The Social Cost of Public Finance

Larry A. Sjaastad
University of Chicago

Daniel L. Wisecarver
Ohio State University

This paper examines the implications for fiscal policy of systematic differences between (social) rates of return on private investment and savings. We show that the social discount rate lies between these divergent rates of return, that the controversy between this result and Marris's stems entirely from different treatments of depreciation, and that reinvestment of net project output has negligible quantitative effects on the social discount rate. Then, within a macroeconomic framework, we derive the stringent conditions for a unique discount rate and demonstrate that, as public-sector consumption must also be charged a shadow price, the social value of a global change in output induced by countercyclical fiscal policy must exceed its (nonzero) social cost. Finally, we examine the Little-Mirrless concept of the social cost of labor and find it unacceptable.

I. Introduction

For decades the most controversial issue in cost-benefit analysis has been the selection of the appropriate rate of discount. The controversy initially focused on the relative merits of two candidate rates: (1) the (relatively low) social rate of time preference, which we shall call the consumption rate of interest (r) and which many writers have identified with the after-tax rate of return on private savings; and (2) the (relatively high)

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513
gross-of-tax rate of return to privately financed investment, which we shall call the investment rate of interest ($\rho$). Recently, however, a number of analysts, the most significant being Sandmo and Dreze (1971), Haberger (1975), and Dreze (1974), have demonstrated that the social rate of discount ($\omega$) should be a weighted average of $\rho$ and $\sigma$. As a result, the debate has been slightly refocused, the candidate rates now being $\sigma$ and the weighted average $\omega$.

This paper is an attempt both to resolve the existing controversy and to indicate the importance of a number of additional problematic issues that have not received previous attention in the literature. In Section II, we rely on a simplified model of a project which generates a perpetuity to demonstrate that, when capital-market distortions are correctly taken into account, $\omega$ equal to a weighted average of $\rho$ and $\sigma$ is the only result that can obtain, even though we also agree that $\sigma$ is the only defensible rate by which alternative future consumption paths should be evaluated. We further demonstrate, again for the case of a perpetuity, that Margin's (1963a, 1963b) well-known analysis yields an investment criterion identical to the net-present-value criterion when $\omega$ is used as the discount rate.

Then, in Section II, we show that a true difference between Margin's analysis and those which result in $\sigma$ does arise for projects with finite lives: the divergence stems from diametrically opposed, implicit assumptions as to consumption behavior toward depreciation of, and net income from, project-created capital. This point leads us, in Section III, directly into the question of reinvestment; arguing that Margin's results, although theoretically correct, depend on an ad hoc treatment of depreciation while the Haberger and Sandmo-Dreze models do not directly address the issue, we present two alternative manners by which reinvestment of the project's net addition to output can be incorporated into the analysis.

At another level of analysis, however, it is our contention that the debate on the social rate of discount has to date neglected a basic issue underlying the evaluation of public expenditures. In particular, concentration on investment criteria and the rate of discount has led, at least implicitly, to an asymmetric treatment of government consumption versus government investment. This has in turn, we argue, obfuscated the one fundamental question that must be resolved: What is the social opportunity cost of any type of government expenditure?

Sections III and IV explore this question within the context of a simple macroeconomic model. We first note the obvious point that when changes in public expenditures alter voluntary, privately financed expenditure decisions—investment or consumption—there is a clearly associated, and in principle directly observable, opportunity cost which may or may not require reference to information from the capital market. We then show how all of the relevant considerations can be combined to formulate a procedure for socially evaluating (any multiplier effects of) general public expenditures, the more convenient manner being a Margin-type shadow price of inputs.

Finally, in Section V, we evaluate the social opportunity cost of labor as expounded by Little and Mirkles in the OECD Manual (1969) and which has remained largely intact in their more recent publication (1974). Our analysis has direct relevance to this seemingly unrelated issue since their shadow price of labor depends crucially on a distinction between the investment and consumption rates of interest. We conclude, as have others, that the Little-Mirkles analysis is wrong, and for a reason even more fundamental than reasons which earlier critics have cited.

II. The Social Rate of Discount, the Opportunity Cost of Public Investment, and Two Distinct Reinvestment Issues

A. Derivation of $\omega$ When Public Projects Generate Perpetuities

Controversy over the appropriate rate of discount would never arise in an undistorted economy, as there would exist only one relevant (market) rate of interest, simultaneously representing the (social) marginal efficiency of investment and the consumption rate of interest. Using this rate to fulfill the net present value criterion for any publicly financed project would therefore guarantee that the project is capable of compensating private investors and consumers for both the privately financed current consumption and the potential consumption that they are induced to forego by the initiation of the government expenditure.

The issue, of course, arises because capital markets are almost universally distorted; a variety of taxes (and other distortions) on the income from capital and on the yields from savings drives $\rho$ upward and $\sigma$ downward, respectively, in relation to "the" market rate of interest. In this context, those who argue for the exclusive use of $\rho$—which in fact represents the social (i.e., tax-inclusive) return to private investment—rely on a pure concept of opportunity cost; discounting public expenditures by any rate of interest less than $\rho$ raises the distinct possibility that a publicly accepted project could yield smaller social returns than would directly investing in the private sector. On the other hand, those who argue for the exclusive use of the consumption rate of interest rely on the following reasoning. The objective of fiscal policy should be to maximize social

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1 Our repeated references to the Haberger approach to the social discount rate will consistently be in terms of the 1973a article.
2 We will generally refer to the Sandmo-Dreze model as the body of analysis contained in both the Sandmo-Dreze (1971) and the Dreze (1974) articles.
3 Haberger implicitly assumes that it is handled elsewhere in the evaluation of the project itself; Sandmo-Dreze avoid it in their 1971 paper by their "sudden-death" assumption.
welfare, approximated by an aggregate utility function which depends only on the time stream of consumption. Since r is defined by the private valuation of current relative to future consumption—that is, r is the supply price of savings—it can be the only relevant rate for discounting the benefits generated by public expenditure.

Both of these positions are correct, in and of themselves, yet each ignores the validity of the other. There can be no doubt that r is the correct rate for discounting positive and negative increments to future consumption. But likewise, there can be no doubt that current public expenditure must be charged not only with current consumption forgone but also with unrealized potential future consumption due to displacement of current investment in other sectors. The obvious implication is that we should seek a social discount rate that encompasses both of these factors; the result will necessarily be a rate that lies between p and r.

To show this result, we assume a closed economy in which all shadow prices, other than the social rate of discount, equal their market prices, and consider a public-sector investment of $\Delta t^f$ that generates a perpetuity. The resultant (permanent) change in output, when private investment also generates perpetuities, is defined as: $\Delta Y = p\Delta t^f + \Delta Y^p$, where $Y$ is output, $t^f$ is private investment, and $\delta$ is the realized rate of return on the public project. The operator $\Delta$ refers to the departure of any variable from the path it would have followed had the public project not been undertaken, rather than to changes that occur from one period to another.

