K \). This demand relation can then be used to measure any benefits or distortions in the capital market.

In which cost benefit analysis can be done on unobservable markets. Another class of cases are those in which prior information fails short of identifying the whole production process, but still enables calculation of the desired magnitudes. Suppose that for simplicity we assume constant returns to scale, perfectly elastic factor supplies and a technological improvement that lowers the cost of input \( L \) from \( r_0 \) to \( r_1 \). How can we measure this benefit? First, we could measure it directly in the input market as a trapezoidal area between \( r_0 \) and \( r_1 \) and the derived (see, e.g., the second diagram in Fig. 6).

demand curve for input \( L \). Alternatively, from Section I we know that we could measure it in the output market by the area composed of a “banana” (the difference between the new and old marginal cost curve) plus a “triangle” representing the additional consumer surplus. Can we ever use information on other observable inputs to derive the relevant measure? The answer is yes—but only under a special though perfectly reasonable assumption. The special assumption is that as the price of the observable input rises the price of the final good must approach \( e^{-1} \). Under this assumption...
tion it is possible to show that the appropriate difference in areas bounded by the new and old input demand curve for any related input can be used to exactly measure the gain to society.

To prove this point suppose that input 1 and input 2 produce an output. Let the wages of inputs 1 and 2 be denoted by $r$ and $s$ respectively. Let $C(r, s)$ be the unit cost function for producing output. Let $Q(u, P_1, P_2)$ be the compensated demand curve for an individual at utility level $u$ facing prices $P_1$ for output 1 and $P_2$ for other outputs. Let $r$ fall from $r_0$ to $r_1$ because of a technological change. Let $s_0$ be the wage received by input 3. The direct measure of benefit is given in the input market by

$$
\int_{r_0}^{r_1} \int_{s_0}^{}\sum \left(\frac{\frac{\partial C}{\partial u}}{\frac{\partial C}{\partial r}}\right) du dr
$$

Using the reasoning from the previous section, the indirect measure of benefit measured in the output market is

$$
\int_{C(r_0, s_0)}^{C(r_1, s_0)} \int_{Q(u, P_1, P_2)}^{P_1} dP_1
$$

which immediately reduces to (4) once the change of the variable $r$ for $P_1$ is performed using the relation $P_1 = C(r, s_0)$.

Now let us look at the demand for input 1. Consider Figure 5 below. The initial derived demand for input 1 shifts in response to the shift in $r$. We want to prove that the shaded area is the relevant area for the measure of benefit and is identical to (4) or (5).

Figure 5

To prove this, notice that the shaded area equals

$$
\int_{r_0}^{r_1} \int_{s_0}^{}\sum \left(\frac{\frac{\partial C}{\partial u}}{\frac{\partial C}{\partial r}}\right) du dr
$$

Now using the relation $C(r, s) = P_1$ we can rewrite the above expression as

$$
\int_{C(r_0, s_0)}^{C(r_1, s_0)} \int_{Q(u, P_1, P_2)}^{P_1} dP_1
$$

since by assumption $C(r_1, s_1) = C(r_0, s_0)$. But (6) is identical to (5) and therefore to (4), hence the result is proved.

The diagram below illustrates the three equivalent measures of gain in the output market, directly affected input (input 1) market, and in a related input (input 2) market. As before, the ability to use related input market measures should be of use when data on directly affected markets are unavailable.

Figure 6

The preceding discussion has shown how prior information on observable input markets can reveal the desired information about an unobservable input market. We now give an example where prior information on observable output markets yields the desired information about an unobservable output market. The example thus shows how information about other output markets can be used to measure benefits in an unobservable output market.
Suppose that we wish to evaluate the benefit of a technological improvement that lowers the price of the $n^{th}$ commodity from $q_0$ to $q_1$. Assume that the price vector $p$ of commodities 1 through $n-1$ are unchanged. The analyst knows the (compensated) demand curve for commodities $1, \ldots, n-1$. Information on demand for commodity $n$ is unavailable. If $m(u, p, q)$ represents the expenditure required to achieve utility level $u$ at prices $p$, $q$, then we wish to calculate $m(u, p_0, q_0) - m(u, p_0, q_1)$ or \( \int_0^{q_1} \frac{2m}{3q} dq \), which is just the area under a compensated demand curve for commodity $n$ between prices $q_0$ and $q_1$ (the derivative of the expenditure function with respect to price yields the compensated demand curve). Since no information on the compensated demand for commodity $n$ is available, the calculation cannot be done.

Now let the analyst be given just a little bit of information about the demand for commodity $n$. Suppose he knows that at prices $p^*$ for commodities $1, \ldots, n-1$, the quantity demanded of commodity $n$ is zero at prices $q_0$ and $q_1$. For example, the demand for tennis rackets can be driven to zero by sufficiently lowering the price of substitutes (golf clubs, squash rackets, etc.) and sufficiently raising the price of complements (tennis balls, sneakers, tennis court fees). The implication of the analyst's information is that $m(u, p^*, q_0) = m(u, p^*, q_1)$ or equivalently that the price of commodity $n$ does not influence consumer expenditure when commodity $n$ is not purchased. Our analyst can use his little bit of information to perform the desired measurement. Using the definition of a line integral and the fact that $m(u, p^*, q_1) = m(u, p^*, q_0)$ we can write that

\[
\begin{align*}
&\left( m(u, p_0, q_0) - m(u, p_0, q_1) \right) \times \\
&\left( m(u, p^*, q_1) - m(u, p^*, q_0) \right) = \int_{p_0}^{p^*} \frac{2m}{3p} \left( m(u, p, q_0)dp - \frac{p^*}{p} m(u, p, q_1)dp. \right)
\end{align*}
\]

