Lecture 1

Measurement of Triangles

Consumer surplus and triangle analysis are useful tools in applying theory to the problems in public finance.

A brief defense of the three basic postulates:

1) Demand price measures the value of the marginal unit of something to the consumer.
2) Supply price measures the value of the marginal unit of something to the supplier.
3) Adding up—we can add up benefits and costs algebraically among members of the group in question.

**Example:** Tax Imposition

Demands forego $x_0$ whose value is area $A_1E0$ which is the gross cost because those units stop being consumed and produced. The benefit is the value of the fixed resources—$CH0A$. Therefore the net cost (the efficiency cost) is $A_1CH0A$.

*Note to remember is that the demand and supply curves measure indifference between $A$ and other alternatives.

When the tax is small, people divert less demand when the tax gets larger, people divert more demand. The demand price reflects the critical point of indifference. The same argument can be made for the supply price of suppliers. So don't really care about indifference vis-a-vis what—motives for taxes are not analyzed here.

**Example:** Analysis of a subsidy

The benefit is the additional demand that is satisfied—$A_1E0$.

The cost is the value of the incrementally used resources—$D_0CH_0A_1$. Therefore the net cost is the triangle $ABC$.

If the curves are linear, a $10\%$ tax or a $10\%$ subsidy will have exactly the same efficiency cost.

Now welfare cost varies with the elasticity of the demand curve:

Assume infinitely elastic supply curves. The more elastic is the demand, the greater the welfare cost of a given tax. Incomes Resources are reluctant to be misallocated when demand is not elastic. If demand gets more elastic resources become scarcer to be misallocated.

Point A is the point at which the tax is prohibitive. $ABC$ is as big as the triangle can get. If demand is still more elastic, the triangle gets smaller because people switch out very easily due to the extreme prohibitiveness of the tax. We recognize all prohibitive taxes to be equivalent, by convention. For purposes of comparison, we use the tax which barely arrives at prohibiting the good as the representative for all prohibitive taxes—this way we can still say the more elastic the demand, the bigger the welfare cost.

Now the welfare cost varies with the size of the tax.
The welfare cost of a tax is the area of the triangle.

\[ T = \frac{1}{2} \text{ base} \times \text{ height} \]

The base is now larger and the height is the same.

**Demand:** \( x = a + c \)

**Supply:** \( x = g + f \)

**Tax:** \( T = P - x = a - \frac{x}{c} \)

\[ P^* = \frac{a}{b} \quad \text{and} \quad P^* = \frac{a}{c} \]

\[ T = T (1/b - 1/c) = \frac{a}{c} (1 - b/c) \]

The area of the triangle is then \( -\frac{1}{2} abx = -\frac{1}{2} ab \cdot \frac{a}{c} \cdot \frac{a}{b} \)

Since the elasticity of supply is \( \epsilon \) and the area of the triangle becomes

\[ \frac{1}{2} ab \cdot \left( \frac{a}{c} \cdot \frac{a}{b} \right) \]

\[ = \frac{1}{2} \left( \frac{a}{c} \cdot \frac{a}{b} \right) \frac{a}{c} \cdot \frac{a}{b} \]

\[ = \frac{-1}{2} \left( \frac{ab}{c} \right) \cdot \frac{a}{c} \cdot \frac{a}{b} \]

And when the tax is evaluated at \( P^* \), the welfare cost becomes

\[ \frac{-1}{2} \left( \frac{ab}{c} \right) \cdot \frac{a}{c} \cdot \frac{a}{b} \]

(Here we interpret prices as being initial prices. Everything is evaluated at \( P^* \). Otherwise one can use another price and define "a" accordingly.)

**Examples:**

- **Supply is not perfectly elastic:**

  The more elastic the supply, the greater the welfare cost.

  \[ \text{Area} = \frac{1}{2} \text{ base} \times \text{height} \]

  The base is now larger and the height is the same.

  **Demand:** \( x = a + c \)

  **Supply:** \( x = g + f \)

  **Tax:** \( T = P - x = a - \frac{x}{c} \)

  \[ P^* = \frac{a}{b} \quad \text{and} \quad P^* = \frac{a}{c} \]

  \[ P^* = \frac{a}{c} \quad \text{and} \quad P^* = \frac{a}{c} \]

  \[ T = T (1/b - 1/c) = \frac{a}{c} (1 - b/c) \]

  \[ \Delta x = \frac{a}{c} (1 - b/c) \]

  The area of the triangle doesn't increase as the elasticity increases since \( \Delta x = \frac{a}{c} \) and \( \Delta x = \frac{a}{c} \) is what will be dependent on the elasticity.
Example: Welfare cost increases with the square of the restriction

\[ \text{The area of the triangle is } -\frac{1}{2} \Delta q \frac{dF}{dq} \]

As the elasticity increases, \( A \) \( P \) declines. The tax which corresponds (in terms of revenue) to the same quota will have to be smaller.

If the elasticity of demand is high, people can easily get along without this good. (They replace it very quickly at almost no cost) So, if we take it away, they don't care very much. There is not much welfare loss. With a tax, the opposite happens if they can replace it easily, they'll react very strongly misallocating resources. If people can't do without it (elasticity of demand is low), they will not move out and just pay the tax out of their consumers' surplus.

With a quota, no matter how much they are willing to pay, they can't get it and the welfare loss becomes great.

**The Monopoly Tax**

Movement from a competitive equilibrium to a monopoly equilibrium is the same as the movement brought about by a tax. This tax is privately imposed and collected.

Example:

**The Monopoly Case:** Same type of analysis

**From the monopolist's view:** Analyzing the tax looking from the price rather than quantity axis

Consumers lose $\Delta P \times \Delta Q$ by paying more; producers' lost surplus is $\Delta P \times q$. The monopoly gets $\Delta P \times q$.

The consumers are losing $\Delta P \times \Delta Q$ but get back in terms of resources to spend elsewhere $\Delta P \times q$ on their net loss in $\Delta P \times \Delta Q$.

The monopolist is gaining back $\Delta P \times q$ which is equivalent to $\Delta P \times \Delta Q$ minus $\Delta P \times q$. APMG is associated with the exercise of monopoly power. MCG is associated with monopoly power.

Therefore the net loss to society remains the triangle $\Delta P \times \Delta Q$. 
Lecture 2

Agricultural price support

Objectives: to keep prices received by producers higher than equilibrium price

What cost does that entail?

The government is accumulating stocks. It can do something with the stocks. Either it can sell them at what the market will bear...

If we give farmers marketing coupons (assuming an arbitrary and free allocation) and permit that they be bought and sold: the coupon price will be...

If we have three classes of farms:

Farm 1 was allocated too few coupons

Farm 2 was allocated too many coupons

Farm 3 had just the right set of coupons

If the farmer is at A because he has not enough coupons, he pays for the coupons in order to bring him to critical level.

If the farmer is at B, he sells his coupons because he would have more coupons than necessary to reach the critical level.

In the U.S., they allocated acreage instead of coupons. Restricting acreage creates problems because acreage is not the only input in agriculture. It will pay for the farmers to use the other inputs beyond the socially optimal efficiency level in order to sell more output at P0 than at P1 to the government.

But now the government has to restrict acreage further to keep output at B, until finally the marginal cost of a unit of other factors reaches P0. This imposes an additional cost to society.
Rationing Situations

Many of the additional costs due to particular schemes can be avoided by allowing further transactions. If it is decided that everybody would get the same rationing and it is a capital offense for transactions of coupons, this creates a welfare cost.

\[ P_1 \] is the price at which excess demand equals excess supply. The price of the ration coupon will be the excess between the rationed price \( P_2 \) and the market clearing price \( P_1 \).

A more interesting problem is being partly subsidized. Each family will be allocated a coupon for one gallon per child which will allow them to buy one gallon at the cheap price. The government thus creates a cheap and an expensive price side by side.

There are three kinds of families:

- Family A
  - The ration is less than they would want to buy
  - They start receiving a subsidy

- Family B
  - The ration is more than they would want to buy
  - They do not receive a subsidy

- Family C
  - The ration is more than they want
  - They do not receive a subsidy

\[ \begin{align*}
1.00 & \quad \text{social cost} \\
0.50 & \quad \text{production cost}
\end{align*} \]

The aggregate costs:
- Old demand curve
- New demand curve

We plotted a new aggregate demand curve following only the relevant segments reflected in the market. (Dark black lines in the previous diagram). There are two costs present now: a) the normal cost of a subsidy and b) the inefficiency cost incurred by giving some people more than what they would buy otherwise at the expense of people who would buy more.

\[ \begin{align*}
\text{social cost} & \quad \text{production cost}
\end{align*} \]

consumption cost which goes to families B and C coming from new incremental production

\[ \begin{align*}
\text{social cost} & \quad \text{production cost}
\end{align*} \]

at the expense of A.

For family A, the rationing is an income transfer, an inframarginal subsidy—there is no social cost.

For family B, they perceive the subsidy but not the rationing—there is a positive social cost.

For family B, they start perceiving a subsidy and the social cost is positive.

The new demand curves fall short of the old. The inefficiency and social cost in this example comes from the normal subsidy cost plus the inefficiency of giving some people more than what they would otherwise buy at the expense of people who would buy more—This is due to the inability to transact with coupons.
Lecture 3

Problems connected with international trade

What is the welfare cost of the tariff?

Production goes up from \( P_o \) to \( P'_o \).
Consumption is reduced by \( P'_d \).
The red rectangle represents the transfer from importers to the government.

\[ P_o \quad (\text{production}) \]
\[ P'_d \quad (\text{consumption}) \]

\( M^e \) are the initial imports. With the tariff, imports are reduced to \( M'^e \).
A is the area equal to \( P'_o \times \frac{1}{2} (P'_d - P'_o) \).
B is the area equal to \( P'_o \times \frac{1}{2} (P'_d - P'_o) \).

\( M'/M^c \) has an area equal to \( A + B \).

The triangles of net cost are \( g \) (producer cost) + \( p \) (consumer cost) = \( \frac{1}{2} (P'_d - P'_o) \).

This last example assumed the world price was given.

Now if the supply facing country \( A \) is rising:

\[ P'_o \quad (\text{tariff}) \]
\[ P'_o \quad (\text{tariff}) \]

when \( A \) faces a tariff, the world price falls. The area under

the excess supply curve has no meaning for country \( A \). The country
which imposes the tariff has lost the value of the product formerly
imported therefore the cost is measured by the blue trapezoid.
To measure the resource saving under the marginal cost curve becomes
the saving in greater than the area under the excess supply curve.
They need to pay \( P'_d \) for \( M'^e \), now they pay \( P'_o \) for \( M' \). The saving may
be greater than the cost, i.e., the red hatched area might be

\[ \text{The optimal tariff} \]

When would a small increase in tariff cause an increment to benefit
equal to an increment to cost?

