Does Competition for Capital Discipline Governments?

Decentralization, Globalization and Public Policy

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Many political economists believe that competition among countries—or regions within them—to attract mobile capital disciplines their governments, motivating them to invest more in infrastructure, reduce waste and corruption, and spend less on non-productive public goods. The result should be convergence on business-friendly policies and clean government. The notion that mobile capital disciplines governments is central to debates on both political decentralization and globalization. We argue that it requires an assumption—countries or regions start out identical—that is unrealistic. If units are sufficiently heterogeneous (in natural resources, geographical location, inherited human capital or infrastructure), we show that capital mobility often \textit{weakens} discipline on the poorly-endowed units and increases policy divergence. While better-endowed units do invest more in infrastructure—and are rewarded by capital inflows—poorly endowed units may actually be less business-friendly or more corrupt than under capital immobility. This may help explain disappointing results of liberalizing capital flows within the Russian federation and sub-Saharan Africa.

October 2003

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1 Introduction

Does competition to attract mobile capital discipline governments? Two literatures contend that it does. The first sees such discipline as harmful. Scholars argue that the fear of capital outflows restricts governments from providing welfare services, environmental regulations, and non-productive public goods that citizens demand. Capital mobility prompts a “race to the bottom” in social and environmental policy, both among subnational governments within decentralized states and among countries competing in world capital markets.\(^1\) By contrast, the second literature views such discipline as salutary. The competition for capital motivates governments to reduce their corruption, waste, and inefficiency and to provide more growth-promoting infrastructure.\(^2\)

Although they disagree about whether such discipline is desirable, authors in both schools agree that it exists. For good or ill, competition for capital is thought to shift government priorities away from non-productive public spending toward business-friendly investments. This

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\(^1\) One of the early statements was in Oates (1972, p.143): “In an attempt to keep taxes low to attract business investment, local officials may hold spending below those levels for which marginal benefits equal marginal costs, particularly for those programs that do not offer direct benefits to local business.” Zodrow and Mieszkowski (1986) modeled how this could occur. Keen and Marchand (1996) showed how capital competition may distort governments’ spending choices, causing them to invest too much in infrastructure (“business centres and airports”) and too little in other public goods (“parks or libraries”). Cumberland (1981) argues that interjurisdictional competition to attract business investment weakens environmental standards. Rom, Peterson and Scheve (1998) discuss the “race-to-the-bottom” in US welfare policies and social services. On globalization, Rodrik (1997) argues that increasing capital mobility has made it harder for national governments to provide social insurance for their citizens (pp.6, 73). Schulze and Ursprung (1999, p.298) contend that states “competing for foreign investment will … restructure their expenditure towards more privately productive public inputs at the expense of transfers and non-productive government consumption.”

\(^2\) Some scholars of federalism argue that interregional competition punishes wasteful or corrupt governments with capital flight (Qian and Roland 1998), inducing them “to provide a hospitable environment for factors,” and to guarantee secure property rights and infrastructure (Montinola, Qian, and Weingast 1995, p.58). In China, competition to attract foreign investment is said to have led many provinces, cities, and townships to adopt pro-business laws, regulations, and tax systems (Ibid, p.77). Others see beneficial effects of capital competition in the international arena. According to Obstfeld (1998, p.10), a “main potential positive role of international capital markets is to discipline policymakers who might be tempted to exploit a captive domestic capital market. Unsound policies—for example, excessive government borrowing or inadequate bank regulation—would spark speculative capital outflows and higher domestic interest rates.” Even one well-known critic of globalization is sympathetic to the argument that “opening the capital account imposes ‘discipline.’ Countries are ‘forced’ to have good economic policies, lest capital flow out of the unit” (Stiglitz 2000, p.1080). The Economist magazine goes further, contending that: “Integration makes it harder to be a tyrant… people can leave and take their savings with them” (The Economist 2001).
view—widespread in both academic and policy circles—inform discussions of both political
decentralization within countries and the liberalization of capital flows between them. Capital
controls are defended by some as vital to preserve national (or regional) policy autonomy, and
attacked by others as shelters for inefficient or corrupt governments.3

In this paper, we argue that the discipline effect invoked by both schools is not as general
as usually thought. The standard model that justifies it relies on a strong assumption that is
unlikely to hold for most real world cases. Critically, scholars assume that regions or countries
(henceforth, “units”) are identical. They then focus only on symmetric equilibria, in which by
definition units converge on the same policies or tax rates. We show that given alternative,
empirically plausible assumptions, almost exactly opposite conclusions follow.

