Answers to Problem Set 2
Part 1: Problems from textbook

Problem 3, chapter 5 (pg. 109)

(a) The demand functions for the two consumer groups are

\[ X_1 = 200 - P \text{ if } P \leq 200, \text{ and } X_1 = 0 \text{ if } P \geq 200. \]
\[ X_2 = 50 - \frac{1}{2}P \text{ if } P \leq 100, \text{ and } X_2 = 0 \text{ if } P \geq 100 \]

First consider the case when \( P \geq 200 \). In this case, \( X_1 = 0 \) and \( X_2 = 0 \), implying \( X_1 + X_2 = 0 \)

Now, consider \( 100 < P \leq 200 \). In this case, \( X_1 = 200 - P \), and \( X_2 = 0 \), implying \( X_1 + X_2 = 200 - P \).

Next, consider \( 0 \leq P \leq 100 \). In this case, \( X_1 = 200 - P \) and \( X_2 = 50 - \frac{1}{2}P \), implying
\[ X_1 + X_2 = 250 - \frac{3}{2}P \]

(b) First consider the case when \( P \geq 200 \). In this case, \( X_1 + X_2 = 0 \), implying \( \pi = 0 \).

Now, consider \( 100 < P \leq 200 \). In this case, \( X_1 + X_2 = 200 - P \). Therefore,

\[ \pi = PX - 40X \]
\[ = (200 - X)X - 40X \]
\[ = 200X - X^2 - 40X \]
\[ \frac{d\pi}{dX} = 200 - 2X - 40 = 0 \]
\[ \Rightarrow 2X = 160 \]
\[ \Rightarrow X = 80 \]
\[ \Rightarrow P = 120 \]
\[ \Rightarrow \pi = (120)(80) - (40)(80) \]
\[ = 6,400 \]

Next, consider \( 0 \leq P \leq 100 \). In this case, \( X_1 + X_2 = 250 - 1.5P \). Therefore,
\[
\pi = PX - 40X \\
= (166.66 - \frac{2}{3}X)X - 40X \\
= 166.66X - \frac{2}{3}X^2 - 40X \\
\frac{d\pi}{dX} = 166.66 - \frac{4}{3}X - 40 = 0 \\
\rightarrow \frac{4}{3}X = 126.66 \\
\rightarrow X = 95 \\
\rightarrow P = 103 \frac{1}{3} \\
\rightarrow \pi = (103.33)(95) - (40)(95) \\
= 6,016.66
\]

But this solution violates the assumption that P is less than 100, so is not viable. If P is 100, then X = 100 and profits will be 6000. This is less than 6400. The maximum is then for the second case with no sales to group 2. Total profits are equal to 6400.

Consumer surplus is obtained by using the first market only. It is given by

\[
CS = \int_{120}^{200} (200 - x) \, dx \\
= \left[ 200x - \frac{x^2}{2} \right]_{120}^{200} = (40,000 - 20,000) - (24,000 - 7,200) \\
= 20,000 - 16,800 = 3,200
\]

(c) For the first market

\[
\pi_1 = P_1X_1 - 40X_1 \\
= (200 - X_1)X_1 - 40X_1 \\
= 200X_1 - X_1^2 - 40X_1 \\
\frac{d\pi_1}{dX_1} = 200 - 2X_1 - 40 = 0 \\
\rightarrow 2X_1 = 160 \\
\rightarrow X_1 = 80 \\
\rightarrow P = 120 \\
\rightarrow \pi_1 = (120)(80) - (40)(80) \\
= 6,400
\]

For the second market:
\[ \pi_2 = P_2X_2 - 40X_2 \\
= (100 - 2X_2)X_2 - 40X_2 \\
= 100X_2 - 2X_2^2 - 40X_2 \\
\frac{d\pi_2}{dX_2} = 100 - 4X_2 - 40 = 0 \\
\rightarrow 4X_2 = 60 \\
\rightarrow X_2 = 15 \\
\rightarrow P = 70 \\
\rightarrow \pi_2 = (70)(15) - (40)(15) \\
= 450 \]

Consumer Surplus in the second market is given by:

\[ CS = \int_{70}^{100} (50 - 0.5x) \, dx \]

\[ = \left( 50x - \frac{x^2}{4} \right) \bigg|_{70}^{100} = \left( 5000 - 2500 \right) - \left( 3500 - 1225 \right) \]

\[ = 2500 - 2275 = 225 \]

Total profits are then 6850 and total consumer surplus is 3425.

(d) Price discrimination has increased total surplus. This is because, without price discrimination, one market is not served.
Problem 3 – chapter 6 p pg. 131

This is an example of second degree price discrimination, involving multipart pricing. Since the consumers are heterogeneous and the company cannot identify who is who, it offers multiple pricing policies and let the consumers self select.

The pricing reflects quantity discounts – when considering price per weekday minutes.