Measurement of increment to cost
Measurement of increment to benefit

The optimal tariff is where the increment to saving equals the increment
to cost. Examples: Point A.

\[ M' (1 + \frac{1}{\epsilon}) \times P'_o \]
\[ \text{marginal cost = demand price} \]
\[ \frac{1}{\epsilon} \]

The optimal tariff is exploitation of monopoly position—the single
justification for a tariff as a first best solution from the standpoint
of the country. This is so only when the country has an upward
rising supply curve when its demand can affect the supply price.

The optimal export tax

\[ A \quad (\text{demand}) \]
\[ P_o \quad (\text{export}) \]

\[ P_o \quad (\text{demand}) \]

\[ \text{tax} \]

\[ \text{export} \]

\[ \text{local} \]

\[ \text{world} \]
The condition for the optimal export tax is when the gain and net loss are equal. This will be satisfied when \( ME = MC \).

\[
P^w (1 + \tau) = P^e (1 - \tau) \]

\( \tau_e = \sqrt{\frac{ME}{MC}} \) (the tax is defined as a percentage of gross price).

The tax comes into the picture when the market organization is competitive or else it won't do any good. It would lead to a monopolistic position if there were a monopoly on which would react to the tax by restricting production by more.

When the export good is a major item in foreign trade of a country A (for example coffee in Brazil), we look at the benefits and costs from the point of view of the country which puts on the tax. In the process of putting on the tax, the foreign exchange price rises and the demand curve is expressed in terms of the new exchange rate.

The tax imposition reduces the supply of foreign exchange and hids up the price of the exported good. The government is getting an extra revenue from the change in the exchange rate— the top rectangle whose area is \( \frac{1}{2} \times H \). \( H \) is the amount of the devaluation due to the imposition of the tax. \( \frac{1}{2} = (1, 1) \).

\[
\text{revaluation of net loss to foreigners: } H \]

\[
\text{uncompensated part of loss of traders: } \frac{1}{2} \]

\[
\text{compensated loss: } \frac{1}{2} \text{ resource saving} \]

In the initial equilibrium, area \( afgd \) is equal to what the country gets from foreigners in local currency. Area \( hfgd \) is the increase in the cost of the exported good due to the rise in the price of the imported good. Area \( adg f \) is the gain in the cost of the imported good due to the rise in the cost of the imported good. Area \( adgb \) is the loss which is unbalanced part of the loss of other traders. Area \( q Resources is what the government gains—the compensated loss.
Lecture 4

The optimal export tax (cont.)

In this case the optimum position is the point at which the modified marginal revenue equals marginal cost—such as point A.

If we have an exchange rate effect, the optimum change in quantity is 

\[ \frac{\partial q}{\partial s} \] 

there is less total restriction on the quantity than if we did not have exchange rate effects.

The optimum position is thus the point where the gain due to released resources—(the trapezoid outlined in blue)—balances off the cost due to marginal revenue forgone—(the block area)—plus the excess burden we impose on the importing group—(the red area).

Distributional weights in cost-benefit analysis

As mentioned earlier, there are two ways of analyzing a subsidy:

1. From the standpoint of the government, the costs are represented by the large trapezoid and the benefits by the small trapezoid. The triangle is the net loss.

2. From the standpoint of the price axis.

The government loses the whole amount of the subsidy, 2 is the producers' gain and 2 is the demanders' gain. The net loss is the triangle.

That duality no longer applies when we use weights.

By convention, we assign the following weights:

-17-

The subsidy doesn't help the egg farmers. The policy to follow is to subsidize the farmers with the most inelastic supply—"more bang for the buck" on \( P \).

If either \( F \) or \( F \) or both are less than one then a subsidy will work

If either \( F \) or \( F \) or both are greater than one then a tax will work optimally.

Optimization:

The optimal tax is where the marginal gain assigned to the transfer just barely counterbalances the efficiency cost.

This leads to high prices being paid to transfers.

In the arena of project evaluation: Consider a sequence of projects whose costs and benefits are independent flows.

If the beneficiary has \( F = 2 \), then applying distributional weights all benefits must be raised by a factor of two. How even project 25 will be approved.

However, suppose unskilled labor supply is infinitely elastic. Even with distributional weights the project would not be accepted. (Fig. 1)

Where the project pays the market wage and hides it up, the real benefit accrues mostly to people in the labor market who are not on the project's wage bill. (Fig. 1)
When we consider projects we deduct from costs the surplus given to workers and when we have higher weights than it is the surplus which is multiplied. The social opportunity cost is now lower and you could accept the project.

Suppose the project pays $1.50 for wages. Suppose also that workers have a weight of two.

If project wage is $1.50, the surplus is $1.50 which is multiplied by the dist. weight of 2 to get a social cost of 3.50. This will allow project four to be undertaken.

Our evaluation ends up depending on purely financial considerations and on the distribuitonal weights of those we are taxing.

We are always counterbalancing a distributinal gain with an efficiency loss.

**Income Taxe**

Does this generate a progressive income tax? But really, if labor supply is completely inelastic we are dealing with pure rents—in efficiency rents. We end up with confiscatory taxation.

If there are administrative costs, there would be a small bend around $y - \lambda$ to $y + \lambda$.

Suppose we have a set of noncompeting groups $\phi_1 < 1$.

If $\phi_1 = \phi_2 = \phi_3$ the tax will be progressive. $\phi_1 < \phi_2 < \phi_3$ the real income tax is a single schedule.

A higher tax on the poor raises tax for all other groups. There is an externality effect: A distributinal externality from changing the tax at the low end of the scale.
Lecture 5

Difference between raising a proportional tax on non-competing versus a tax affecting that group within a context of global income tax structures:

Assume the weight of demands in l-neutrality:

\[
\text{Loss to the government} = \text{Gain} (1 - \beta) a \lambda L
\]

Optimum tax = \( \lambda = \frac{1}{1 + \beta} \frac{(1 - \beta)}{(1 - \beta) a L}
\)

The social value of the tax will depend on which group it will affect

\[
\eta (1 - \beta) a \lambda L
\]

The total effect of the tax is the prior effect on \( L_1 \) plus later effects (externalities):

\[
\eta (1 - \beta) a \lambda L + \frac{\eta (1 - \beta) a \lambda L}{1 + (1 - \beta) a L}
\]

\( L_2 \) does not change because we are dealing with the inframarginal groups.

Dividing both sides by \( \eta a L \):

\[
\frac{\eta a L}{\eta a L} \left( \frac{1 - \beta)}{1 + (1 - \beta) a L} \right) \left( \frac{1}{1 - \beta) a \lambda L} \left( \frac{1}{1 - \beta) a \lambda L} \right)
\]

If \( (1 - \beta) a \lambda L \approx 1 \) Group 1, has \( \beta = \frac{1}{2} \)

Quantifying the externality: assume \( a \lambda L = 2 \)

\[
\eta a L = \frac{1}{1 + \frac{1}{2}} \lambda = \frac{1}{1 + \frac{1}{2}} \lambda = \frac{1}{3} \lambda
\]

We think of an income tax affecting only the upper tail of the distribution where we have a Pareto law applying. See very simple kind of representation of this sort of distribution would be an exponential or geometric progression.

---

<table>
<thead>
<tr>
<th>Group</th>
<th>( L_2 / L_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>0.24</td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
</tr>
</tbody>
</table>

For group 1 the tax would be \( \beta = 1 + \frac{1}{2} (0.24) = 0.50 \%

For group 2 the tax would be \( \beta = 1 + \frac{1}{3} \frac{1}{2} (0.24) = 0.33 \%

It becomes a regressive tax.

This shows how distributional weights lead to nonsense because we are applying the tax to where marginal efficiency cost is equal to the external benefit. It is "like throwing away the baby with the bath water".

If you want to use distributional weights, you must put a limit to the price you are willing to pay for any implicit transfer. For example the Heavens function: He suggested, in Pahlavi 1973, that countries with same rate of growth in income should get the same weight. If the recipient group has a \( \beta \) of 10 times that of another group, you can transfer up to the point where \( \beta \) of the transfer is less in the transfer and still justify your transfer. "It's like carrying ice cream across the desert until 90% melts."

The difference in marginal weights is the point to which we push efficiency losses in counterbalance the marginal distributional gains.

The problem with the Heavens function is that the factors can go to infinity—it invites incredible marginal inefficiencies in generating transfers. It is a member of a general class of functions:

\[
\log \beta = c - \log \frac{L}{L_1}
\]

An alternative: Draw a poverty line and give extra weight to transfers which would benefit people below this line. The distribution function should be of the sorts:

\[
\text{poverty level, } \% \text{ inefficiency cost we are willing to bear}
\]

Economists should not get involved in political opinions but should be professional econometricians and stick to the conventions of the three postulates—is \( \beta \) opinion.
Lecture 6
Demand and Supply curves

What we are working with are substitution effects—compensated demand curves. However it does not have to be so; we can modify the analysis to include income effects.

So private sector is constrained by the expenditure lemma.

If $x_1$ and $x_2$ are the only relevant goods—an equal rate excise tax on both goods would look like the expenditure lemma. But, an excise tax on $x_1$ will have the property of generating a change in the slope of the constraint. One can use both and raise the $y$ intercept by income subsidies and change slopes by excise tax on one good.

Assume you start at an equilibrium like point $A$—$E_1$. $E_2$. The constraint. The true compensating variation in income is bounded by using the two measures. At point $B$ we give people income to enable them to get the old bundle of goods at the new prices. It cuts through the first budget constraint and the consumer is on a higher indifference curve. This is an overstatement.

$E_1$ $E_2$ overstates the total compensation in income from going from $A$ to $B$.

If we consider $E_3$, $E_4$, this measure results in an understatement of the true compensation $p_1 E_3 E_4$.

So we take the average:

$$\frac{E_1 E_3 + E_2 E_4}{2}$$

If we were to put an equal percentage tax on $x_2$. Then you would go back to $A$ from $B$.

The effect of a tax on good $2$ in the presence of an already existing tax on good $1$.

There is no welfare commutative to a shifting demand curve for $x_2$.

You put a tax on $x_1$—welfare changes: $\frac{1}{2} \Delta p_1 < 0$

You put a tax on $x_2$—welfare changes: $\frac{1}{2} \Delta p_2 > 0$, plus $1 \Delta x_1 > 0$

Sum of the welfare changes is 0.