If some units start out better endowed than others with characteristics that make them
attractive to investors (e.g., natural resources, geographical advantages, inherited human capital),
symmetric equilibria will not exist. If differences in endowments are sufficiently large, the worse-
endowed units will actually have less business-friendly policies in equilibrium under capital
mobility than if they had effective capital controls. Rather than being disciplined, officials of such
units will spend a larger share of the budget on non-productive public goods or on their own
consumption than when capital is immobile. By contrast, better-endowed units will invest more in
business services and will suck capital out of their poorly-endowed counterparts. The result will
be not convergence but polarization of both policies and government quality.

To put it concretely, even if Chad’s government were to invest massively in business
infrastructure, it would not be able to attract much money out of the capital markets of New York
or compete in productivity with the industrial zones of East Asia. Even if the Russian republic of
Buryatia were to install high-speed fibre-optic cables, it would not divert much business from

3 In this paper, we focus on questions of capital mobility and do not consider the effects of increasing trade
openness on government policies. Even if capital market liberalization does not discipline governments,
trade liberalization might.
Moscow and St Petersburg. Under capital immobility, governments have some incentive to increase the productivity with which domestic savings are invested—they will be able to tax the profits. Under capital mobility, domestic savings will flee the unit’s undeveloped infrastructure and political risk in search of more secure returns. Knowing they cannot compete, governments in poorly-endowed units will give up on pro-business policies and focus instead on either predation or satisfying the demands of local citizens. They will face less, not more, effective discipline.

To demonstrate our point, we develop a general model of competition for capital among heterogeneous units. In this model, governments allocate spending between investments to improve the business environment (“infrastructure”) and non-business-promoting activities (either public goods or officials’ consumption.) Infrastructure increases capital productivity; other types of spending do not. The units may differ in initial endowments. When units start out identical, a symmetric equilibrium exists in which the governments spend a larger share of their budget on infrastructure and less on public consumption under capital mobility than under capital immobility. However, if the units do not start out identical, no symmetric equilibrium exists.

When units are heterogeneous, better-endowed units always invest more in infrastructure in equilibrium than poorly-endowed ones. If initial asymmetry is high, poorly-endowed units invest less in infrastructure and receive less capital under mobility than under immobility. In general, competition for capital exacerbates initial inequality. In the extreme, only a polarization equilibrium exists in which poorly-endowed units make no infrastructure investment and get no capital.

Recognizing this casts light on some otherwise surprising empirical cases. Internal capital flows have been liberalized recently in both China and Russia. While competition among the more developed coastal provinces of China and among cities within them is impressive, there is
little evidence of any salutary effect of competition on the inland provinces. In Russia, as we illustrate in section 5, capital appears to have flowed out of poorly-endowed regions into a few well-endowed ones, exacerbating interregional inequality. Many developing countries liberalized their capital accounts in the 1980s and 1990s. Some—usually the upper middle-income ones—experienced large inflows of capital, which helped stimulate growth. However, others—in particular, some Sub-Saharan African countries—suffered net capital outflows. During these decades, there was no noticeable, general improvement in the quality of African governance, and the continent continued to fall further behind the rest of the world in infrastructure and in output. We argue that these observations are consistent with our model, but not with the standard view of capital competition.