By assumption, the analyst knows the compensated demand curves $2m/3p$ for commodities $1, \ldots, n-1$. Therefore both integrals in (7) are calculable since they are just integrals under these compensated demand curves. Moreover, it follows that if the analyst has enough prior information about $p^*$ so that he can calculate (7) for any $q_0$, $q_1$, then the analyst can let $q_1 = q_0 + \epsilon$ for small \( \epsilon \) and thereby calculate $2m/3p$ which is precisely the compensated demand curve for commodity $n$.

III

Missing data hamper many practical applications of cost-benefit analysis. This paper has shown that even when data on directly affected markets is unavailable, it is often still possible to perform the correct calculations by using available data from related output or other input markets. Such "indirect" calculations of costs and benefits should be a powerful and useful tool in applied cost-benefit analysis.
REFERENCES


FOOTNOTES

1. I thank Arnold C. Harberger for encouraging me to write up these results and for perceptive comments. I also thank John Panzar, Robert Willig, the referee and George Borts for helpful comments.

2. It is also true that an input market benefit is completely reflected in the output market. This result follows from a proof almost identical to the one given below.

3. The method of proof is similar to that used by Schmalensee (1976).

4. Whenever cost-benefit analysis is done, compensated demand and supply curves must be used. The input demand curves are the demands for input that arise when the amount of output produced is given by a compensated demand. With a nonlinear production possibilities frontier, the relevant curves to use are compensated "general equilibrium" demand curves which incorporate price changes in other markets into the demand curve.

See Harberger (1964) and Peter Diamond and Daniel McFadden (1974).

5. For simplicity of notation, the cost of producing output $Q(t)$ is not written as $C(Q(t))$ but as $C(t)$.

6. This result makes use of the well known fact that the rents earned can be calculated as the area between the supply curve and the horizontal price line.

7. To a first approximation $\frac{d}{d\theta} IC(Q_e) = C_0(Q_0)T_e - Q_0$. 

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Here we draw \( \bar{c} \) as a function of output. The notation in Figure 1 is the same as in Figure 2.

This statement appears to conflict with Wisecarver's feeling that 'practical measurement of the substitution component in the output market is intractable' and that output measures 'do not fit into the framework of applied welfare economics.' Wisecarver is, however, correct to insist on the simpler input market measure when it is available.

The recent paper by John Passar and Robert Willey (1978) also appears to imply that only direct input market measures are correct. However, a careful reading of their paper shows that this implication would be a misrepresentation of their views. Passar and Willey argue correctly that consumer surplus in input and output markets will differ, but do not discuss how taking account of a 'banana' shaped area can enable use of output market measures. This paper is in agreement then with Passar and Willey. Without taking the 'banana' into account, it is fruitless to perform cost benefit analysis in the output market.

Input prices may of course change with the quantity of input demanded, but the (unobservable) supply curve of inputs must remain stable over time (or else the change must be parameterized in terms of observable variables).

All inputs are assumed to be used to produce only this one output. Otherwise, changes in factor price ratios will reverberate into other output markets. In such a case, each output market would have to be separately analyzed as in Figure 2, and the total deadweight loss would equal the sum of the deadweight losses across all affected output markets.

With no rents, this "banana" is a rectangle.

A Cobb-Douglas and a constant elasticity of substitution production function with a substitution elasticity below one satisfy this criterion.

We use Shepherd's Lemma that the demand curve for input 1 equals

\[
\frac{\Delta}{\Delta r} C(r, s) = \frac{\partial C}{\partial r}.
\]

Again we are making use of Shepherd's Lemma that the demand curve for input 2 is

\[
\frac{\Delta}{\Delta s} C(r, s) = \frac{\partial C}{\partial s}.
\]

If the values of \( C(r_0, s) \) and \( C(r_1, s) \) are finite and unequal but known, then we would have to subtract from (6) the expression

\[
\int_{r_1}^{r_0} \frac{\partial C}{\partial r}s_1\, dr_1.
\]

If we further assume that we know an \( s_0^* \) and \( s_1^* \) such that \( C(r_0, s_0^*) = C(r_1, s_1^*) \) and \( C(r_0, s_1^*) = C(r_1, s_0^*) \), then this last expression can be written as

\[
\int_{s_0}^{s_1} Q \frac{\partial C}{\partial s}(r, s),
\]

which is a calculable area under the demand curve for input 2.

The case when factors earn rents is a straightforward generalization of the proofs of Section 1. When factors do earn rents, an interesting question is how the rent change in a factor is related to the benefit measure to society. For example, a frequent issue in urban economics is how the change in land rents reflects a transport improvement. Because of space limitations, it is left to the reader to prove that in general when land is in fixed supply the benefit of a transport cost change will be totally reflected in land rents, while if land and transport are used in fixed proportions at each distance from the city. (This proof is available on request from the author.)
This same idea is discussed in detail by Karl-Göran Miller (1974), David Bradford and Gregory Hildebrand (1977), and A. Myrick Freeman (1977) in the context of identifying demands for public goods.