There is a welfare commutative to shifting the demand curve for $x_1$ (because it already has a tax). Going back to $A$ from $B$ involves a subsidy or giving back money to the people.

In a three commodity world:

- The own effects: $- \Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
- The own effects: $\Delta x_1$ $\Delta x_2$ $\Delta x_3$
So, \( S_{i} = S_{i} + S_{i'} = S_{i} + S_{i'} = \frac{\partial V}{\partial x_{i}} = \frac{\partial V}{\partial x_{i}} \).

Can also be written as, \( \frac{\partial V}{\partial x_{i}} = \frac{\partial V}{\partial x_{i}} \) or \( \frac{\partial V}{\partial x_{i}} \).

Generally: 
\[
L = V(x_{1}, x_{2}, x_{3}) - \lambda P_{1} x_{1} - \lambda P_{2} x_{2} - \lambda P_{3} x_{3} - \lambda x_{4}
\]

First order conditions from differentiating the Lagrangian:

- \( \frac{\partial L}{\partial x_{1}} = \lambda P_{1} = 0 \)
- \( \frac{\partial L}{\partial x_{2}} = \lambda P_{2} = 0 \)
- \( \frac{\partial L}{\partial x_{3}} = \lambda P_{3} = 0 \)
- \( \frac{\partial L}{\partial \lambda} = -x_{4} = 0 \)

Restrictions:

- \( S_{i} = S_{i} \) or \( S_{i} = S_{i} \)

Rica- Stolichny substitution terms:

- \( x_{1} = x_{1} \)
- \( x_{2} = x_{2} \)
- \( x_{3} = x_{3} \)
- \( x_{4} = x_{4} \)

So the change in welfare is

\[
\frac{\partial V}{\partial x_{i}} = \frac{\partial V}{\partial x_{i}} \]

Dividing by \( \lambda \) converts utility into money by subjective judgment on the marginal utility of income.

This is the formula for the generalized triangle which when approximated by a Taylor's series can be rewritten as:

\[
\frac{\partial V}{\partial x_{i}} = \sum (z_{i}) \frac{\partial V}{\partial x_{i}}
\]

where \( z \) is the distortion.

A change in \( I \) will only have a cost or a benefit if \( x^{*} \) is a distorted activity.

**Lecture 7**

**Derivatives of the welfare cost formula**

If you have a tax on all goods, then the welfare cost \( C \):

\[
W = \frac{1}{2} \sum_{i} x_{i}^{2}
\]

The own effects are negative, cross effects are positive but own effects dominate—proofs semifiniteness of bordered hessian.

\[
L = \sum_{i} \frac{1}{2} S_{i} x_{i}^{2}
\]

From \( \sum_{i} \frac{1}{2} S_{i} x_{i}^{2} \), we derive \( \sum_{i} \frac{1}{2} S_{i} x_{i}^{2} = \frac{1}{2} \sum_{i} x_{i}^{2} \).

Using the adding up property: \( \sum_{i} x_{i}^{2} = \frac{1}{2} \sum_{i} x_{i}^{2} \).

The number of independent substitution terms is \( n(n-1) \) (the number of off-diagonal elements divided by two).

If \( t_{1} = t_{2} \), then the welfare cost is 0.

Implications: Only the relevant distortions that count. Equivalent subsidies and taxes have the same effect.

These three ways of measuring welfare cost are equivalent but may be more convenient depending on how the question is posed.

Applications: A tax on good 2 in the presence of a tax on good 1.

Assume a different percentage tax (if they are equal there is no net effect).

The effect on welfare compared to optimum.
Lecture 8

Conditions under which an income tax will be better than an excise tax—

The income tax is a suboptimal way of raising revenue, a second best solution. The income tax does not tax leisure.

The excise tax on goods 1 and 2 is equivalent to a subsidy on leisure and a subsidy on goods tax 2 is equivalent to a tax on leisure, given linear demand and supply curves.

Welfare cost of an excise tax on good 1: $W_{e1} = -\frac{1}{2} \left( \alpha_1 T_1^2 \right)

Welfare cost of an excise tax on good 2: $W_{e2} = -\frac{1}{2} \left( \alpha_2 T_2^2 \right)

If $T_1^2 = \alpha_1 T_1^2$, then $T_2^2 = \alpha_2 T_2^2$, becomes initial prices $= 1$

Then $W_{e2} = \frac{1}{2} \beta_2 \alpha_2 T_2^2$

$x_1, x_2$ (in $(x_1, x_2)$)

$x_1 = -\frac{x_2}{1 - \beta_1}$

$x_2 = -\frac{x_1}{1 - \beta_2}$

$s_1, s_2$ = $\frac{1}{2}$ where $s_1$ is the fraction of good one in National Income

$W_{e1} = W_{e2} = -\frac{1}{2} \left( \frac{1}{s_1} \frac{\beta_1}{\beta_1 + \beta_2} \right)$

$\frac{\beta_1}{\beta_1 + \beta_2} = \varepsilon$ The elasticity of the supply of labor

$x_2 = s_2 \frac{\beta_1}{\beta_1 + \beta_2} X_2$ is the slope of leisure demand function or the labor supply function

$\frac{1}{\beta_1 + \beta_2} = \varepsilon$ $s_2 = 1 - \frac{\beta_1}{\beta_1 + \beta_2} \frac{\beta_1}{\beta_1 + \beta_2} - \frac{1}{\beta_1 + \beta_2}$

Therefore, $W_{e1} = W_{e2} = -\frac{1}{2} \left( \alpha_1 T_1^2 \right) (1 - \alpha_2 \frac{\beta_1}{\beta_1 + \beta_2})$

If labor were completely immobile, $\alpha_1^2 = \varepsilon$, would exactly measure the WC of the excise tax and the WC of the income tax = 0.

Examples: Tobago

$\sqrt{\alpha_1} = 0.08$ $\alpha_1 = 0.08$

$W_{e1} = W_{e2} = \text{Triangle} \left( 1 - \frac{0.27}{0.3} \right) = \text{Triangle} \left( 0.99 \right)$

Value added tax

Consider $\int_{1}^{0.9} I_1^2 dI$ $\alpha_1 = 0.8$
For a aggregate goods $\theta_1 \theta_1 + \theta_2 \theta_2 - \theta_1 \theta_2 - \theta_1 \theta_2$

So WC \_ WC \_ Triangle \_ (- \_ 6)

Triangular analysis does not do any good anymore. What lies behind this is that good one is a complement to leisure, or less substitutable for leisure than good 2 so the elastic tax is better than an income tax. On $\theta_1 \theta_1 + \theta_2 \theta_2$

The optimum tax on goods 1 and 2: Given that a certain revenue is raised.

Min WC = - $\lambda$ (s_1 s_1 + s_2 s_2) - $\lambda$ (s_1 s_1 + s_2 s_2) = 0

$\lambda$ = $\frac{s_1 s_1 + s_2 s_2}{s_1 s_1 + s_2 s_2}$

$\lambda$ = $\frac{s_1 s_1 + s_2 s_2}{s_1 s_1 + s_2 s_2}$

If labor were a complement, then we would have a gain:

Keeping revenue constant, as we increase the tax on $x_2$, we decrease the income tax. We start with an income tax $\theta_2$, then we reduce the rate of the income tax assuming both $x_1$ and $x_2$ are substitutes for leisure. Now we put a tax on good 1. This increases the cost because good 1 is less than average substitute for leisure. $\theta_1 \theta_1$.
Lecture 9

Labor-leisure and saving decision

Complements to leisure: If people react the same way when income decreases by 25% due to a tax or when income decreases by 25% due to a reduction in working hours then

\[ \frac{1}{1 + \frac{1}{17}} \]

then the best candidate for complements to leisure are inferior goods. And they should be taxed more heavily.

With enough assumptions, you can show that an income tax is better than an excise tax. We introduce complications when we introduce leisure. We also introduce more complications when we introduce the savings decision.

Income tax doubles the taxation of savings—income is taxed before it is saved and the income gotten from the savings is also taxed. In the presence of the tax, the MFF becomes different from the MFF among present and future consumption.

\[ \rho = \frac{c}{p} \]

\[ \eta = \text{after-tax rate of return on saving} \]

\[ \frac{1}{\eta} \]

The welfare cost is \( \frac{1}{\eta} \).

When we have a world with a labor-leisure choice (a life cycle model with a retirement period) \( C_2 \), \( C_3 \), and \( L \). In a way the model allocates 8760 hours per year in period 0. Consider the neutral tax

\[ T = T_1 = T_2 \]

and \( T_1 = T \).

It creates a distortion between \( C_2 \) and \( L \) equal to that between \( C_2 \) and \( L \) but no distortion between \( C_1 \) and \( C_2 \).

Treating both \( C_2 \) and \( C_3 \) as substitutes for leisure, the income tax is worse than the consumption tax on two grounds:

- It introduces a distortion between \( C_2 \) and \( L \) greater than before.
- To the extent that \( L \) is a substitute for \( C_2 \) the expansion of \( L \) causes an excess of social cost over social benefit.

When \( C_2 \) and \( L \) are substitutes the consumption tax is preferable.

The income tax yielding the same revenue as a consumption tax will be at a lower rate, including, as it does, savings.

(This lecture was taken from someone else's notes since I was absent.)
Lecture 10
Measurement of Wages

In the Three Postulates, \((Y - Y') = f(X)\) is the distortion due to activity \(A\). It can be put in the form of a line integral:

\[
\int_{x_1}^{x_2} f(x) \, dx = \Phi \\Delta X
\]

Up to now we have been assuming constant costs. When costs are not constant, the \(X_i\) are altered and we use \(X_i^*\). We want a reaction coefficient which shows how the level of activity \(A\) changes with the tax on \(X_i^*\).

When solving demand and supply equations for any commodity in order to get tax effects, you get a jumble. The formula of the Three Postulates simplifies it all. Using

\[
\int_{x_1}^{x_2} f(x) \, dx = \Phi \\Delta X
\]

As we have the linear approximation (linear reaction model). For small changes, the world is quadratic (in the neighborhood of an equilibrium position).

Quadrate approximations are useful and practical to approximate changes. Below, the curves are drawn with respect to \(x_i\), we put on a graph. Then \(x_i\) is changed. Here we are assuming that the demand for \(X_i\) does not change with the tax on \(X_i\) and the supply of \(X_i\) does not change due to the tax on \(X_i\);

All aside, this stuff sort of flows out, a graphical representation of the Three Postulates leading to the same quadratic form as before. The thing that is sort of neat about this kind of formulation is that the elasticity-dynamics properties of substitution terms can be extended to these \(x_i\) as well as to the \(X_i\).