Our argument is related to several others. Students of economic growth noticed some time ago that countries’ incomes were not converging in the way that simple neoclassical models predicted (see, for instance, Romer 1994, Barro and Sala-i-Martin 1995). Common explanations posit that capital is more productive when combined with high levels of human capital, infrastructure, or property rights protection (Lucas 1990, Mankiw, Romer and Weil 1992, Sachs, Borensztein, De Gregorio, and Lee 1998). Most previous treatments have not noted, however, that such initial differences in endowments also undermine the argument that capital mobility disciplines governments. The novelty of our analysis is to show that competition for capital will not necessarily cause governments to converge on capital-friendly policies and clean government.

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4 For instance, Jian, Sachs and Warner (1996) found that the more developed coastal provinces began diverging in output from the less developed inland provinces in the 1990s after international trade and investment flows were liberalized.

5 Another implication of our analysis is that the more attractive the better-endowed countries are to foreign investors, the more they will focus on infrastructure at the expense of non-productive public goods or services. Although the parallel to our model is not exact (since we assume for simplicity that all “well-endowed” units are alike), it is striking that among industrial democracies the most vigorous campaigns to roll back the welfare state came in the two countries that were already the most attractive to investors—the USA (since the 1980s) and the UK (under Thatcher) (see, e.g., Piven 2001).
In fact, poorly-endowed countries will often choose to invest less in growth-promoting policies, even knowing they will lose capital as a result.6

In a fascinating recent essay, Rogowski (2003) makes an argument similar to ours. He uses a spatial model of policy preferences to explore the extent to which the median voter (worker) will vote for environmental or labor policies that accommodate—and thus attract—mobile capital. He finds that for some kinds of initial asymmetry, the two countries’ policies diverge further under capital mobility than under immobility.7 Our paper differs in several ways. First, in Rogowski’s model policies are costly only in the sense that they may scare away investors—for instance, restrictive labor or environmental regulations; the model does not include the tax cost of financing policies, and so does not extend readily to policies that involve public spending. In our model, governments choose consumption and investment levels within an endogenously determined budget. Second, since the median voter dictates policy in Rogowski’s model and there is no room for agency slack, it cannot be used to analyze the effect of capital competition on corruption—one major focus of our paper. Third, in Rogowski’s model the actors do not anticipate the reactions of others; in ours, the players are fully strategic.

Several previous papers analyzed asymmetric tax competition. Bucovetsky (1991) presented a model in which smaller countries have lower tax rates at equilibrium because the benefit from capital has a larger per capita impact than in larger countries. Kanbur and Keen (1993), in a model with commodity taxes and transportation costs, found that governments of geographically small countries should set the tax rate lower, because the shorter distance for arbitrageurs to travel reduces the rents the government can extract. We do not examine the effects of country size. Wilson (1999) and Wilson and Wildasin (forthcoming) review the formal literature on tax competition.

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6 The ineffectiveness of such uneven competition to motivate players echoes a result of the literature on tournaments (e.g. Nalebuff and Stiglitz 1983). We thank a referee for pointing out this parallel.

7 However, for another kind of asymmetry—specifically in the initial capital/labor ratio—he finds policy convergence.
Finally, Besley and Smart (2001) also study the effect of competition for mobile capital on government policies. They introduce asymmetric information about the type of incumbent officials, where “type” denotes the official’s relative preference for public goods and rents. In their model, the intensity of capital competition affects how officials allocate funds between public goods and rents. They derive the interesting result that competition for capital is most likely to increase voter welfare not when officials are most predatory but when they care most about providing public goods. There are two main differences with our approach. First, officials can spend on three things—public goods, their own rents, and productivity-enhancing infrastructure. We lump public goods together with rents in a single variable, $c_i$, and study how capital competition affects the tradeoff between $c_i$ and infrastructure. Besley and Smart ignore infrastructure and examine how capital competition affects the tradeoff between public goods and rents. Thus, the two papers study different parts of a larger problem, and the results should be viewed as complementary. Second, our focus is on the way competition for capital interacts with initial asymmetries in endowments, and we therefore leave details of the electoral game in the background. Besley and Smart’s focus is on the way capital competition interacts with the electoral game, and so they abstract from questions of endowment asymmetry.