In any economic system, with or without an organized capital market, each dollar of $\Delta t^f$ takes place at the expense of some fraction (possibly zero) $\delta$ of investment and $\delta$ of consumption. Without loss of generality, we set $\Delta t^f$ equal to unity; hence $\Delta t^f = -p$, and $\Delta Y = \delta - p$. Clearly, the criterion for accepting the project must be that total income available for consumption be at least as great with the project as without it. Since we are explicitly utilizing marginal analysis, it is appropriate to compare the present value of $\Delta Y$, discounted at r, with consumption forgone, $(1 - \delta)$. That is, the investment criterion is:

$$\int_0^\infty (\delta - p)e^{-\delta t} dt \geq (1 - \delta).$$

Solving for $\delta$, we obtain:

$$\delta \geq p + (1 - \delta)r = \omega. \quad (1)$$

Obviously the social rate of discount, $\omega$, is a weighted average of $r$ and $r$, the weights being the aforementioned shares of increments to public expenditure that come at the expense of investment and consumption, $\delta$ and $(1 - \delta)$.

The one remaining issue is the definition of these weights, for which we fall back on Harberger's pioneering analysis, and on its subsequent confirmation and extension by Sandmo and Dreze. That is, departing from an initial equilibrium position in a distorted capital market, Harberger demonstrates that the (perpetual) stream of income required to compensate private-sector investors and consumers for a (permanent) extraction of $1.00 from that market is:

$$\omega = \left[ p \left( \frac{\partial t^f}{\partial t} \right) + r \left( \frac{\partial Y^p}{\partial t} \right) \right] \left[ \left( \frac{\partial t^f}{\partial t} \right) + \left( \frac{\partial Y^p}{\partial t} \right) \right].$$

This weighted average formulation highlights Harberger's major contribution toward reaching the discount-rate controversy; given a perfectly but distorted capital market and rational economic behavior, it is the interest rate which allocates output between consumption and investment. Therefore, the capital market interactions that occur in response to government-borrowing-induced changes in the interest rate define the shares of each dollar of public investment that come at the expense of privately financed investment

$$\left[ \frac{\partial t^f}{\partial t} \left( \frac{\partial t^f}{\partial t} + \frac{\partial Y^p}{\partial t} \right) \right]$$

and consumption

$$\left[ \frac{\partial Y^p}{\partial t} \left( \frac{\partial t^f}{\partial t} + \frac{\partial Y^p}{\partial t} \right) \right].$$

These capital-market-determined weights, then, are our $\omega$ and $(1 - \omega)$, respectively.

Harberger's analysis has been criticized for two alleged inadequacies, first, that it shares the same problems of all analyses based on concepts of consumers' and producers' surplus. This point is obviously invalid; even a cursory reading indicates that Harberger's graphical analysis is merely a pedagogical guide to the intuition behind his derivation of $\omega$. A second criticism is that the model is not of a sufficiently "general equilibrium" nature. There are three senses in which this latter point has some substance. First, Harberger assumes private-sector maximization behavior, rather than explicitly incorporating it into his analysis. Second, and more important, the approach is concerned only with "sourcing"—the raising of funds—although the manner in which they are to be spent can clearly

4 Of course, Harberger's results extend to any number of distorted investment and/or consumption activities that might be affected by the change in government borrowing. For our purposes, however, such an elaboration is unnecessary.
affect private expenditure decisions. And third, no account is taken of income-saving relationships.

Recent and independent work by Sandmo and Dreze (SD) has produced results identical to Harberger’s, eliminating, in their 1971 paper, the first substantive objection above. Using a two-period analysis of utility-maximizing consumers and profit-maximizing firms, SD show that, if the government’s objective is to choose its level of investment so as to maximize the economy’s utility function, subject to the government’s second-period budget constraint, the resulting first-order condition is:

\[
(1 + r)\left[ \frac{g(C'y)}{[g(C'y) + g'(C'y)]} \right] + \frac{1 + r(1 - t)}{[g(C'y) + g'(C'y)]} = 1 + \rho,
\]

where (in their notation) \( g(c) \) is the public-sector investment function, \( r \) is the consumption rate of interest, \( C \) is first-period consumption, \( s \) is the rate of tax on profits, \( y \) is the level of privately financed investment, and \( \rho \) is defined as the social rate of discount. Thus, relying on the same capital-market mechanism, SD also find that \( \omega \) is the same weighted average (in our notation, \( \rho = r(1 - t) \) and \( 1 - \beta = \int [g(C'y) + g'(C'y)] \left[ \frac{g(C'y)}{[g(C'y) + g'(C'y)]} \right] \). However, the existence of Harberger’s demonstration (to take the first) of this fact has not removed the controversy, as proponents of the exclusive use of \( s \) have not yet been convinced. Perhaps the analyst most often cited in support of this latter position is Stephen A. Marglin, particularly with respect to his two papers, “The Social Rate of Discount and the Optimal Rate of Investment” (1965a) and “The Opportunity Costs of Public Investment” (1963b).

One substantive difference—the definition of \( s \)—between models that generate \( \omega \) and Marglin’s approach can be dismissed as immaterial in the current context. For Marglin (1965a), \( r \) is the “social rate of time preference” whereas Harberger and SD presume that \( r \) is best approximated by the net-of-tax yield on savings. Whether the social rate of time preference systematically differs from (an appropriately weighted average of) after-tax rates of return to saving will not be examined in what follows as the precise choice of \( r \) can affect only quantitative results. As we are concerned with the conceptual issue of defining the social discount rate in the context of capital market distortions, it is sufficient for our purposes simply to view \( r \) as the appropriate rate for discounting future consumption. For simplicity, however, we shall continue to treat \( r \) as the after-tax yield on savings.

Aside from this relatively minor point—minor in terms of the controversy as to the appropriate social rate of discount—reliance on Marglin’s system as an alternative to analyses which yield \( \omega \) is misplaced. For perpetuities the two approaches yield identical investment criteria. Marglin’s general expression for the net present value of a public-sector project is:

\[
\int_{0}^{\infty} B(x, t) e^{-\beta_x} dt = \alpha K(x),
\]

where \( B \) measures future (net) project benefits as a function of the scale of investment \( x \) and time, \( K \) is the initial period capital cost, and \( \alpha \) is the social opportunity cost per dollar (shadow price) of public investment. If the economy is undistorted, \( \alpha = 1 \); if there are capital market distortions, however, \( \alpha > 1 \) is an “adjustment factor” to reflect the fact that each
Marglin's analysis and those that generate the appropriate discount rate for public projects, yield results very similar to those of the private sector methods. This is because public projects are, in most cases, subject to the same conditions that govern private projects. The difference lies in the relative size of the public sector's capital stock and the amount of public investment financed by the government. In order to determine the present value of a public project, one must first determine the present value of the capital stock. The present value of the capital stock is then added to the present value of the benefits of the project to obtain the total present value of the project.