Considering the adding-up property:

An equiproportional tax on all goods (including leisure) will not alter the equilibrium quantities. It is like giving money back to the people through the back door somehow.

\[
\Delta X_j = \frac{\partial f}{\partial X_j} \Delta X_j = \frac{\partial f}{\partial X_j} \Delta X_j = 0 \quad \text{when} \quad X_j = 0 \quad \text{for all} \quad j
\]

This will work like a neutral tax if we exclude the problems of depression, different economic levels, and different amounts of capital inputs.

\[
\frac{\partial f}{\partial X_j} \Delta X_j = 0 \quad \text{so} \quad \Delta X_j (\frac{\partial f}{\partial X_j} \Delta X_j) = 0
\]

which becomes \(\frac{\partial f}{\partial X_j} = 0\) since it is.

Proving symmetry:
Consider the expressions for welfare cost when \(T_1\) is imposed first and when \(T_2\) is imposed first:

\[\begin{align*}
T_1 \text{ first action:} & = \delta \bar{X}_1 \bar{T}_1^2 \\
T_2 \text{ second action:} & = \delta \bar{X}_2 \bar{T}_2^2 + \bar{X}_1 \bar{T}_1 T_2
\end{align*}\]

and

\[\begin{align*}
T_2 \text{ first action:} & = \delta \bar{X}_2 \bar{T}_2^2 \\
T_1 \text{ second action:} & = \delta \bar{X}_1 \bar{T}_1^2 + \bar{X}_2 \bar{T}_2 T_1
\end{align*}\]

The two are equivalent if \(\delta \bar{X}_1 = \bar{X}_2\)

The general quadratic form for the change in welfare is...

\[
\frac{1}{2} \delta \bar{X}_1 \bar{T}_1 \bar{T}_2 \leq 0
\]

It follows that \(\delta \bar{X}_1 \bar{T}_1 \bar{T}_2 \leq 0\) if we only have one tax and \(\delta \bar{X}_1 \bar{T}_1 \leq 0\)

These reaction coefficients have all the properties which follow from the quadratic form—negative semidefiniteness; each principal minor is 0.
We should go a step further than the A's because sometimes the distortions are in the factor markets. We set up a framework capable of handling which is just an extension of the same thing we have done and supply for the factor and distortions create discrepancy among supply and demand prices.

We have: $A_i = \text{tax on capital in activity } i$
$B_i = \text{tax on labor in activity } j$

We define four reaction coefficients:

$$
\begin{align*}
G_{ij} &= \frac{A_i}{A_j} = \frac{\partial x_i}{\partial x_j} \\
H_{ij} &= \frac{A_j}{A_i} = \frac{\partial x_j}{\partial x_i} \\
L_{ij} &= A_i / x_j \\
N_{ij} &= A_j / x_i
\end{align*}
$$

Testing for symmetry:

When the tax on capital comes first you have a welfare cost of...

$$
T_A = \frac{1}{2} \frac{G_{ij} L_{ij}}{H_{ij} N_{ij}}
$$

When the tax on labor comes first:

$$
T_B = \frac{1}{2} \frac{H_{ij} N_{ij}}{G_{ij} L_{ij}}
$$

If there is to be symmetry, then:

$$
\frac{L_{ij}}{H_{ij}} = \frac{N_{ij}}{G_{ij}} = \frac{x_j}{x_i}
$$

i.e. $G_{ji} = H_{ij}$ and $L_{ij} = N_{ji}$ but $H_{ij} \neq N_{ji}$ and $L_{ji} \neq N_{ji}$

Because the last two relations are effects of one tax factor affecting the other factor in the other industry.

Problems in building up some tax equivalency in terms of $A_i$ and $B_j$.

Take our engine as an example where $b_k = \text{tax on X} = \text{tax on X}' = \text{tax on X}''$.

where $x$ are labor costs, $X$ are capital costs including depreciation and $X'$ are materials costs.

If materials enter in fixed proportions then $b_k$ is equivalent tax:

$$
\begin{align*}
b_{Xk} &= \frac{X}{X' + X''} \frac{b_k}{X'} \\
b_{Xk} &= \frac{X}{X' + X''} \frac{b_k}{X''}
\end{align*}
$$

Examples: If material is 40% of total cost then a 10% tax on X is a 16.75% tax on gross values added (i.e. $b_{X1} = 16.75$).

If you can't substitute away from capital X, then $b_{X1}$.

Lecture 11

Tax equivalency (cont.)

We can find a true equivalence with $b_k = \text{tax on capital}$ only if materials enter in fixed proportions.

$$
\begin{align*}
b_{Xk} &= \frac{X}{X' + X''} \frac{b_k}{X'} \\
b_{Xk} &= \frac{X}{X' + X''} \frac{b_k}{X''}
\end{align*}
$$

$X_k$ is the tax on gross earnings of capital
$T_A$ is the tax on net earnings of capital

$$
\begin{align*}
T_A &= \frac{T_A}{T_A} \\
T_k &= \frac{T_k}{T_k}
\end{align*}
$$

$T_A$ is the net return on capital
$T_k$ is depreciation or user cost of capital
$T_A$ is the rate of gross earnings of capital

When is a tax on net earnings the same in effect as a tax on gross earnings? When something blocks the normal response to the tax. The normal response is to shift the lives of the assets towards short end. (Even if there is a shift from gross to net earnings tax due to depreciation which is higher for short than for long term assets.)

If a production relationship were to prevent this kind of shift (example: trucking activity—the design of trucks will not be changed) then equivalence between tax on net earnings and net earnings. There is equivalence depends on $T_A / T_k$ remaining constant.

Tax Neutrality

A flat rate tax on labor earnings is neutral if $A_i$ is zero elastic. A flat rate tax on labor earnings imposing the same hourly rate on leisure time will be neutral no matter which supply elasticity labor has.

With respect to capital it is only sensible to talk of $T_A$ when we talk of a neutral tax. When the capital stock is independent of the rate of return—then the tax on capital is neutral.
How much efficiency would be reduced if we put capital earnings tax? It is hard to measure by figuring the dynamics of capital stock growth. Our method assumes a given capital stock so that calculation becomes less hairy than simulating growth. If equal across all activities, $\text{FS}_g$ and $\text{K}_g$ might be neutral.

An equal rate on products cannot be neutral unless there are special assumptions: first we must distinguish between final products or inputs—we must define the base.

The value added tax—the base of the tax is sales minus material purchases. That tax has potential for suffering from depreciation business (depreciation at different rates for different assets). But output on capital will be taxed more heavily when it has longer-lived limitations. To get around this you should have a TID of the consumption type.

Three types of TID:

Product type—the base is sales minus materials purchased.

Income type—the base is sales minus materials purchased minus depreciation.

Consumption type—the base is sales minus materials purchased minus investment.

The product type will be discriminatory against activities with shorter asset lives. The other types solve this problem—the difference between the two others is with their incentives for saving.

The income type, when aggregated, is like the income tax.

The product type, when aggregated, is like the GNP type tax.

The consumption type, when aggregated, is like a tax on consumption expenditure.

The income tax entails a double taxation of savings—it is nonneutral. The consumption type is neutral except for the labor-leisure choice. The labor-leisure choice is important only in a certain sector which has an alternative to work... examples: students, housewives... high income elasticity types. The consumption type is therefore not too bad compared to other taxes. It comes as close as possible to neutrality.

The base of the income type tax is higher than the base of the consumption type TID.

The corporation income tax

Analyzing the tax on factors of production in terms of the product market The extent to which the welfare cost of a tax on the factor can be transferred to the market for the product...

Joint demand for a factor of production

The effects would be identical if we tax the means or the brick or the house if there are fixed proportions. But you must look at the tax only in one market otherwise you are double counting. There is an equivalent tax on the producer which will have the same effect as a tax on a factor.

What happens when you have variable proportions?

The welfare cost of a tax on factors will be greater than that of a corresponding tax on the product (assuming variable proportions) when you change relative prices the tax on the house minimizes monetary cost of the house not the resource cost.

Unit resource costs are higher.

Elasticity of demand for a factor: example: means

$$\text{Ell}_m = \frac{\text{Ell}_m}{\text{Ell}_m} = \left(1 - \frac{\text{Ell}_m}{\text{Ell}_m} \right) \frac{\text{Ell}_m}{\text{Ell}_m} \quad \text{Ell}_m > 0$$
Factor tax vs. Product tax raising the same revenue

The impact of the tax includes the extent of use of the factor, the product and the amount of the revenue.

Factor A is used in the production of product X, \( A^{0} \) is the amount of factor A that is used when there is no distortion

\[ \frac{A^*}{A} = \frac{A^0}{A} \]

Bringing about the same reduction in the use of factor A by adding a tax on \( Z \), what happens to welfare cost? We want a \( b = b^* \) so that will have the same effect on reducing \( A^* \) to \( A^0 \)

Since we have 2 factors, they are substitutes if \( \frac{1}{A} \) \( \frac{dA}{h} > 0 \) and \( \frac{1}{h} \) \( \frac{dh}{A} > 0 \)

\( T_A \), \( T_b \)

\[ \text{will lead to a substitution effect which dominates the scale effect.} \]

The welfare cost will be greater than the one from the scale tax on \( A \). If \( S_f A + 0 \) then the welfare costs are the same for the factor and product tax.

\[ 4 \log A = (\frac{1}{A} \frac{dA}{h} + 1 - \frac{d}{h} S_f A) \log A + (\frac{1}{h} \frac{dh}{A} - (1 - \frac{1}{h}) S_f A) \frac{dA}{h} \]

1) Then the \( (\frac{1}{A} \frac{dA}{h} + 1 - \frac{d}{h} S_f A) > 0 \) then the welfare cost of the product tax is less than the factor tax.

2) Then the \( (\frac{1}{A} \frac{dA}{h} + 1 - \frac{d}{h} S_f A) < 0 \) then the welfare cost of the product tax is equal the welfare cost of the factor tax. \( F_{PA} = 0 \)

\[ (W - b)_{T_A} < (W - b)_{T_B} \]

\( T_B \) is more desirable.

Revenues are equal in the case of zero substitutability between \( A \) and \( B \) inputs. One should not make the mistake of picking the rising portion of the revenue curve; it may give the false conclusions.

The corporation income tax

The important lesson to be drawn from B's work in this field is that capital bears the full burden of the corporation income tax is not an extreme statement.