2 The model

2.1 The economy

The economy is divided into $N + M$ regions or countries (“units”), indexed by $i$, each of which has a government, $G_i$. There is a fixed amount of capital, $K$, in the whole economy. Let $k_i$ be the amount of capital invested in unit $i$. Each government can invest in infrastructure to improve the business environment in its unit. Let $I_i$ be $G_i$’s infrastructure investment. “Infrastructure” should be interpreted broadly here: it represents anything governments do to increase the productivity of
capital in their units. Thus, it includes physical infrastructure (transportation, telecommunications, etc.), education, public health, and a system of well-enforced property rights and legal protections.8

Our purpose is to study the effect of exogenous differences in the endowments of countries or regions on their governments’ policies. We therefore treat certain variables (the stocks of natural resources, human capital, or infrastructure) as fixed at the moment governments decide on policies, although of course such stocks result in part from previous endogenous choices. Of the \( N + M \) units, \( N \) are “well-endowed” and \( M \) are “poorly-endowed”. The “well-endowed” units have characteristics (resources, human capital, inherited infrastructure) that increase the marginal productivity of capital invested in them. For simplicity, we suppose that all well-endowed units are identical and all poorly-endowed ones are also identical.

To study the effects of exogenous asymmetry in the most direct and transparent way, we assume that the degree of asymmetry can be captured by a single parameter, \( \rho \), in an otherwise very general and standard production function. The aggregate production function of unit \( i \), \( F_i \), is given by

\[
F_n = f(k_n, I_n; \rho), \quad n \in N; \\
F_m = f(k_m, I_m; 0) = f(k_m, I_m), \quad m \in M
\]

where \( n \in N \) denotes a generic well-endowed unit, \( m \in M \) denotes a generic poorly-endowed unit, and \( \rho \geq 0 \) in well-endowed units, but is normalized to zero for poorly-endowed ones. For any \( \rho \), the production function \( f \) is strictly increasing and concave in the amount of infrastructure investment \( I_i \) and of capital \( k_i \) in unit \( i \): \( \partial f / \partial I_i > 0, \partial f / \partial k_i > 0 \), \( \partial^2 f / \partial I_i^2 < 0, \partial^2 f / \partial k_i^2 < 0 \). We also assume that infrastructure and capital are complementary: for all \( i \), infrastructure investment in unit \( i \) increases the marginal productivity of capital in unit

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8 Infrastructure investment in one unit may also create externalities for other units. For example, better infrastructure in one unit reduces local transportation costs, benefiting firms in other units that trade with it. For simplicity, we ignore such investment externalities in our analysis.
$i: f_{ik} > 0$. (To simplify notation, we use subscripts to denote derivatives of $f$ with respect to $k$ and $I$, for example, $f_k = \partial f / \partial k$ and $f_{ik} = \partial^2 f / \partial k \partial I$.) Given $k_i$ and $I_i$, well-endowed units have a higher marginal productivity of capital: $\partial^2 f(k, I; \rho) / \partial k \partial \rho > 0$. We also assume that better endowments do not themselves reduce output or reduce the productivity of infrastructure: $\partial f(k, I; \rho) / \partial \rho \geq 0$, and $\partial^2 f(k, I; \rho) / \partial I \partial \rho \geq 0$. The latter assumption serves to exclude implausible cases in which better endowments—while useful to attract capital—perversely harm production itself.

The formulation in (1) is consistent with a wide variety of common production functions. One specification that allows a considerable degree of generality is $f(k, I; \rho) = g(k, I) + \rho k$, where $\partial f(k, I; \rho) / \partial k \partial \rho = 1$, $\partial f / \partial \rho = k \geq 0$ and $\partial^2 f / \partial I \partial \rho = 0$. Another specification is $f(k, I; \rho) = A(\rho) g(k, I)$, where $A(0) = A_0 > 0$, $A'(\rho) > 0$, and $g(k, I)$ is a standard production function. Then, $\partial f(k, I; \rho) / \partial k \partial \rho = A'(\rho) g_k > 0$, $\partial f(k, I; \rho) / \partial \rho = A'(\rho) g(k, I) > 0$ and $\partial^2 f(k, I; \rho) / \partial I \partial \rho = A'(\rho) g_{II} > 0$.