The present value of the capital stock can be calculated by discounting the future stream of benefits. The discount rate used is the rate of interest that equates the present value of the benefits to the present value of the capital stock. If the discount rate is too high, the present value of the capital stock will be too low, and the project will be rejected. If the discount rate is too low, the present value of the capital stock will be too high, and the project will be accepted.

The benefits of the project consist of the present value of the increased output, increased employment, and increased consumer surplus. The present value of the increased output is calculated by multiplying the annual increase in output by the marginal revenue product of the increased output. The present value of increased employment is calculated by multiplying the annual increase in employment by the marginal benefit of employment. The present value of increased consumer surplus is calculated by multiplying the annual increase in consumer surplus by the marginal benefit of consumer surplus.

The discount rate used in calculating the present value of the capital stock is the real rate of interest. The real rate of interest is the nominal rate of interest minus the inflation rate. The inflation rate is the rate at which the price level is expected to increase in the future.

Marglin's analysis also takes into account the fact that public projects are subject to political influences. The political process can affect the discount rate that is used to determine the present value of the capital stock. For example, a project that is politically popular may be given a lower discount rate than a project that is politically unpopular.

Marglin's analysis also considers the fact that public projects are subject to negative externalities. A negative externality is a cost that is imposed on third parties by an activity. For example, a project that generates pollution will impose a cost on those who are affected by the pollution.

The present value of the benefits of a public project must be compared to the present value of the costs of the project. The costs of a public project include the present value of the capital stock, the present value of the increased output, increased employment, and increased consumer surplus, and the present value of the negative externalities. If the present value of the benefits of the project is greater than the present value of the costs of the project, the project should be accepted. If the present value of the benefits of the project is less than the present value of the costs of the project, the project should be rejected.

In the end, Marglin's analysis provides a framework for evaluating public projects. The analysis takes into account the economic and political factors that affect public projects. The analysis also considers the benefits and costs of public projects.

References:
Simplified allocation of depreciation between consumption and investment. If Marglin's assumption holds, the above implications clearly follow. On the other hand, it is our contention that individuals maximizing welfare over time will attempt to distinguish net income from depreciation, and hence will not intentionally consume depreciation simply because it is available. It is this explicit behavioral assumption that distinguishes (and extends) our understanding of a well-functioning capital market from that of Harberger and Sandmo-Dreez.

At least two qualifications are in order at this point. First, general access to the capital market as a means of disposal of depreciation is an important restriction on the finding that $w$ is the social rate of discount. Second, while it is perhaps relatively simple for owners of private firms to distinguish depreciation from net output, it is by no means obvious that consumers can do so with the same ease when the output is produced in the public sector. If consumers have no way of identifying depreciation and if the government does not intervene with reinvestment, then consumers will likely, if unwittingly, consume depreciation in its entirety, and we find ourselves in the world of Marglin's simplest model. At the other extreme is the idea implicitly underlying the social-discount-rate approach, that consumers do find it possible to identify correctly the depreciation component of gross output. Persons who argue that the truth lies somewhere in the middle must be prepared to accept the direct implication that the rate of discount must be higher the shorter is the life of the project, as short-term projects will generally lead to capital consumption at an earlier date than will long-lived projects.

Whatever position is accepted, our analysis indicates that reinvestment does critically influence the outcome when projects have finite lives; the reason is not that saving is superior or inferior to consumption (at the margin), but rather that saving is transformed by the capital market into private-sector investment which earns a rate of return greater than the consumption rate of interest. The reinvestment issue is clearly

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11 Although we are purposely abstracting from distribution issues in this paper, we do recognize that some or all of the recipients of public-project benefits may not avail themselves to the capital market. If some beneficiaries are consuming all of their income and would consume more if possible, presumably at least some of the project's depreciation will be consumed in its entirety without being channeled back to the capital market.

12 The ambiguity concerning depreciation is precisely the problem that comes up in the context of project profiles with multiple internal rates of return. This issue is treated in the appendix referred to in n. 11.
twofold; first, how the community intends to distribute—and ultimately does distribute—depreciation between consumption and (re)investment; and second, the treatment of increments to net income created by a public-sector project. We now turn to this second issue.

C. Reinvestment of Net Project Output

While our discussion of the first reinvestment issue—one that has apparently not received previous attention in the literature—has revealed that the difference between the Margin and the Harberger-SD approach is at once superficial (for perpetuities) and fundamental (for finite-lived projects), our examination of the second issue—the allocation of "new" income between consumption and saving—exposes a conceptual weakness in the social-discount-rate approach. Although both Harberger and SD posit positive private-sector saving and investment, neither explicitly explores the implications of the fact that an acceptable public-sector investment must produce a change in income—and hence potential investment—in at least one period. On the other hand, Margin devotes a great deal of his formal analysis to this issue. We find it pointless, however, to examine his models in detail because he treats privately financed investment (and saving) as being totally independent of capital-market phenomena; that is, in his model(s) private investment passively conforms to saving, which in turn is determined in an ad hoc manner without reference to the capital market. 12

It has been argued that a well-functioning capital market renders current public-investment decisions logically independent of future investment possibilities. However, if project-induced reinvestment has measurable capital-market effects—if it alters the path of privately financed investment—then the social rate of discount can be required to take these effects into account. We shall illustrate an appropriate procedure for doing so, again with the two polar cases of a perpetuity and a two-period project. For both cases, we assume that the public sector invests $1.00 in period zero and nothing thereafter.

For the case of a perpetuity, the public project generates the following effects in the investment period: $\Delta C = -(1 - \delta), \Delta Y = 0, \Delta I = 1$, where the subscripts refer to the time period. 13 Continuing, $\Delta Y = \delta - \delta p, \Delta Y = \delta, \Delta I = \delta I = 0(ma), \Delta C = \Delta Y(1 - ma), \frac{\delta}{\delta}$

and such a choice is obviously voluntary rather than dictated by a budget (or by some other, artificial) constraint—then the presumption must be that they consider themselves to be better off as a consequence. Hence, our procedure of comparing the present value of change in future income with current consumption forgone implies a sufficient condition for project acceptance. 12 Jedidah Margin at one point identifies a reinvestment parameter with the marginal propensity to save (1965a, p. 282).

13 Recall that the operator & refers to differences measured from the values that would have existed in the absence of the publicly financed project.
social rate of discount. Letting \( c^* \) be the social discount rate adjusted for the effect of reinforcement on output, we derive:

\[
(5) c^* = \frac{(1 - \rho) - (1 - \phi)(1 - \rho) + \phi}{(1 - \phi)(1 - \rho) + \phi}.
\]

Having demonstrated that reinforcement does have a qualitative effect, we now measure the adjustment that must be made to the social rate of discount to account for the effect of reinforcement on output. We have seen that the adjustment for the effect of reinforcement is likely to be quite small, possibly smaller than the adjustment for the effect of capital stock. The adjustment for the effect of reinforcement is also likely to be small, and the adjustment for the effect of capital stock is likely to be relatively large. The adjustment for the effect of reinforcement is larger than the adjustment for the effect of capital stock because the latter is based on the change in capital stock, while the former is not. The adjustment for the effect of reinforcement is also larger than the adjustment for the effect of capital stock because the latter is based on the change in capital stock, while the former is based on the change in output.