In the subject the traditional trichotomy

- Burden borne by capital \( \leq 0 \)
- Burden of the corporate income tax \( \geq 0 \)
- Burden borne by capital \( \leq 0 \)

The two sector analysis method used by B comes up with:

\[ \begin{align*}
\text{capital's share} & = \left( \frac{K}{K_x} \right) \times (\text{Total K income}) \\
\text{in natural income} & = \left( \frac{K_x}{K} \right) \times (\text{K in corp. sector})
\end{align*} \]

The 2 sector model is a closed economy model. Since capital is fixed in several countries across national boundaries, this assumption is unrealistic and can lead to erroneous conclusions. However, the corporate income tax is a general tax across countries. The model can be applicable when you can talk of capital in developed countries as a whole—the effect of these taxes reduces the rate of return on capital across countries. In the case where one country reduces its rate of tax, the analysis is a bit different and the results of the
model are erroneous.

The traditional trichotomy

The burden of the tax is divided between shareholders, labor and consumers. If this is the way it is then the burden borne by capital cannot be less than of equal to one because everyone who owns capital in this market must share the burden borne by capital.

Shareholders are thought of in the short run as residual recipients but the argument does not take into account resource allocation due to restoration of equilibrium.

Consumers, we can consider them as the other face of workers and shareholders. When we speak of the corporation income tax we are speaking of the tax on capital in the corporate sector.

We do not allow for debt issues because the firm cannot finance itself with 100% debt. A tax of 30% on equity will be approximately a 50% tax however if the firm finances with debt it can lower this tax rate by lowering the amount of equity. Since firms have not been successful in avoiding the tax in this way to any large extent, we will neglect it.

The traditional trichotomy

The third mistake—whatever be the categories we are separating out, the fractions of the tax burden that they bear does not have to be positive.

$E$—the corporate sector

$F$—the noncorporate sector

We are driven down the after tax rate of return; the gross rate of return rises.

What is the burden of the tax? The difference between the gross and net returns on taxed activity times the amount of initial capital in that activity, $D$. Burden borne by capital in the reduction in yield times $E$ so $F$ bears more burden than the burden of the tax. Therefore labor must be bearing a negative burden.

If demand of $F$ is infinitely elastic: $E_{\gamma} = \frac{E}{F}$

The burden? Not labor because the wage to labor is taken as numerically on the vertical axis. The price of corporate product prices relative to noncorporate product—consumers bear the tax.

Examples: $T_{L}$, $T_{K}$, Gov't revenue

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_{L}$</th>
<th>$T_{K}$</th>
<th>Gov't revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>60</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>Case II</td>
<td>60</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>Case III</td>
<td>60</td>
<td>60</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Real income of labor and capital has fallen by their share of total income losses.

$E_{\gamma} = \delta_{\gamma}$
There is a straight line contract curve due to the fact that factor proportions are the same in the two activities. Both industries have constant returns to scale so there are parallel tangencies. We start at point A and as a consequence of the tax, we go to point B, where X receives the same return gross of tax. Industry I will not be able to shift more capital and labor. The price of capital will rise by T at point B. If point B is $P_0$, then point A is $P_0 - T$. If $P_0 = P_L$, then the case where consumers bear the tax.

If $\frac{K_L}{L_L} > \frac{K_I}{L_I}$, the $X$ industry is capital intensive, while fixed proportions in industry $I$.

$E_0 / E_L$ falls due to the tax and capital bears more of the burden.

If $E_0 = 0$, industry $I$ is labor intensive, $E_0 / E_L < L_L / L_I$, the burden borne by capital decreases.

Explanations:

The interesting range of the ratios: Burden borne by capital

\[
\frac{K}{L} = \frac{K_0}{L_0} + \frac{\Delta K}{L} = \frac{K_0}{L_0} + \frac{\Delta K}{L_0} \cdot \alpha + \frac{\Delta L}{L_0} \cdot (1 - \alpha)
\]

Traditional $\Delta K / L_0 = -1$ (see graph)
The Cobb-Douglas Case

\[ Q = X^\alpha Y^{1-\alpha} \]

Production function:

\[ X = L^\alpha \]  
\[ Y = L^{1-\alpha} \]

Tax on X:

\[ \text{Tax on } X = 0.10 X \]

Tax on Y:

\[ \text{Tax on } Y = 0.40 Y \]

Capital goes exactly as the govt's setting—that is what we mean by burden being borne completely by capital.

The general case:

\[ X = L^\alpha, \quad Y = L^{1-\alpha} \quad \text{Cobb-Douglas Case} \]

So that is not an extreme position but a middle of the range effect.

Corporation income tax (cont.):

Partial factor taxes in a two factor two product world

1. Demand for the product \( P_x = E_x (P_x/P_Y) \)
2. Supply for the product \( Q_x = Q_x (P_x/P_Y) \)
3. Equilibrium in the market \( Q_x = E_x \)
4. Factor substitution in X
5. Factor substitution in Y
6. Price formation in X
7. Price formation in Y
8. Labor market equilibrium
9. Capital market equilibrium

1. \( 4X/X = E_x (4P_x/P_y - 4P_x/P_Y) \)
2. \( 4X/X = f_x 4P_x/P_y + f_x 4P_x/P_Y \) homogeneous of first degree prod. fun.
3. \( 4X/X = 4X/Y \)  
   \( 4P_y/P_x = 0 \) (choice of numeraire is arbitrary)
4. \( r_{xy} (4P_x/P_y - 4P_y/P_x) = 4X/Y = 4X/Y \)
5. \( r_{xy} (4P_y/P_x - 4P_y/P_x) = 4X/Y = 4X/Y \)
6. \( 4P_y/P_x = f_x 4P_x/P_y + f_x 4P_x/P_Y \)
   Since \( P_y, P_x, P_a = \)
6. \( 4P_y/P_x = f_x 4P_x/P_y + f_x 4P_x/P_Y \)
   \( T \) is in $ of the initial price of capital
7. \( 4P_y/P_x = 4P_x/P_y + 4P_x/P_Y \)
8. \( L_x = L_y \)  
   \( 4P_x = 4P_x \)
There are nine unknown variables: \( dX_1, dX_2, dX_3, dL_1, dL_2, dL_3, dX_4, dX_5, dX_6 \)

Assume no distortion in the market for labor

And in the capital market:

\[ dX_7 = dX_7 / P_7 \]

and \( dX_8 = dX_8 / P_8 \)

General equilibrium problem: when looking at efficiency costs we assume the government gives money back in the form of neutral tax or transfer--a nice way to handle the problem on a theoretical level. However, when we talk of incidence--we bear the direct burden? Then how do they suffer if they are getting back the money? So to handle the incidence of taxation--we assume the government imposes the tax (we don't look at how it is spent). So we look at how people do without this money, i.e., assume the government buys the goods and services and throws them away. We don't contemplate any benefits from government's use of those goods and services. So we assume government spends the money as the citizens would have (we lump to the government--the same tastes and utilities) for the easiest solution.

Now working through the model:

From equations 1, 2, 4, 7

\[ E_s (\frac{dX_1}{X_1} + \frac{dX_2}{X_2} - \frac{dX_3}{X_3}) = \frac{dX_4}{L_4} + \frac{dX_5}{L_5} + \frac{dX_6}{L_6} \]

From equations 4, 5, 8, 9

\[ \frac{dX_7}{X_7} + \frac{dX_8}{X_8} = - \frac{dX_9}{L_9} + \frac{dX_9}{L_9} \]

\[ \frac{dX_7}{X_7} = \frac{dX_7}{X_7} - \frac{dX_9}{L_9} + \frac{dX_9}{L_9} - \frac{dX_9}{L_9} \]

Now the three equations are:

\[ E_s (\frac{dX_4}{L_4} + \frac{dX_5}{L_5} + \frac{dX_6}{L_6} + \frac{dX_7}{L_7} + \frac{dX_8}{L_8}) \]

\[ \frac{dX_7}{X_7} = \frac{dX_7}{X_7} - \frac{dX_9}{L_9} + \frac{dX_9}{L_9} - \frac{dX_9}{L_9} \]

\[ 0 = - \frac{dX_9}{L_9} + \frac{dX_9}{L_9} - \frac{dX_9}{L_9} \]

What is \( dX_7 \)? When \( dX_9 = 0 \) capital bears \( \frac{dX_7}{X_7} \) of the burden because \( E_s \) is the numerator.

When \( dX_9 = 0 \) capital bears \( \frac{dX_7}{X_7} \) of the burden.

To determine the incidence of the tax, \( dX_7 \) is critical.

Using Cramer's Rule we can drive \( dX_7 \) from the three equations above in terms of \( X_s \) and \( X_s \):

1. When \( X_s \) is very large \( (\rightarrow \infty) \), then \( dX_7 = 0 \)
2. When \( X_s \) is very large \( (\rightarrow \infty) \) then \( dX_7 = -1 \) when we have equal factor proportions.
3. When \( X_s = 0 \) then \( dX_7 = -1 \)
4. When \( X_s = 0 \) then \( dX_7 = 0 \)

When we don't have equal factor proportions, we have capital bearing more burden if we are dealing with the capital intensive industry and capital bearing less of the burden if we are dealing with the labor intensive industry.

When \( dX_7 > 0 \) and we put a tax on the labor intensive industry, capital will bear less of its share of the burden of the tax.
Our model looks at point A—full employment and only one set of relative prices \( P_r / P_y \) which prevails. \( \Delta P_r = \Delta P_y = 0 \)

Examples:

\( Y = 50\% \) of price of capital

\[
\begin{align*}
\delta_y &= .6 \\
\delta_k &= .4 \\
\Delta P_r &= 1 \\
\Delta P_y &= 1 \\
\delta_r &= 2 \\
\delta_k &= 2
\end{align*}
\]

\[
\begin{align*}
P_r &= 1.6 (1) + .4 (2) = 1.6 \\
P_y &= 1.6 (1) + .4 (1) + .2 = 1.34
\end{align*}
\]

If \( \delta_y = .8 \), \( \delta_k = .2 \), then:

\[
\begin{align*}
\Delta P_y &= 1/6 \\
P_y &= 1.167
\end{align*}
\]

\[
\begin{align*}
P_r &= .8 (1.167) + .2 (1) = 1.15 \\
P_y &= .2 (1.167) + .8 (1) + .152 = 1.15
\end{align*}
\]

Differences in factor proportions act as a shock absorber for effects of \( Y \). If factor proportions are close it is hard.

---

Lecture 15

The corporation income tax (cont.)

The model explores the condition of full employment (i.e. point A of the previous graph).

That factor which will be more hurt is that which is more intensive in the taxed industry. If factor proportions are very different, then the burden borne by the intensive factor is less than if the factor proportions were relatively close together.