2.2 Government objectives

Some economic analyses assume benevolent governments which maximize social welfare in their jurisdictions; others assume Leviathans which maximize their own consumption or tax revenues. In this paper, we model governments as partially self-interested actors, which may care about both social welfare and their own consumption.9 A government’s objective function includes: (1) total output within the unit net of taxes, and (2) utility from government consumption. The latter can include both spending on public goods and services demanded by the population and the

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9 For a similar approach, see Edwards and Keen (1996).
government’s own legal or illegal consumption of public funds. Specifically, government $G_i$ has the quasilinear payoff function:

$$U_i = (1 - t)F_i + \lambda v(c_i)$$  \hspace{1cm} (2)$$

where $t$ is the tax rate on output, $c_i$ is government spending, and $\lambda \geq 0$ measures the government’s preference for public spending relative to private consumption (assumed to be the same across units). The variable $c_i$ can be interpreted in either of two ways: as budget-funded consumption by incumbent officials or as spending on public goods and services. The utility function $v(c)$ is strictly increasing and concave. Equation (2) is quite general, including for instance the case of pure benevolence, in which case $c_i$ represents public good provision, as well as the case of extremely predatory government, when $c_i$ represents government consumption and $\lambda$ approaches infinity. Each government is endowed with initial fiscal revenue $S \geq 0$. The budget constraint of government $G_i$ is $I_i + c_i = S + tF_i$.\(^{10}\)

Government payoff functions of this kind are extremely common in political economy analyses (see for example Persson and Tabellini 2000), and are consistent with various models of voting. Three types of model are currently popular. First, a Downsian spatial model would predict that policies chosen by government are simply those most preferred by the median voter, assuming a Condorcet winner exists.\(^{11}\) Interpreting our $c_i$ as government spending on public

\(^{10}\) The formulation in (2) ignores the complication that under capital mobility some capital owned by citizens of unit $i$ may be invested elsewhere and that some of the capital invested in $i$ may belong to citizens of other units. Thus after-tax output in $i$ will not correspond exactly to consumption of citizens of $i$. An alternative simplification would be to replace $F_i$ in (2) with $L_i$, where $L_i = f(k_i, I_i) - \frac{\partial f}{\partial k}$ is labor income in $i$, and to make the concavity assumptions about $L_i$ that we made about $F_i$. Qian and Roland (1998, p.1148) take this approach. It seems reasonable to assume $L_i$ would increase concavely in both $I_i$ and $k_i$. In any case, if government $i$’s payoff function is not concave in $I_i$ and $k_i$ there will be no competition for capital: the only equilibria under mobility will be corner solutions in which all capital flows to one unit. Our argument will hold even more strongly.

\(^{11}\) Persson and Tabellini (2000, pp.24-5) show that a Condorcet winner is likely to exist if voter preferences are linear in private consumption and concave in the preference for public spending.
goods, and supposing that each voter has preferences that are linear in after-tax output and strictly concave in public spending, we arrive at a version of (2). Second, “citizen-candidate” models, which assume that candidates cannot commit themselves to policies before elections, predict that the winner will simply implement his most preferred policies (see Osborne and Slivinski 1996, Besley and Coate 1997). If we again assume all citizens’ preferences are linear in after-tax output and concave in public spending, the winning candidate will again maximize some version of (2). Besley and Coate assume no rent extraction by government because candidates’ true preferences are common knowledge and so no candidate with a preference for rents would be elected. However, the analysis would not change much if we assumed that all candidates have positive, concave preferences for rents and, in the terms of equation (2), $\lambda$ is drawn from a known distribution. The winning candidate would have a preference for rents drawn at random from this distribution, and we could interpret $c_i$ in our model as rents rather than public goods.