In the case of project (a), for example, the adjustment for the effect of capital stock is relatively small, and the adjustment for the effect of reinforcement is relatively large. The adjustment for the effect of capital stock is smaller than the adjustment for the effect of reinforcement because the latter is based on the change in capital stock, while the former is based on the change in output. The adjustment for the effect of capital stock is also smaller than the adjustment for the effect of reinforcement because the latter is based on the change in capital stock, while the former is based on the change in output.

In the case of project (b), for example, the adjustment for the effect of capital stock is relatively small, and the adjustment for the effect of reinforcement is relatively large. The adjustment for the effect of capital stock is smaller than the adjustment for the effect of reinforcement because the latter is based on the change in capital stock, while the former is based on the change in output. The adjustment for the effect of capital stock is also smaller than the adjustment for the effect of reinforcement because the latter is based on the change in capital stock, while the former is based on the change in output.

In the case of project (c), for example, the adjustment for the effect of capital stock is relatively small, and the adjustment for the effect of reinforcement is relatively large. The adjustment for the effect of capital stock is smaller than the adjustment for the effect of reinforcement because the latter is based on the change in capital stock, while the former is based on the change in output. The adjustment for the effect of capital stock is also smaller than the adjustment for the effect of reinforcement because the latter is based on the change in capital stock, while the former is based on the change in output.
Nevertheless, it is to be emphasized that this latter procedure is a strictly equivalent alternative to discounting the nonaugmented benefit stream by \( \omega \). Discounting by \( \omega \) is appropriate if the benefits from reinvestment are not included in the calculation of a given project's net benefit stream; if these additional benefits are included, discounting by \( \omega \) is inappropriate. The important point is that our extra term in the social rate of discount or in the net benefit stream—arises solely because the project causes consumption to fall in the initial period and income and consumption to rise in at least one following period; reinvestment arises, and cannot be ignored, precisely because (and only because) not all of the increase in income need be immediately consumed.

The last point to be made in this section, one that is rather obvious but nevertheless suffers neglect, concerns the permanency of the effects of public-sector investment. One of the key difficulties of fiscal policy in general is that the public nearly always has the capability to offset or negate any effect of government action by altering its own consumption and/or investment behavior. In the case of public-sector investment, a marginal project (\( \delta = \omega \)) will alter the voluntarily chosen paths of present consumption and investment only if the project is a公益活动 or if it generates reinvestment. Otherwise, consumption will fall by \((1 - \delta)\) and private investment by \(\delta\) in the initial period; subsequently, consumption will increase by precisely the amount necessary—\((1 + r)(1 - \delta)\) in the two-period case—to compensate consumers, and private investment will be replenished by \(\delta\), leaving the path of future income unaltered. That the effects of such a project are purely transitory is guaranteed by our concept of gross and net income, plus the existence of a well-functioning capital market.

III. The Social Opportunity Cost of Public Funds

One of the more important results from the previous section is the general equivalence of shadow pricing capital-good inputs & la Marglin and discounting future flows at the social opportunity cost of capital. We now turn to an examination of these concepts within the broader framework of public finance and expenditure in general, in contrast to the more narrow focus of both prior approaches as far as they have been developed in the literature. The process reveals the additional costs and benefits to be included in a comprehensive cost-benefit analysis of all public expenditure, be it for consumption or investment purposes.11

While neither the social-discount-rate approach nor the shadow pricing of inputs has an a priori claim as the theoretically superior

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11 For simplicity we now abstract from the reinvestment issue in order to concentrate our attention on other aspects.

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12 There are two types of shadow pricing in this context; the first is always applicable to project inputs and outputs and is necessary because of distortions in the markets in which these goods are traded. The second type of shadow pricing—the one we associate with Marglin—is in our framework a further adjustment on prices of inputs arising from distortions in the market for financial capital. Only this second adjustment is of concern in this paper.
Finally, the investment and consumption rates of interest are assumed (for simplicity) to be linked by a positive constant such that $dp = dr$.

In the context of cost-benefit analysis, it is generally assumed that public expenditure comes at the expense of private consumption and investment at a given rate of real output. Thus, when considering conventional cost-benefit analysis in this macroeconomic context, we assume that the authorities adjust the money supply to maintain contemporary aggregate demand ($\gamma_y$) at a constant level, even in the face of changes in public finance and expenditure.

Now, given the above model, the effect on the interest rate of simply extracting funds from the capital market can be found by differentiating equation (6) partially with respect to $b$ and setting the result equal to minus unity.\(^{12}\)

$$\frac{dy}{db} = -1 = f(\gamma' - \phi') + \frac{f_0(b)}{f_0'(b)}; f_0 > 0, f' < 0.$$  

Thus:

$$\frac{dy}{db} = (\psi f_0 - 1)/f_0,$$ \hspace{1cm} (9)

where $\psi$ is the marginal propensity of "spend" and $f_0 = \frac{dc}{dr} + \frac{d\gamma}{d\gamma}$.\(^{13}\) The interest rate effect of tax finance is obtained in the same manner: $dy/d\alpha = -1 = -f_0 + f_0(b/\alpha)$, or:

$$\frac{dy}{d\alpha} = (f_0 - 1)/f_0.$$ \hspace{1cm} (10)

Equality of equations (9) and (10) is tautological when $\psi = 1$; that is, if there is no "burden" of the debt. The proposition that tax and bond finance of government expenditure may have identical effects is one that has long been a part of public-finance literature and tradition.\(^{14}\)

If increases in the public debt are to pose no "burden" for future generations, it must be true that the citizens fail to distinguish between tax and bond finance in determining its consumption and investment behavior. If this be the case, it obviously follows that tax multipliers are exactly zero.\(^{15}\)

Returning to equation (9), it is central to all three derivations of the social discount rate, \(\alpha\) (Section 11), that the effect of sourcing on the rate of interest be \(-(1/\alpha)\); we see that this will be true only if $f_0' \equiv 0$ is zero.

\(^{14}\) In order for the funds to be obtained at a constant real output, saving must rise relative to investment by exactly one unit, and hence aggregate expenditure must decline by one unit.

\(^{15}\) We assume for simplicity that public-sector spending is completely interest inelastic so that we can continue to interpret $\alpha$ and $\alpha$ as privately financed consumption and investment, respectively. Note also that $\phi = (d\gamma(b))/\alpha$.