Reasons:

\[
\begin{align*}
P_{r} &= P_{k} + \frac{\delta_{k}}{\delta_{r}} P_{y} \quad \text{then} \\
P_{r} &= P_{y} \left( \frac{\delta_{k}}{\delta_{r}} + 1 \right) \\
\end{align*}
\]

The closer together are factor proportions, the closer to zero will be the numerator.

When \( \delta_{y} = \delta_{k} \), capital bears the whole burden of the tax: \( P_{r} = P_{y} \frac{\delta_{k}}{\delta_{y}} \).

The case where the capital market is equilibrated across national boundaries:

\[
\begin{align*}
\dot{E}_{xy} (\dot{X} - \dot{Y}) &= \frac{\dot{X}}{X} \left( \frac{\delta_{x}}{\delta_{y}} \right) + \frac{\dot{Y}}{Y} \\
\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} &= \delta_{y} (\Delta P_{y} + 1 - \Delta P_{r}) \\
\end{align*}
\]

where \( \delta_{y} = \frac{1}{\dot{E}_{xy}} \)

\[
\begin{align*}
\dot{E}_{xy} - \frac{\dot{X}}{X} \frac{\dot{Y}}{Y} &= \delta_{y} (\Delta P_{y} - \Delta P_{r})
\end{align*}
\]

If \( \Delta P_{r} = 0 \) then \( \dot{X}_{r} = \dot{X}_{y} \)

\[
\begin{align*}
\dot{X}_{r} &= \delta_{y} \\
\dot{X}_{y} &= \delta_{y} \\
\end{align*}
\]

\[
\begin{align*}
\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} &= \delta_{y} \\
\delta_{y} &= \frac{1}{\dot{E}_{xy}}
\end{align*}
\]

\[
\begin{align*}
\dot{X}_{r} &= \frac{\dot{X}}{X} \\
\dot{X}_{y} &= \frac{\dot{Y}}{Y}
\end{align*}
\]
Incidence when the elasticity of demand for the corporate sector’s product is large relative to other items:

\[ \frac{\Delta c}{\Delta x} = \frac{\frac{\partial E_p}{\partial x} \left( E_p - \frac{C_p}{E_p} \right) + \frac{\partial E_p}{\partial y} \left( \frac{C_p}{E_p} \right)}{E_p (\frac{C_p}{E_p} - \frac{C_p}{E_p})} = \frac{\partial E_p}{\partial y} - \frac{\partial E_p}{\partial x} \]

If \( \frac{\partial E_p}{\partial x} = 0 \) and \( \frac{\partial E_p}{\partial y} = 0 \) you have fixed proportions.
You will have a smaller corporate sector if \( E_p > \frac{C_p}{E_p} \).

That means we will be allocating labor as well as capital from the corporate sector as if it were a fixed proportions sector—in this case labor and capital are pushed out in exactly the same proportions but its first approximation is the fixed proportions case. Capital will bear more than its share of the corporate sector is more capital intensive.

It is probably true that the corporate sector is more labor intensive—in a geographical setting. Therefore the burden will depend on the nature of the corporate sector in the region. To the extent that there are affects on relative prices of labor and capital, there will be affects throughout the entire economy—the essence of the two sector approach. This might not be the right way to handle the problem of the small geographical area. As we go to small geographical regimes, the models we have looked at don’t make sense. A partial equilibrium model makes more sense. A disaggregative approach for example: The effect of the corporate income tax will be felt by some tangible factor, in part by consumers in the area (regionally specific demand will be insensitive and a rise in prices will work however if demand can be elastic it is movement down the demand curve of a monopoly). Besides that, there is an outline in activities with high demand elasticities.

If the world were really Cobb-Douglas, the aggregate approach will account a small decrease of the rate of return to all the capital in the world.

Frances Modigliani and Norton Miller, The Cost of Capital, Corporation, 
Finance and the Theory of Investment, AER June 1958, p.292
Lecture 16

The Financing Effect

The corporation income tax is a tax on return denominated to equity capital plus the residual share (rent, monopoly profits, return to the owned factor).

The question of financing—debt capital is considered an expense while equity is taxed at 50% marginal tax rate. Why not finance 100% by debt?

1) There must be some residual claimant
2) There is a given optimum debt/equity ratio for a given class of firms—-lenders will change that optimum and cause a welfare loss

![Graph showing marginal cost of debt increasing as debt increases.]

M. cost of debt increases as you move to more debt financing and there arises a discrepancy between the supply of debt and the W.

There is also a rising supply price of equity due to higher leverage factors as W/E ratio rises you cause a negative externality in the equity market.

$D_0$ is a hypothetical demand curve for debt, net of this externality

The Miller-Modigliani theory is correct but the homogeneous perceptions assumption of theirs is a stable oversimplification. But there are preferences for bonds, for stocks etc… in a capital market people have different attitudes apart from institutions (intrinsic funds, and insurance companies which prefer bonds). They may want to hedge against inflation, therefore they would not want stable stocks (TVT).

There is a division in the market on the individual level; bondholders are disjoint from stockholders. If feels this contradicts MM theory.

The rising supply curve is due to different individuals having different appreciation of the future earnings of particular companies.

Reason for $(W_{debt} - R_{debt}) > 0$.

Larry Fischer explains risk premium on bonds from cross-sectional data on American Companies in 1947 and 1954, risk premium R/R, last default, given of excess outstanding, variance in earnings. He finds $R^2 = .72$

As found the same coefficients for depression years and boom years $R^2$ for pooled regression was .73.

Tambini’s thesis—brought Fischer’s work up to date. The same regression file the data from the 1950’s.

Recent problems from the developing countries:

We usually associate rising supply curve as behavior of a monopolist. A firm can be behaving like a monopolist although it has no effect on world capital because of default risk.

The marginal participant thinks market perceives default risk and as you move out to more debt more and more individuals which perceive higher risk enter into the market. If the firm perceives its own default risk, it will not look at the risk free rate as the cost of capital because it doesn’t believe that the probability of default exists but its suppliers have a different idea as the firm looks like a monopolist, i.e., acts such that $W_C$ is higher than $D_e$ rather than at the risk free rate. If the company believes that the probability of its default is an $R^2$ please as well as the suppliers do, then it will take the risk free rate as its cost of capital.

![Graph showing two cases, Case I and Case II, regarding the range of disagreement between the market and the firm.]

Case I: The firm feels the suppliers of funds are unfair and takes its cost of capital as the W.

Case II: The firm feels that the suppliers of funds are fair and takes its cost of capital at the risk free rate.
Case III
Fleeting the public—Doesn’t happen often. If people (the market) think the probability of default is less than the firm thinks.

Lecture 17
Stimulation of investment

Tax stimulus to investment—a feature of legislation common in all countries

The simple income tax already entails the double taxation of savings in the sense that it is proportional; the third time by property taxation and the fourth time by corporation income tax. Then we say we want to stimulate investment—special treatments for capital gains, special tax stimuli to invest etc... It cannot be a rational process out of which this sort of pot-pourri emerges.

We can go into the political arena and say ‘We think that the earnings of labor are hard earned and that the earnings of capital are fruits of idleness, represent no true effort on the part of the people. Shouldn’t we tax capital more heavily...? Oh yes, yes, yes... But we need investment for our economic growth, so shouldn’t we have some special incentives...? Yes, yes, yes... You see it’s like that! Economics is a wonderful playground for a bored charlatan... You can use people up and down the avenues and they don’t know it. Really it’s true. You can sell ideas under some labels and not under other labels although they’re exactly the same. For example, in Latin America, if you say ‘I’m a free trader’ there’s no better way to have yourself never taken seriously again but if you say here we are under import tariffs on a lot of different industries, protecting these activities. Now if we calculate the effective protection we find it varies among activities wouldn’t it be sensible to make more equal so that it could make it easier to handle? Oh yes... However we are neglecting exports. Oh yes, we never thought about that... But if we put a subsidy to exports equal in magnitude to the tax that we are putting on imports so that we equalize effective protection, then we are giving the same treatment to imports and exports... Oh yes that’s a great idea... Bull that free trade. The same is true with taxation on capital.”

There are subtleties in the political process but certainly what we see is not rational economics.

The tax incentives for investments are various kinds: there are tax holidays, accelerated depreciation, direct expensing of investment costs, and there are interest tax credits etc... Each has a special role.

We can also analyse them in more than one place. If we have a corporation income tax that already discriminates between corporate and noncorporate users of capital: Examples accelerated depreciation to certain industries.

We might see the global picture between the corporate and noncorporate sectors as a basis of comparison. So effectively we are reducing the degree of discrimination imposed by the corporation income tax. In any way that we give tax stimulus to investment in the corporate sector it will have that effect. It is, any income under which the corporate tax, where the source affects the marginal activity, will have the effect of reducing the overall disincentive in the corporate sector.
Our area of interest however lies in more specific properties of investment incentives. If you are stimulating, what particular ways can be recommended? What are their attributes? Can we attach advantages and disadvantages?

We begin with two propositions:

Expensing—a total expensing of investment costs against the corporation income tax is neutral in all relevant respects—what that means is that the government is getting a certain piece of the action.

If you have an investment with this profile:

and the tax rate is 50%, the government pays 25% and gets . . . . . . .

i.e. when there is a loss government gets 50% and when there is a benefit, the government takes 50% of the net benefit. It is equivalent to the government being a 50% partner—an automatic partner.

Investing 75% rather than 100% does not change the investment decisions concerning risk or life of the investment. If investment from now on were expensed, there would be a net tax revenue. It is modern growing economy the income from capital would be greater than gross investment. (income from capital gross of tax and depreciation that is)

In a stable economy, gross investment equals depreciation in a growing economy, the difference is profits. Investment = profit in the Golden rule world—which is optimal if capital in the initial position is used. However, since nature is not in the habit of giving us the initial capital stock we choose, we are usually short of being in that world. So you would make money—it isn’t exactly equivalent to summing the corporation income tax but it is similar in all effects. (resource allocation effects are essentially zero)

There is another kind of neutrality which is possible in the corporation taxation. Full expensing is equivalent to government partnership—the other treatment that has a certain neutrality to it is income taxation of true economic income i.e. we tax income after true economic depreciation is calculated. There is no bias in favor of or against long lived or short lived assets. Many ways of seeing this: the elegant way is as a series of negative elements in the vector of outputs in the firm. True economic depreciation viewed as wanting to be charged to costs of operation. Then what you end up with is a concept of income such that a proportional tax on that income will not bias the choice of investment. Consider assets of 1, 0, 20 years—associate with each asset some kind of typical profile of an investment. There are several different kinds of investment interventions that starting with a bunch of these investment prospects which are exactly at the margin of the acceptance decision—due to the tax they will no longer be at the margin of indifference. The proportional taxation of income with full correction for true economic depreciation will not have that effect.