The third type of voting model in widespread use is the retrospective voting model (Barro 1973, Ferejohn 1986). The incumbent is assumed to maximize rents, and the voters (usually assumed identical) have a utility function that increases in private income and perhaps also public good provision. The voters coordinate to reelect the incumbent only if he chooses policies that provide them a level of utility above a threshold, $k$, which is determined endogenously. If the incumbent is voted out, he is replaced by an identical candidate. To prevent the incumbent from giving up on reelection and consuming all income in the current round, voters must set $k$ such that the present discounted value of rents if he is reelected at least equals the payoff from consuming all output in one round and getting voted out.

It is easy to see that for some plausible functional forms this can yield a version of our equation (2). Suppose the game consists of two periods. The incumbent has a concave utility function over his own consumption in each period. He maximizes: $v(c_i) + \delta p v(R)$, where $c_i$ is
his first period consumption of the budget, $\delta$ is the discount rate, $p$ is the probability he will be reelected to a second (and final) term, $v(\cdot)$ is concave, and $R$ is his second-period rents—which, if he is unconstrained, equal the whole of output. The voters derive linear utility each period equal to $(1-t)F$, and coordinate to set a threshold, $k$, for reelecting the incumbent. However, there is some uncertainty, so reelection is stochastic: he is reelected if and only if $(1-t)F > k + \epsilon_1$, where $\epsilon_1$ has a known distribution. For simplicity, we assume a uniform distribution with density $\phi$. So $p = \Pr[\epsilon_1 < (1-t)F - k] = \frac{1}{2} + \phi[(1-t)F - k]$. We can therefore rewrite the incumbent’s objective as: $v(c_i) + \frac{1}{\lambda}(1-t)F + \mu$, where $\mu \equiv \delta v(R)(\frac{1}{2} - \phi k)$ and $\frac{1}{\lambda} \equiv \delta v(R)\phi$ are constant given $k$. Maximizing this is identical to maximizing $(1-t)F + \lambda v(c_i)$, which is our (2).

Finally, another way to justify the concavity of $v(c_i)$ is to suppose that $c_i$ represents embezzlement by officials, and that the probability of detection and/or the severity of punishment increase rapidly with $c_i$. If one assumes, in addition, that officials derive some positive benefit from after-tax private consumption (perhaps they also have some private investment income), this could again rationalize a version of (2).

2.3 The game

We study a game in which all governments simultaneously choose how much infrastructure investment to make. Our focus is on how capital mobility affects the governments’ investment incentives. In particular, we compare two polar cases: (1) capital is completely immobile and the

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12 Besley and Smart (2001) combine a retrospective voting model with asymmetric information about the incumbent official’s type. “Good” incumbents maximize voter welfare (assumed linear in public goods and concavely decreasing in taxation); “bad” incumbents care only about rents, but are constrained by the voters’ retrospective voting strategy, and—in pooling or hybrid equilibria of the signaling game—by the incentive to pass as “good”.
allocation is fixed at some historically determined level; and (2) capital is perfectly mobile and can cross borders costlessly. Perfect mobility implies that the after-tax marginal productivity of capital should be equalized across units. Of course, the real world lies somewhere in between these polar cases. Nevertheless, by comparing them, we hope to shed light on what happens as capital becomes more mobile. To present the main ideas in the simplest way, we first assume that the tax rates in all units are exogenously fixed at the same level: \( t_i = t \geq 0 \), for all \( i \). Then, in Section 4, we relax this assumption.

3 Characterizing equilibria

3.1 Capital immobility

Suppose each well-endowed unit has a fixed capital allocation of \( \bar{k}_n > 0 \) and each poorly-endowed unit has a fixed capital allocation of \( \bar{k}_m > 0 \), where \( N \bar{k}_n + M \bar{k}_m = K \). Each government \( G_i \) chooses \((c_i, I_i)\) to maximize \( U_i = (1 - t)F_i + \lambda v(c_i) \) subject to its budget constraint \( c_i = S + tF_i - I_i \). Substituting the budget constraint into the objective function, we get the first order condition:

\[
\frac{dU_i}{dI_i} = (1 - t) \frac{\partial F_i}{\partial I_i} + \lambda v'(c_i)(t \frac{\partial F_i}{\partial I_i} - 1) = 0
\]

which can be rewritten as

\[
\frac{\partial F_i}{\partial I_i} = \lambda v'[1 + (\lambda v' - 1)t] \quad (3)
\]