\(^{16}\) This proposition has recently been brought once again to the attention of monetary economists by Barro (1974). While Barro's technical treatment of the issue is no less than novel, it leads to no new insights (see, e.g., Bailey 1962, or Hauser 1964).

\(^{17}\) The equivalence of tax and debt finance is also argued by Harberger and Sandmo-This view is in connection with the social opportunity cost of capital. Their position arises from the explicit (or implicit) assumption that investment in the capital market is a permissible use of tax revenues.
Assuming \( f_r = 0 \), there appears to be no compelling logic nor empirical evidence supporting the extreme assumption that \( f_r \) is zero. On the other hand, if \( f_r = f_c = 1 \), then \( dR/db = 0 \) because all new bond issues are voluntarily purchased at the expense of private consumption and/or investment without changes in the interest rate. This extreme sensitivity of spending behavior with respect to the size of the public debt places debt and tax finance on equal footing insofar as interest-rate effects are concerned and, if this were indeed the case, the social rate of discount would change drastically. Assuming that the direct effect of bond finance falls entirely on consumption, we have: \( dC/db = -f_c f_r + (dC/db)(dR/db) \), and from this result it is straightforward to derive the social discount rate: \( r = \rho(1 - f_c f_r) + \rho(1 - f_c) \) if \( f_r = 1 \), the weight received by \( \rho \) is \( \rho(1 - f_c) \) rather than \( \rho \); the effects are dramatic as presumably \( f_r \) is a rather large fraction, perhaps near unity, \( \nu \).

At this point it is convenient to examine the Harberger (1973, p. 111) and Dreze (1974, p. 60) contention that tax and bond finance have equal social opportunity costs. Their argument does not require identical interest-rate effects associated with the two sourcing operations but rather is based on the idea that the option of debt retirement or direct investment in the capital market constitutes a viable alternative use of tax revenue, and that this option would yield a social benefit identical with the social cost of extracting funds from the capital market. That is, while the total social cost of raising funds may well depend upon the mode of finance, the opportunity cost of the use of those funds is determined solely in the capital market. \( \nu \) For this argument to be correct, it is necessary that the interest-rate effect of taxation be zero—that is, that the cost of using tax funds for a project consist only of the loss of benefits that could be obtained by investing those funds in the capital market. \( \nu \) In terms of equation (10), the requirement is that \( f_c \) be exactly unity; if this is not the case—if taxation itself has interest-rate effects—then the option of placing tax

\[ dR/db = -(1/f_c)(f_r(1 - f_r) dG/db - (1 - k)) + 1 + f_r - k. \]

\( \nu \) The interpretation of all terms of (11) is obvious, apart from \( f_r \). That term represents the direct effect of public outlays on privately financed expenditure; that is, it reflects the broadening of the opportunity set for both private consumption and investment as a consequence of increased public-sector expenditure. It does not reflect any competitive displacement of privately financed consumption or investment by public-sector activity; that effect is fully captured in the final term \( -k \). \( \nu \) Hence, \( f_r \) is assumed to be nonnegative.

\[ \nu \] This is easily seen by assuming that the public sector produces a good already privately purchased; if tax financed, we expect that total expenditure will remain constant. Holding output \( J \), the interest rate \( f \), and government borrowing \( b \) constant, the change in total expenditure is \( f_r + f_c + b + k \) if the government is perceived to produce its output as efficiently as the private sector, then \( f_r = 0 = 0, f_c = 0. \]
The social cost of unavoidable expenditure, defined as the cost of inapplicable or non-reducing expenditure, is widely recognized as a key factor in the determination of private costs and the optimal allocation of resources.

The formula for calculating the social cost of unavoidable expenditure is:

\[ \frac{\text{Social Cost}}{\text{Unavoidable Expenditure}} = \frac{\text{Cost of Inapplicable Expenditure}}{\text{Non-Reducing Expenditure}} \]

This formula reflects the idea that the social cost of unavoidable expenditure is directly proportional to the cost of inapplicable expenditure and inversely proportional to the non-reducing expenditure.

The social cost of unavoidable expenditure, however, is not only influenced by the cost of inapplicable expenditure and non-reducing expenditure but also by the economic conditions and the historical context of the expenditure.

In summary, the social cost of unavoidable expenditure is a complex and multifaceted concept that requires careful consideration in the allocation of resources and the optimization of economic activities.

References:


JOURNAL OF POLITICAL ECONOMY

or a weighted average of $r$ and $w$. Similarly simplified, expression (15) becomes:

$$\delta \geq r + w(1 - \phi),$$

(15')

a weighted average of $r$ and $w$.

The critical coefficient is $\phi'$, which lies in the zero-unit interval. If we assume that $\phi' = 0$, then our conditions become merely $\delta \geq \phi$, which are the Marglin and Harberger, Sandoz-Dreze definitions of the shadow cost of capital and the social rate of discount, respectively.

The striking aspect of this result is that it refers to a consumption project: the conditions that lead us to accept $w$ as the social rate of discount are precisely those that require us to use $r$ as the shadow price of bond-financed, public-sector consumption. This conclusion is a direct but apparently

inappropriately appreciated implication of both the shadow-price and the social-rate approaches.

If, on the other hand, $\phi' = 1$, expressions (14) and (15) become:

$$\delta \geq r + \phi,$$

and

$$\delta \geq w.$$

In this case, there is no need to shadow price inputs and the social rate of discount becomes merely the consumption rate of interest. The investment rate of interest is irrelevant, as bond finance does not in this case affect private investment.

In concluding this section, we return to the issue of the equivalence of debt and tax finance. We have seen that, if investment in the capital markets is a permissible use of tax revenues and if society in its current spending behavior is insensitive to the size of the public debt, then both debt and tax finance of public expenditure for any purpose must bear an inordinate cost. A direct implication of that cost is that taxes should be increased with the proceeds being invested in the capital market until the marginal social collection cost just balances the benefits of expanding the distorted activity (privately financed investment). Neglecting the burden of tax-collection costs, the benefit of this operation, measured in terms of present value (at constant real output), is $[\delta p(1 + \phi r)] + [3C(\phi)]dr = [\delta p(1 + (1 - \phi))f, dr$ per dollar of debt retirement. The resultant change in the interest rate is given by subtracting equation (9) from (10) which reduces to $f, (1 - \phi)f,\delta r$, and hence the benefit is $f, (1 - \phi)\delta r$ per dollar of debt retirement.

But this result carries an immediate implication which, in a very real sense, brings us back to the true meaning of second-best analysis: a

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17 If, e.g., $r = 0.05$ and $\phi$ being such that $w = 0.10$, then $\delta = 2$, implying that bond-financed consumption expenditures must register an excess of costs over costs of at least 100 percent.

18 This is equivalent to the case analysed earlier in this section in which all bonds are voluntarily purchased at the expense of consumption.