Average prices

<table>
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<tr>
<th>Plan</th>
<th>Per weekday minutes</th>
<th>Per total minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan 1</td>
<td>0.175</td>
<td>0.010</td>
</tr>
<tr>
<td>Plan 2</td>
<td>0.114</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Problem 4 – chapter 6 – pg. 132

(a) If the club owner cannot price discriminate, she will consider the aggregate demand, which is

\[ Q^{S+A} = (18 - 3P) + (10 - 2P) = 28 - 5P \]

The corresponding inverse demand function is \( P = \frac{28}{5} - \frac{1}{5} Q^{S+A} \) and the marginal revenue is \( MR = \frac{28}{5} - \frac{2}{5} Q^{S+A} \).

Equate marginal revenue with marginal cost, that is, \( \frac{28}{5} - \frac{2}{5} Q^{S+A} = 2 \) \( \Rightarrow Q^{S+A} = 9 \)

Therefore, \( P = \frac{28}{5} - \frac{1}{5} Q^{S+A} = \frac{28}{5} - \frac{9}{5} = \frac{19}{5} \).

Her profit without price discrimination: \( \pi = (P - 2)Q = (\frac{19}{5} - 2)(9) = \frac{(9)^2}{5} = 16.2 \)

(b) If the club owner can price discriminate, she will equate marginal revenue and marginal cost for each group. That is,

\[ 6 - \frac{2}{3} Q^S = 2 \] \( \Rightarrow Q^S = 6 \) \( \Rightarrow P^S = 4 \)

\[ 5 - Q^A = 2 \] \( \Rightarrow Q^A = 3 \) \( \Rightarrow P^A = 3.5 \)

Hence, total profit with price discrimination is

\( \pi^S + \pi^A = (4 - 2)(6) + (3.5 - 2)(3) = 12 + 4.5 = 16.5 \). Therefore, her profit is higher with price discrimination.
(c) In this scenario, she can practice two-part pricing. For each group, the number of token will be equal to quantity demanded at price $2, which is the marginal cost of a drink.

Number of tokens for students $= 18 - 3(2) = 12$, and the number of tokens for the adults $= 10 - 2(2) = 6$.

Now, for each group, the cover charge should equal the consumer surplus received at the given number of tokens. That is,

Cover charge for a student $= \frac{1}{2}(6 - 2)(12) + 2(12) = 48$

Cover charge for an adult $= \frac{1}{2}(5 - 2)(6) + 2(6) = 21$

Therefore, her profits $= \text{Total revenue} - \text{total costs of drinks}$

$$= \frac{1}{2}(6 - 2)(12) + 2(12) + \frac{1}{2}(5 - 2)(6) + 2(6) - 2(12) - 2(6) = 33$$

**Problem 5 – ch. 6 – pg 132**

Suppose the monopolist offers two entry fees, unit price combinations. One is targeted towards the low demanders and the other is targeted towards the high demanders. Let $(F_1, p_1)$ be the entry fee and unit price combination paid by the low demanders and $(F_2, p_2)$ be the entry fee and unit price combinations paid by the high demanders.

Now, from the discussions in the text, it follows that $F_1 = \frac{1}{2}(12 - p_1)^2$. Also, the monopolist needs to adjust $p_2$ so that the high demanders do not buy the combination intended for the low demanders. Therefore, it follows that

$$\frac{1}{2}(16 - p_2)^2 - F_2 = \frac{1}{2}(16 - p_1)^2 - \frac{1}{2}(12 - p_1)^2$$

$$\Rightarrow F_2 = \frac{1}{2}(16 - p_1)^2 - \frac{1}{2}(16 - p_1)^2 + \frac{1}{2}(12 - p_1)^2$$

Thus, the profit of the monopolist is given by

$$\pi_1 = N_h[(p_2 - 4)(16 - p_2) + F_2] + N_l\left((p_1 - 4)(12 - p_1) + \frac{1}{2}(12 - p_1)^2\right)$$

Differentiate $\pi$ with respect to $p_1$ and $p_2$, and equate each expression to zero to obtain

$$p_1 = 4\left[1 + \frac{N_h}{N_l}\right], \quad p_2 = 4$$

Substituting the optimal prices in to the profit expression, we obtain the maximum profit the monopolist can earn by serving both groups. Observe that

$$\pi_1 = 72 - \frac{1}{2}(12 - 4\frac{N_h}{N_l})^2 + \frac{1}{2}(8 - 4\frac{N_h}{N_l})^2 \left(\frac{N_h}{N_l}\right) + \left[4\frac{N_h}{N_l}\left(8 - 4\frac{N_h}{N_l}\right) + \frac{1}{2}(8 - 4\frac{N_h}{N_l})^2\right]$$
On the other hand, if the monopolist serves the high demanders only, its profit will be
\[ \pi_2 = N_h \frac{1}{2} (16 - 4)^2 = 72N_h \]

Therefore, the monopolist serves both groups if and only if \( \frac{\pi_1}{\pi_2} \geq 1 \).

It can be verified that (remark: use a program such as MAPLE)
\[ \frac{\pi_1}{\pi_2} \geq 1 \text{ if and only if } \frac{N_h}{N_l} \geq 1 \]