Suppose a stationary state, a bunch of assets of 3 years and

Investment of each year replaces one third of your investments. Every year we replace investment and we are earning 25% after cost of replacement (which is exactly the true depreciation). Now we have a tax of 50% on income from capital as depreciation. We now earn 125%. That will apply all across the board. If you were indifferent between assets of types ABC you will again be indifferent except now the rate of return will be lower. We have a wedge going between the gross rate of return and the net rate of return on capital which is equivalent to all of the activities in the corporate sector. The introduction of such a wedge is going to maintain whatever relationships might exist between the gross of tax rate of return of projects and the net of tax rate of return. Therefore the projects that survive the corporation income tax will be projects that yield 65 at 125 gross. The ranking of these projects which are marginal at the pre-tax rate of return is the same when considering their post-tax rates of return.

The main case where there is a bias towards short lived assets is the investment tax credit which is the one which has been most recently used in the United States. How it has become so popular device? Obviously a mistaken policy measure. The government pays a certain fraction of investment to the investor and allows you not to deduct but allows you to take say 75%. It is not like being able to write off because you can now write off the full amount of the investment. It favors short lived assets.

(This is a partial transcription from a tape which was used due to my absence)
Lecture 13

Accelerated depreciation makes long-lived projects look artificially better.

Tax credits have artificial incentives for short-lived projects.

Why investment tax credits create artificial incentives for short-lived projects?

\[ \text{Let } \beta \text{ be the yield and } \delta \text{ be the rate of depreciation and } p \text{ the price of equipment.} \]

\[ \frac{p}{(1 + \beta)} = \frac{p}{1} \times \frac{1}{1 + \delta} \]

\[ \text{is the rental value of the asset in equilibrium.} \]

You can think of a tax credit which pays 10% of the price of the asset when you acquire it as being equivalent to paying 10% deductions on depreciation as it depreciable.

Depreciation is smaller for long-lived assets than for short-lived assets as you get a bigger subsidy when using short-lived assets.

If you produce the same economic rent with long-lived assets as you do with short-lived ones, shouldn’t we be comparing the generation of the same economic rent? That does not mean to have equalized economic rent?

Subsidy: 10% x S

\[ \text{Econ. Rent } 30 \times 120 = (0.1 x 0.2) \times 100 \]

\[ = 120 \times (0.1 x 0.2) = 24 \times 0.2 = 4.8 \]

It is the rate or return that should be equalized:

\[ \frac{r}{1 + \delta} = \frac{r}{1} \times \frac{1}{1 + \delta} \]

Investment in short-lived assets will go until the marginal return is lower than investment in long-lived assets in the presence of an investment tax credit.

One way to get a 10% return:

\[ 2 \text{ yr. asset } 100 \times 100 \times 100 \times 100 \]

\[ \text{Before tax return } 20 \times 20 \times 20 \times 20 \]

\[ \text{After tax return } 10 \times 10 \times 10 \times 10 \]

\[ \text{Steady state replacement } 50 \times 50 \times 50 \times 50 \]

\[ \text{depreciation} \]

\[ \text{For a 5 yr. asset the before-tax return is 20, the after-tax return is } 10 \text{ and depreciation is 20.} \]

\[ \text{For a twenty yr. asset the before-tax return is 20 the after-tax return is 10 and depreciation is 5.} \]

\[ \text{Due to the tax credits: } 2 \text{ yr. asset } 30 \times 30 \times 30 \times 30 \]

\[ \text{Before tax return } 20 \times 20 \times 20 \times 20 \]

\[ \text{After tax return } 10 \times 10 \times 10 \times 10 \]

\[ \text{Depreciation } 50 \times 50 \times 50 \times 50 \]

\[ \text{Tax credit } 10\% \times 20 = 10\% \times 20 = 10\% \times 5 = .5 \]

\[ \text{This is not an equilibrium situation. This will lead to allowing short-lived assets to be excerpted to equal after tax returns. It is} \]

not to be assumed that quantities of investments will be very different.

\[ . \text{ It depends on the elasticities of the rate of return schedule.} \]

\[ \text{The equilibrium tax: } 2 \text{ yr. asset } 100 \times 100 \times 100 \times 100 \]

\[ \text{Before tax return } 10 \times 10 \times 10 \times 10 \]

\[ \text{After tax return } 10 \times 10 \times 10 \times 10 \]

\[ \text{Depreciation } 50 \times 50 \times 50 \times 50 \]

So we can see that short lived investments are favored by the tax credit. It enhances demand for short-lived assets. It bids up the price of funds (surplus). It reduces the tax rate on capital, it reduced a pre-existing distortion but it introduces other distortions.

If you want to stimulate investment in a sector subject to income tax, one way to do it is to reduce the income tax rate. But it is not popular, because not enough "bang for the buck" since it reduces tax on all investment. A way to reduce tax on new investment—allow expensing of new investment 10% and allow depreciation of 10%. This is the same as allowing a reduction of income tax from 10% to 5% on new investment.

\[ \text{Tax Holidays.} \]

\[ \text{Tax holidays are exemption from income tax on an investment for a} \]

\[ \text{specified amount of time. (usually 4-5 yrs.)} \]

\[ \text{A five-year holiday will make incentives for investments which are} \]

\[ \text{positive flows during the period of the holiday. If it is to take} \]

\[ \text{place after a certain date—it gives incentive to have losses until} \]

\[ \text{then and then have tremendous profits for the specified period.} \]

\[ \text{Tax holidays have very different affects on different firms which are} \]

\[ \text{at different stages of their profitability schedule which have} \]

\[ \text{different ages of assets etc... it introduces tremendous distortions.} \]

\[ \text{Percentage Depreciation (related to corporate tax)} \]

\[ \text{Percentage depletion is only part of the tax treatment of the mineral} \]

\[ \text{industry. Capital gains treatment to mineral industry activities takes} \]

\[ \text{place with respect to an individual.} \]

\[ \text{It's work on } \% \text{ depletion is now 23 yrs. old, assumes capital makes} \]

\[ \text{a normal profit and says what was insufficiency introduced by the} \]

\[ \text{depletion allowance. Depletion laws have have form now that all} \]

\[ \text{prices have risen as much.} \]

\[ \% \text{ depletion amounts to the same thing as a tax credit—the problem is} \]

\[ \text{that you still have to tax the whole depletion. No operating allowance.} \]

\[ \text{Percentage depletion allows over depreciation of taxed good in question.} \]

\[ \text{History: Depreciation of costs of wells (new wells) will be allowed;} \]

\[ \text{however it will not apply to cost of already existing wells but their} \]

\[ \text{fair market value. New well explorers complained that they were getting} \]

\[ \text{unfair treatment. So the discovery depletion was created. True economic} \]

\[ \text{value which is depreciated over time of the cost of discovery, whichever} \]

\[ \text{is greater, can be written off for each well. This was a type of double} \]
Incentive because you write off all bad wells and the value of the good wells. Problems arise in determining the value. There was a legal battle in the percentage depletion case instituted. 27%—The fraction of the value of oil which the well yielded which was permitted to be written off. The intention was an administrative simplification of discovery depletion. Percentage depletion amounted to double depletion—more was written off because of costs than costs incurred.

How do you analyze percentage depletion?

You make a comparison between what conditions would be if normal taxation had been instituted instead of percentage depletion.

The value of the oil well subject to percentage depletion is not different from the value of the oil well subject to cost depreciation. What is the price that an investor would pay for an operation?

\[ t = \text{present value of future gross income stream} \]
\[ (\text{before taxes, depreciation and depletion}) \]
\[ d = \text{depreciation (discounts B)} \]
\[ B = \text{price you pay for an asset (undiscounted)} \]

Taxable income: \( T - dB \)

Taxes: \( t(\bar{D} - dB) \)

After tax income: \( T - t(\bar{D} - dB) \)

What you are willing to pay for the asset assuming you depreciate it with cost depletion:

\[ R_1 = \frac{T}{1 - d} \]

What you are willing to pay for the asset assuming you depreciate it with percentage depletion:

\[ R_2 = \frac{T}{t(1 - d)} \]

\( p = 35\% \) which corresponds to 27% of gross income (which includes asset expense etc...)

\( d \) depends on the time shape of the asset. Assuming a 10 yr. life and 10% rate of discount then \( d = 0.5\% \)

\( R_2 \) comparing \( R_1 \) with \( R_2 \):

\[ R_2 = \frac{.35}{.01} = (.35)/(.3) = .742 \]

\[ R_2 = \frac{T}{(1 - .35 + .5[.35] = .679 T} \]

From the data it looks like people chose cost depletion when they have bought the well and when they have discovered the well they prefer percentage depletion.

If they purchase a well they consider an alternative project—cost of a machine for example:

\[ C_1 = .7T \]

If they discover the well they consider the total cost—cost of bad wells, looking for wells as well as that of the good well. The cost associated with the well is greater than the cost of a bad well.

\[ C_2 = 4(.05) = .67T \]

\[ C_2 = 1.1T \]
Lecture 19

The Choice Between Percentage Depletion and Cost Depletion (cont.)

Theoretically percentage depletion is less favorable for firms than cost depletion except when firms discover wells. With percentage depletion you attach depletion expense which are far greater than the cost of the successful well because you write off the cost of dry holes. So firms who discover wells prefer percentage depletion.

Capital gains is a special case which is connected with the same phenomenon. You decide to write off dry holes expenses from personal income and pay capital gains tax on the net profit from the successful well. For example:

<table>
<thead>
<tr>
<th>Expenses</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Successful well</td>
<td>200</td>
</tr>
<tr>
<td>Dry hole</td>
<td>800</td>
</tr>
<tr>
<td>You receive</td>
<td>900</td>
</tr>
</tbody>
</table>

**Tax treatment on the dry hole**

Rate: 100% written off

Rate: 70%

Rate: 50%

Y: Value of oil

\[ Y = 1 \times (0.25) \times (0.5) \]

C: Total costs

\[ C = Y - C_1 \times (1 - .25) \times (0.5) \]

Cost of dry hole

Cost of successful well

\[ C/Y \text{ is approximately } 1.45 \]

It's suggestion for reform: Instantaneous write off of all costs is equivalent to a subsidy to costs which can be written off instantaneously—it gets around the problem.

N.B., if you are given special treatment (happens now for research expenditures) & want to analyze instantaneous expensing. We would still have the problem of capital gains. There are two possibilities:

- Some assets that have special treatment, they are not eligible for the capital gains tax (which is less than the income tax) so that if \( C = 1 \times (1 - .25) \times (0.5) \), then you just skip over the tax altogether.