19 Equations (9) and (16) were derived assuming $\delta f, = \delta r[1 - (\delta f,)] - 1$. The interest-rate effect obtained above for tax-financed debt retirement requires only that $\delta f, = \delta r[1] - 1$; hence it is consistent with constant real output.

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IV. A Cost-Benefit Analysis of Fiscal Policy

We now turn our attention to the social value of fiscal policy and its influence on the level of economic activity. The central point in what follows is that the appropriate measure of the efficiency of fiscal policy is not the magnitude of any fiscal multiplier but rather the social value to be attached to incremental output—current and future—attributable to fiscal measures. To maintain the framework of the preceding analysis, we will focus primarily on the impact of bond finance but also investigate the differential effects of other modes of fiscal policy.

To confine the analysis to pure fiscal policy, the passive money introduced in Section III is replaced by a monetary policy independent of both public finance and public-sector expenditure. In particular, the nominal stock of money is taken as fixed. We assume further that real output can expand or contract freely at a constant price level in response to fiscal stimulus, subject only to the constraint that the demand for money equal the supply. We accept the dubious realism of this assumption in order to examine the case most favorable to fiscal policy.

Utilizing equations (4)–(8) and fixing $dt = 0$ and $dw = \delta r$, we obtain the following expression for the change in expenditure and output:

$$\phi' = \phi - f, (1 + \phi) + f, (1 - \phi)\delta r + f, (1 - \phi)\delta r.$$  

As the nominal (and hence real) money stock is constant, equality of demand for and supply of money implies: $dr = (L_2 - L_1)f, \delta r$, where $L_2$ and $L_1$ are the partial derivatives of the demand for (real) cash balances with respect to output and the rate of interest, respectively. Combining these two equations, we have:

$$\phi(\delta r) = \phi - f, (1 + \phi) + f, (1 - \phi)\delta r + f, (1 - \phi)\delta r.$$  

where $\phi^* = f,$. The response of the interest rate to changes in public-sector spending is:

$$\delta r = \phi^* - \delta r = \delta r(L_2 - L_1)f, \delta r - \delta r(L_1 - L_2)f, \delta r.$$  

and $f, \delta r = (f, (L_2 - L_1))\delta r - \delta r$ where $\beta$ is a positive constant. We can now evaluate fiscal policy in terms of this simple model. The approach

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46 Harberger (1979a, p. 121, n. 13) makes essentially the same point, although in a rather different context.
uses the decline in privately financed expenditure, evaluated at shadow prices, to measure the "cost" of the project output. This cost can, of course, be negative if both private and public output could expand. We emphasize at the outset that this measure of cost does not include the nonmarket opportunity cost of the resources engaged to produce an increase in global output.

The change in output \( \Delta y \) associated with an increase in public-sector spending \( \Delta d_0 \) is obviously divided between global consumption and global investment. The change in privately financed expenditure is \( \Delta f \) less the change in the value (to users) of the public-sector output, \( \Delta k = k'(\Delta d_0) \). In terms of (the differentiated form of) equation (7) of Section III:

\[
\Delta y = (df - k'd_0) + \Delta d_0
\]  

(7)

The term in parentheses is the change in privately financed consumption and investment, which expands to:

\[
df - k'd_0 = f_s db^* + f_d dt + f_d + f_d k'd_0 = f_s db^* + f_d dt + f_d + f_d k'd_0
\]  

(18)

The bracketed term of equation (18) is the change in privately financed spending directly induced by the change in perceived income, corrected for the direct substitution of public-sector for privately financed output: this change in output is assigned the shadow price of \( b^* \) as defined in Section III. The second term of equation (18) is privately financed spending brought about by the interest-rate effect and is assigned the usual shadow price of \( d_0 \). The final term captures spending induced by a broadening of the opportunity set, whose shadow price is \( d^* \), also defined in Section III.

The shadow price of the project output, \( k' \), is now defined as \( a_k^* \):

\[
a_k = a_k^*(f_s (db/d_0) + f_d (d'E - d') - k' + f_d (db/d_0) + a_k^* r d_0
\]  

A definitive evaluation of \( a_k^* \) is probably impossible, owing to the large number of coefficients to be evaluated; but some illustrative calculations, based upon plausible orders of magnitude, yield rather startling results.\(^{34}\) An assumed value for \( \beta \) can be obtained by writing that coefficient as follows:

\[
\beta = \frac{f_s L_t}{L_t} = \frac{f_s}{f_s + f_d + f_d k'd_0 - \Delta k_0
\]  

(19)

To evaluate eq. (19) we used estimates of \( a, b, c, \phi, \phi', \phi^*, f_s, f_d, K, \phi' \) and \( \beta \). Not only is the estimation problem formidable, but also the magnitude of none of the coefficients will respond to the specific nature of the public-sector expenditure.

\[^{34}\text{To evaluate eq. (19) we used estimates of } a, b, c, \phi, \phi', \phi^*, f_s, f_d, K, \phi' \text{ and } \beta. \text{ Not only is the estimation problem formidable, but also the magnitude of none of the coefficients will respond to the specific nature of the public-sector expenditure.}\]

\[^{35}\text{The value of } f_s \text{ is approximately unity (exactly so if } h(g'(z) = g) \text{ and the income elasticity of demand for real cash balances is widely thought to be at least unity—some estimates are well in excess of unity. It is plausible that the semielasticity of expenditure and of the demand for money with respect to the interest rate are of the same order of magnitude; therefore we find it reasonable to expect that } \beta \text{ has a value of at least unity. We also assume that } f_s, \text{ the marginal propensity to spend, is unity. This rather high value of } f_s \text{ reflects a liberal allowance for accelerator effects and in addition, for immediate purposes, has the side benefit that } \phi' \text{ disappears from the multiplier } \mu. \text{ Evaluating } a_k^* \text{ accordingly, we obtain:}

\[
a_k^* = (a - a^*) (1 + \phi + a^* \phi' - \Delta f_0
\]  

(19')

\(^{36}\text{We now proceed to obtain the range of values for } a_k^* \text{ in terms of } a, a^*, \text{ and } a^* \text{ for various limiting values of } \phi' \text{ and } f_s. \text{ The coefficient } \phi' \text{ clearly lies in the zero–unity interval and we make the same assumption with respect to } f_s \text{ although clearly only an extraordinary } \phi' \text{ will create an equal amount of private-sector expenditure in response to a broadening of the opportunity set for consumption and/or investment. Taking all combinations of the limiting values for } \phi' \text{ and } f_s \text{ we have calculated } \mu, \phi_k, \phi_k^* \text{ and } a_k^* \text{ in terms of } a, a^*, \text{ and } a^*; \text{ the results are presented in table 1.}

\[a_k^* = (a - a^*) (1 + \phi + a^* \phi' - \Delta f_0
\]  

(19')

\[^{36}\text{The shadow price of } a_k^* \text{ cannot exceed } a_k^* \text{ as the latter is defined for constant (contemporaneous) output, whereas the former permits global consumption and investment to respond to fiscal stimulus. The two differ, of course, by exactly the social value, } a_k^*, \text{ of the increase in global spending. It is again emphasized that neither } a_k^* \text{ nor the difference between } a_k^* \text{ and } a_k^* \text{ represented in table 1 include the social value of the resources required to produce the additional output. Further evaluation of } a_k^* \text{ and } a_k^* \text{ requires knowledge of } a, a^*, \text{ and } a^*, \text{ all of which range from unity to } \rho h. \text{ Given the distortions in the capital market in most countries, it is plausible that } \rho \text{ is at least twice } r \text{ and hence}
\]
the range for these coefficients is assumed to be from one to two. Recalling
the definitions of these coefficients, we can now write them as: $a = 1 + \beta$; $a' = 1 + \beta'$; and $a'' = 1 + \beta''$.

In the case of public-consumption projects it is reasonable, as was
explained in the previous section, to assume that $\theta^* \approx 0$.

Further, such projects will be likely to offer but limited scope for new
privately financed investment; hence $\theta' \approx 0$. For
such projects, then, we assume that both $a'\theta$ and $a''\theta$ are approximately
unity. If in addition $\theta$ is approximately unity (consumption very interest
inelastic), it follows that $a$ is approximately unity for all four cases con-
templated in table 1. As $a$ is to be interpreted as the minimum acceptable
social value of the direct output of a project, we conclude that consumption-
oriented public spending must at least "pay its own way," in the sense
that the value of the output in the eyes of the beneficiaries must
at least as large as the costs (measured at market prices), even if the shadow
price of resources is zero.\footnote{The prices here referred to are already corrected for the traditional distortions such
as taxes and monopoly/monopsony, but are not corrected for differences between-
market opportunity cost and market prices arising from involuntary unemploy-}

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But the central result obtained by LM does not depend upon the reason for a wedge between the investment and consumption rates of interest, only that a gap exist. Given the widespread taxation of income in general and income from capital in particular, their concept must be examined further.

The LM analysis rests on their contention that (at least part of) the projects-induced increment to the aggregate wage bill is consumed when it could be invoiced. **Defining units of labor such that its opportunity cost (designated "w" in the LM equations) is unity, their formulation of the shadow wage is:

\[ w = 1 + (c - 1)(r - 1)/r, \]  
(20)

where \( r \) is the per-worker consumption by project employees and \( c \) is the value of investment relative to consumption (i.e., the value of the consumption stream, discounted by \( r \), that can be generated by investing \$1.00 today) (Little and Mirrlees 1969, p. 162). As it is assumed that \( c - 1 \) is positive, the shadow wage exceeds the opportunity cost of labor whenever \( r \) exceeds \( r \); note, however, that the LM shadow wage is bounded by the marginal product of labor in agriculture (unity) and the project wage (the assumed upper limit for \( r \)).

An obvious and natural interpretation of equation (20) is to treat \( c - 1 \) as the change in the level of a distorted activity and \( r - 1 \) as the distortion. The coefficient \( r \) is, of course, equal to \( p/r \) — the present value of a perpetual stream of consumption equal to \( p \) discounted by \( r \) — and thus is equal to Marglin's \( x \) when saving is completely interest inelastic (\( \delta = 1 \)), which would appear to be the implicit LM assumption.

Thus we can write equation (20) as:

\[ w = 1 + (c - 1)(p - r)/r. \]  
(21)

It is clear that LM are measuring wages in terms of current consumption goods, but from equation (21) it is equally clear that they are normalizing the distortion (\( c - 1 \)) by \( a \), the consumption value of investment and are

**The rise in the aggregate wage bill comes about because the project wage, for unspecified reasons, exceeds the marginal product of labor.**

**It is unlikely to be unidimensional that the LM adjustment to the forgone marginal product of labor in alternative pursuits was inspired by a clearly perceived need for a shadow wage rate of an order of magnitude similar to that of the market wage.**

As Harberger (1973a) has ingeniously pointed out, the difference between the market and the shadow wage is, from the social point of view, a transfer from capital to labor and thus is part of the (social) return to capital. If that difference is large, as occurs when the social cost of labor is the (assumed zero) marginal product of agricultural labor, the discount rate becomes very high; indeed so high that the more capital-intensive, showpiece projects so much in vogue in certain backward countries are impossible to justify with conventional cost-benefit analysis.

**The correct measure for the shadow wage emerges from our approach in Section III in which consumption is the numerator. Given \( \delta = 1 \), the externality associated with consumption is \( (p/r - 1) \), which is simply the product of the associated change in investment (minus one) and the present value of the distortion associated with that activity measured in terms of consumption goods. The negative sign indicates that consumption reduces potential welfare and hence can be dropped when we view the externality as a cost. Thus the corrected LM shadow wage is:

\[ w = 1 + (c - 1)(p/r - 1) \]

\[ w = 1 + (c - 1)(a - 1), \]  
(22)

which differs from equation (20) only in that the distortion is measured in current consumption rather than investment.

Now \( x \), the value of investment measured in consumption, is presumably at least unity (cases where \( p < r \) are uninteresting) but, since it has no upper limits, the correct representation of the LM result is unbounded from above. Thus the corrected result does not possess the highly convenient but curious property of the original formulation (eq. [20]) that, as the consumption rate of interest approaches zero, the distortion approaches unity and the shadow wage approaches the project wage (\( c \)). With the revised formulation (eq. [22]), both the distortion and the shadow wage increase without limit as \( p \) approaches zero. This is what it should be; if in fact there is no time preference and if there exist opportunities to invest at a positive rate of return, then to detract from such investments poses a cost the present value of which can be arbitrarily large.

Finally, the corrected result (eq. [22]) does not depend upon the value of \( \delta \). Although LM argue that all increments in consumption are at the expense of investment, they posit no realistic argument to validate the assumption. If the project is unsubsidized, measured output of the economy is increased by the return to capital plus the excess of the wage bill over the marginal product forgone elsewhere, permitting an increase in consumption with no reduction in investment. But the wage bill must be financed by selling the output, increasing taxes, or selling bonds. As is clear from the analysis of Section IV, each of these alternatives has effects on both consumption and investment, and hence we should replace \( c - 1 \) in equation (22) with the net change in consumption, or alternatively attach to \( c - 1 \) an externality which reflects the fact that some fraction of the funds generated to pay the wage bill come at the cost of additional debt.
expense of investment. Clearly $\theta$ is that fraction, so we obtain: $w = 1 + (c - 1)\theta(p/r - 1) = 1 + (c - 1)(\alpha - 1)$, which is identical to equation (22).

Note that when $c = 2$ (project-induced consumption is exactly unity), we obtain the Marglin shadow price of capital inputs: $w = \alpha$. This result should surprise no one; the LM approach penalizes a project for employment of labor which increases consumption at the expense of (potential) investment. The Marglin approach penalizes the employment of capital because, in the absence of a specific reinvestment strategy, the depreciation of that capital will be consumed at the expense of investment.