- The problem is that the two pieces of the balance sheet are affected with different tax rates that is why you end up with different after-tax income.
Lecture 20.

Capital Gains tax (cont.)

We don't have data on the annual return on capital gains but we have data on realized capital gains. The richer will prefer longer term capital gains.

Assume a portfolio of $1000:

<table>
<thead>
<tr>
<th>Capital gain</th>
<th>Net gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bought at</td>
<td>Value</td>
</tr>
<tr>
<td>A 100</td>
<td>95</td>
</tr>
<tr>
<td>B 200</td>
<td>200</td>
</tr>
<tr>
<td>C 400</td>
<td>400</td>
</tr>
<tr>
<td>D 200</td>
<td>200</td>
</tr>
<tr>
<td>E 100</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>1100</td>
</tr>
</tbody>
</table>

65 (65/100) 95 = 92
10 (10/100) 200 = 20
20 (20/100) 400 = 80
10 (10/100) 200 = 20
5 (5/100) 50 = 2.5

You can choose judiciously which assets to liquidate in order to affect capital gains with capital losses. If the portfolio is growing (at 10/yr) you may run out of losses.

This postponement effect reduces the taxes—the gain is realized long after it is accrued and discounted the tax is much less. A reform which will never be implemented is taxing capital gains as they accrue. It would eliminate this locked-in problem without eliminating the revenue.

Lecture 21

Tax Treatment of Housing: Negative Income Tax averaging

Rents

Tax exceptions on income from owner-occupied housing is one of the most important subsidies both in the United States and the rest of the world. It seems that there is a kind of inescapable political barrier associated with this particular provision. No economist has made a good case for it. The government is in effect paying a regressive share of the rent: f(t) = t

Why? "Home ownership is a good thing" Poor people look forward to it while rich people enjoy it. It is a deduction which goes up with income rather than as a tax.

From the rent which you figure out, you deduct real estate taxes, maintenance and depreciation charges. If the owner who occupies the house does not declare a rent he cannot deduct maintenance and depreciation: he can deduct real estate taxes and mortgage interest. Hence a tax offset. Industry income from owner-occupied housing is non-biased regression, with reform being within the range of feasibility. It is truly difficult to redistribute income by taxation.

If we took all income over $50,000 and spread it over the poor, they wouldn't be very much less poor. It is unlikely that there is a system which actually redistributes. If it does, most countries don't want to emulate it for example the U.K. There are all sorts of problems which come up such as braindrain, etc...

The tax system is not strong enough to accomplish income redistribution.

Negative Income Tax

Experiment in New Jersey—but people knew they were a special program divorced from the rest of income. Windfall vs. permanent effect. Evidence shows in the direction that income maintenance payments were treated as windfalls. Looks nice, uncomplicated but it doesn't do what people want it to do. Does it make sense to impose higher marginal rates on our poorer than our richer people? In the main, there is no happy solution to the problem.

Perhaps this explains the unlikelihood of much of our tax expenditure.

Averaging under personal income tax

There is a discrimination between transitory and permanent income. This is a problem especially for athletes. In the U.S. it is not automatic, not everyone must or can average their income.

The simple concept behind it: a moving average 1/2 of income this year gets income and 1/3 in the future. This is equity for people with variations around the mean which rises.

Cumulative Income Averaging

The best of the varieties: it simply averages the marginal rate for windfall gains and a higher rate for losses.

(This lecture was copied from someone else's notes due to my absence.)
Lecture 22
Income averaging: Property taxation: Consumption taxes

Moving average vs. contemporaneous taxation

Contemporaneous taxation is better from built in stabilizing view

The moving average

\[ T_{t-1} = \frac{T_{t-1} + T_{t-2} + \cdots + T_{t-n}}{n} \]

You pay less extra tax when income rises than with contemporaneous tax.

You get a bigger reduction in tax by going from \( T_0 \) to \( T_t \) twice than by going from \( T_0 \) to \( T_t \) once.

Cumulative taxation with interest

\[ b \] is the beginning date

\[ \frac{(1 - b)(T_{t-1} + T_{t-2} + \cdots + T_{t-n})}{n} = T_{t-1} \]

all the income earned until time \( t-1 \) of periods over which he has earned

Average tax due associated with average incomes:

\[ T_{t-1} = T \left( T_{t-1} \right) \]

Total tax payments minus what is already paid = what one owes

\[ (1 - b)(T_{t-1}) = (1 - b)(T_{t-1} + \cdots + T_{t-n}) = T_{t-1} \]

We should discriminate on timing of relevant payments of the tax. We should have a certain present value of taxes paid and income earned so that any two people at same time and if they paid the same amount and present value of income should pay the same present value of tax.

Present value of all income to date: \( T_{t-1}(1 - b) = (1 - b)T_{t-1} + (1+b)T_{t} \)

\[ T_{t-1} = \text{present values of all income up to } T_{t-1}/(1-b) \]

The tax bill:

\[ (1-b)T_{t} - (1-b)T_{t-1}(1+b) = T_{t} \]

Equating the tax payments in terms of present value in terms of the diagram:

Property tax

It is hard to value things—self assessment is unrealistic.

Variants of self assessment—government assesses your property with your help.

In Latin America for example, the government uses a tax to assess his land and then taxes on that assessment

Beyond valuation, what is to be said for property tax?

If you have a fixed stock of capital in an economy, and you put on a property which takes away a certain fraction of income generated by that capital:

\[ F = \text{gross of tax rent} \]

Property tax is set at \( T = 0.2 \cdot F \)

We don't change scarcity or distribution of capital when we put on a property tax. What happens is that the interest rate falls (closed economy assumption) by just enough to reflect the tax.

If there is an inelastic supply, the tax will come out of the rent. The way it happens is that the after tax yield increases, and the capital value of existing assets falls and of reproductible assets falls.

As the economy grows the tax raises house rents and user cost of capital equipment.
In a closed economy, a dynamic analysis:

S is the supply curve of funds or the flow supply of savings or the long-term supply of stock of capital.

D is the flow demand for increments of capital stock or the HU stock.

D' is the demand affected by the tax.

The tax is imposed and if the equilibrium moves to point C it means that the interest rate remained constant and the value falls by the full amount of the capitalized value of all future taxes.

Bent on land stays the same all along curve D' because it is a fixed component.

If the interest rate falls, the value stays the same for example pt. B.

At point D we have an intermediate position when there is a response from capital value and interest rate.

Consumption Taxes

Kaldor-Ohlin Expenditure Tax

Earlier, Irving Fisher proposed a consumption-expenditure tax. Milton Friedman proposed one during the war and now Martin Feldstein is proposing it.

If you value savings because it is useful in promoting growth in the economy, then you are in favor of a consumption tax.

The consumption tax makes what people take out of the economy, the income tax makes what people put into the economy.

The consumption tax gets rid of one distortion, the labor-leisure choice distortion, of a tax.

With equal revenue, the welfare cost will be less.

Now administrable is the expenditure tax. In order to administer the consumption-expenditure tax you need more data than for an income tax.

Consider a household—you need to know:
Receipts from labor services + new debt + profits in interest, rents + sales of eligible assets + current costs of doing business + old debt repayment + interest paid on debt + tax paid + purchases of eligible assets = consumption.

Consumer durables are hard to deal with. And housing is it considered as an investment and you impose a rent on housing.

If the house is held as an asset— you receive rent from the house.

Treatment of depreciation—you can maintain a sliding fund and put it into some bank account the amount that would be depreciation.

Depreciation would not appear in taxation.

Tax would have a charge on over-occupied housing and real estate.

Would you want to have an inheritance tax along with the consumption tax? With the consumption tax, Kaldor would like to have wealth taxes.

Difference between an inheritance tax and a wealth tax:

An inheritance tax—the person did not himself generate income—he should be taxed.

A wealth tax—wealth carries economic power, therefore has more taxable capacity. Wealth is not worth—what would accrue to an estate if the person were to die, does not cover human wealth.

For the consumption tax you need to know:

Where the money comes from

What assets were acquired and sold.

It can't just be feasibility of administration which blocks the consumption tax. The problem is the force with which administration would have to come down with the tax; the enforcement of taking down people's portfolios is difficult; the information requirements are not greater but you have to keep a running account of assets faithfully.
Lecture 23

Value added tax and Border tax adjustments

No group is the beneficiary of the tax

The French first instituted the Value-added tax in 1954.

Each level pays the tax on what it sells and gets an offset for all that it buys which has already been taxed. Redistributions buying from outside the system get no deduction.

Farmers are out of the VAT system—they are the exempt sector because they are the early part of the production chain. Users of farm products pay the tax for them. Actually farmers pay more because they don't deduct taxes on fertilizer etc... (Inputs in farming) Because farmers are left out—administrative reason.

Retailers at final sales are sometimes left out—not because of the tax goes down for them and this stimulates the activity. Many people try to hide activities under this retailing label to reduce their taxes.

All types of VAT sales but differ in what you can deduct from sales a tax of the product type—taxes gross value added i.e. they deduct current material inputs.

A tax of the income type—it covers whole economy, the net national product by allowing sales minus current material inputs minus depreciation.

A tax of the consumption type—added up over all activities, it would end up taxing consumption in national income accounting language. It is sales minus current material inputs minus capital goods acquisition.

All existing VAT are of the consumption type.

The most neutral and easiest to administer in the income types it is problematic because deducting depreciation creates ambiguity.

The product type is hard to administer because it is hard to distinguish between capital and current inputs.

Border tax adjustments in the VAT

Border tax adjustments is a mechanism which levies indirect taxes in addition to tariffs on goods which are imported and which retains those taxes when they are exported.

With a fixed exchange rate—VAT makes internal prices higher but once goods go in and out they are adjusted. Border tax adjustments do not give an advantage to a country in international trade, they permit a tax to be accommodated without causing a change in the exchange rate or internal prices. They are a trick to avoid a reaction to the tax by factor prices.

<table>
<thead>
<tr>
<th>Initial Situation</th>
<th>20% VAT</th>
<th>VAT &amp; PTA</th>
<th>VAT &amp; No Border tax adjustments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange Rate</td>
<td>1</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>Internal factor</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Domestic Prices</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Price of imports</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Price of import</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Substitutes</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Price of exports</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

Another way we can look at border tax adjustments—Suppose we decide to have an excise tax where should we collect it? PTA convert tax on production to tax on consumption. It makes it possible to tax consumption while we tax producers.

If Value added fall predominantly on national versus international goods, there will be distortions and overheating in other markets however no direct interference with trade patterns if all sectors of production are affected similarly—there is no distortion.