In summary, Little and Mirrlees have confused the debate over the social cost of labor by (a) failing to post sufficient conditions for a shadow wage different from the market wage and (b) incorrectly defining the shadow wage even when those conditions are met.

VI. Conclusions

Since this paper has covered a great deal of territory, we shall devote this concluding section to a summary and recapitulation of our major findings.

1. Within the narrow confines of traditional cost-benefit analysis, given that such analysis is based on (a) the consumption rate of interest as the only truthfully relevant rate of discount for calculating the present value of future net-benefit flows directly attributable to public-sector projects, but also on (b) the concomitant recognition of the fact that, with the almost universal prevalence of capital market distortions, the investment rate of interest exceeds the consumption rate and therefore that the social opportunity cost per dollar of private investment forgone in order to finance the public project exceeds $1.00, the only possible result as to the social rate of discount ($\omega$) is a weighted average of $p$ and $r$. The requisite weights are, of course, the shares in which each dollar of financing for public-sector investment projects comes, respectively, at the expense of privately financed investment ($\theta$) and private consumption ($1 - \theta$). Our demonstration of this result mirrors (and simplifies) the previous analyses of Harberger and of Sandmo and Dreze.

2. The net-present-value criterion, using $\omega$ as the social rate of discount, and Marglin’s well-known “alternative” criterion of discounting solely by the consumption rate of interest, but adjusting the initial capital cost by the shadow price of capital-goods inputs ($\alpha$), we are shown to be exactly equivalent (a) when the public project generates a perpetuity, and (b) if we cast a capital market onto Marglin’s model in order to impart a modicum of economic behavior to the shadow-price procedure, particularly with respect to the definition of $\omega$. In at least these terms, then, a great deal of the continuing debate that has taken place between the partisans of the two approaches has been meaningless.

3. However, these approaches do significantly differ for public projects with less than infinite lives; yet the crucial difference lies not in the rate at which future net-benefit flows should be discounted—the focus of the last 12-15 years of debate in the literature—but rather in their divergent, implicit assumptions as to the general public’s treatment of the depreciation deriving from governmental financed investment. The philosophy of Marglin’s basic model denies a distinction between depreciation and net output; hence, contrary to the usual interpretation, his criterion is far more stringent than is the social-discount-rate approach for finite-lived projects. On the other hand, the criterion based on $\omega$ as the appropriate rate of discount implicitly assumes that, one way or another, the general public recognizes public-sector depreciation as such and therefore attempts to save rather than consume it; through subsequent capital-market effects, the attempt will be partially unsuccessful, such that the end result will be a global stock of capital that is unaltered (for marginal projects) after the complete execution of the project. Acceptance of the Marglin position requires the heretofore lacking explanation as to why private expenditure behavior should be so widely divergent with respect to private-sector versus public-sector depreciation, in view of the fact that the option of consuming a portion of society’s capital stock always exists with or without government expenditures. On the other hand, acceptance of the social discount rate approach requires a conviction that it is in fact feasible for the public to distinguish between gross and net output of public projects.

4. The foregoing thus represents a fundamental reinvestment issue that has not received previous attention in the literature. The second, and more widely recognized, question concerns the effects that arise from the possibility of reinvesting all or part of the net throw-off from public-sector projects. Owing to the fact that an acceptable public-sector investment must cause global output to be greater than it would have been in at least one subsequent period, we find that the option of investing all or part of that incremental output creates a qualitatively (though in all likelihood not a quantitatively) important adjustment to the social opportunity cost of capital as envisaged by Harberger and by Sandmo and Dreze.

5. When we extend the analysis to a broader, macroeconomic framework, it becomes clear (eq. [12]) that the uniqueness of $\omega$—as is asserted by Harberger and Sandmo-Dreze—depends upon three quite stringent conditions: (i) real output is exogenously determined during the investment period; (ii) there is no direct effect of project execution on privately financed expenditures; (iii) the general public exhibits equal perceptiveness to the value of public output, on the one hand, and to the present value of future tax liabilities arising from governmental bond finance, on the other. Since, even given i, there is no obvious reason to accept ii and iii as generally valid, we relax them in Section III in order to explore
the implications for the general shadow price of public finance. First, we find that, although both the social discount rate and the shadow price of inputs continue to generate equivalent investment criteria, the latter is the more easily extended approach, being applicable to both investment and consumption projects. More importantly, as our equations (14) and (15) indicate, relaxing conditions ii and iii has potentially dramatic effects on either social-evaluation concept. There is no presumptive case to be made for the uniqueness (across varieties of public expenditures) of either concept. However, given that this nonuniqueness depends, in turn, on rather poorly estimable differences—(a - a*) and (a - a) on the one hand, and (b - b*) and (b - b) on the other—it can be argued that assuming conditions (i)-(iii) is not inappropriate for defining a or a*, especially since the implications so conveniently eliminated thereby can be reinstated through the incorporation of other (perhaps "external") costs and/or benefits for each project. Nevertheless, our analysis highlights the importance of identifying and explicitly including all such additional effects within the project-evaluation procedure.

6. One of the important conclusions arising from the macroeconomic framework of analysis is that in general there is no basis for asymmetrical treatment of publicly financed investment and consumption projects. In other words, if we are justified in discounting returns to investment projects at a rate higher than the consumption rate of interest, then we are also justified in charging a shadow price for consumption-project inputs that is higher than the market price. (Ironically, this conclusion follows directly from the Marglin approach as well.) This conclusion is of considerable quantitative significance as the currently encountered arguments for discount rates in the neighborhood of 10 percent (in real terms) also justify a surcharge on consumption project inputs in the neighborhood of 100 percent.

7. In Section IV we relax the assumption of exogenously determined real output in order to evaluate, from the social cost-benefit viewpoint, the effects of anticyclical fiscal policy. The requisite expression for this evaluation (eq. [19]) is too complex for precise estimation, but by choosing plausible values (or ranges) for the underlying parameters we conclude that the case for anticyclical fiscal policy is anything but strong. Even under the most favorable (Keynesian) assumption—an infinitely elastic supply of resources at a zero social cost— we find that it will generally be true that both investment and consumption projects must "pay their own way" in the sense that project-specific benefits must be sufficient to justify project-specific costs.

8. Finally, almost as a digression in Section V we briefly consider—and quickly dispense with—the Little-Mirrlees notions concerning the shadow price of labor. While the Little-Mirrlees analysis can be recast to conform with the conventional treatment of distortions, their case for consumption being a distorted activity is lacking. In addition, they erroneously use investment as the numerator, thus compromising both the qualitative and quantitative aspects of their central result.

References

Marglin, Stephen A. "The Social Rate of Discount and the Optimal Rate of Investment." Q.J.E. 77 (February 1965): 95-111. (c)