Let \( \tau = \lambda v'[1 + (\lambda v' - 1)t] \), which can be interpreted as the opportunity cost of infrastructure investment for the governments. Equation (3) simply says that the marginal product of infrastructure should equal its marginal cost. Note that \( \tau \) is a monotonic function of \( v' \) and also increases in \( \lambda \): the greater the taste for public consumption, the larger the opportunity cost of infrastructure investment.
For \( \bar{k}_n \) and \( \bar{k}_m \), we can solve \( I_i \) from Equation (3) to get the optimal infrastructure investments in well-endowed and poorly-endowed units. Denote the solution \( \bar{T}(k; \rho) \).

**Proposition 1:** Under capital immobility, \( \bar{T}(k; \rho) \) is strictly increasing in \( k \) and weakly increasing in \( \rho \).

**Proof:** See the Appendix.

Under immobility, the level of governments’ infrastructure investment depends on two things. First, since infrastructure and capital are complementary, units with more capital will invest more in infrastructure. Second, if better endowments increase the productivity of infrastructure, then units with better endowments will build more infrastructure for this reason.

### 3.2 Interior equilibrium under capital mobility

Now suppose capital is perfectly mobile across units. Under mobility, capital will flow from units with lower after-tax marginal rates of return to capital to units with higher rates. In an interior equilibrium in which all well-endowed and poorly-endowed units have positive capital, i.e. \( k_n > 0 \) and \( k_m > 0 \), the rates in all the units must be equalized. Let \( r \) be the economy-wide net return to capital. We suppose for now that each unit is small relative to the whole economy (both \( N \) and \( M \) are large), so each takes \( r \) as given and ignores potential effects of its decisions on \( r \).

We show in Section 4 that our main results generalize to situations in which each unit anticipates all strategic effects of its decisions on other units.

If government \( G_i \) makes infrastructure investment \( I_i \), the capital inflow to \( i \) is given by

\[
(1-t)\frac{\partial F_i(k_i, I_i)}{\partial k_i} = r
\]  

(4)
The capital allocation to unit $i$, $k_i(I_i, r)$, can be derived from Equation (4). It is easy to see that

$$\frac{\partial k_i}{\partial I_i} = -\frac{f_{iI}}{f_{kk}} > 0$$

$$\frac{\partial k_i}{\partial r} = \frac{1}{(1-t)f_{kk}} < 0$$

(5)

This says that under capital mobility, capital flows into units that invest more in infrastructure. It captures the widely recognized disciplining effect of capital competition. On the other hand, given unit $i$'s infrastructure investment, capital inflow to $i$ is lower, the higher is the world-wide net return to capital, $r$, because higher $r$ means that other units are making larger infrastructure investments.

Thus, one key factor affecting the allocation of capital across units is their levels of infrastructure investment. But there is another important factor that has not been widely noticed and whose effect may completely offset the first—the initial asymmetry in endowments. By the Implicit Function Theorem, from Equation (4) we get

$$\frac{\partial k_n}{\partial \rho} = -\frac{\partial^2 f(k_n, I_n; \rho)}{\partial k_n \partial \rho} = 0$$

(6)

For any given infrastructure investments, the capital inflow to well-endowed units is greater, the greater is their advantage in initial conditions, $\rho$. Investors are attracted by both infrastructure investments and better endowments. However, endowments do not just affect the allocation of capital directly. As we show below, they also influence the calculations of governments when deciding on infrastructure investments. When asymmetry is large, governments in well-endowed units invest much more than those in poorly-endowed ones. For the latter, the disciplining effect of capital competition is blunted: the incentive to invest in infrastructure may actually be weaker than under capital immobility.

The lemma below summarizes the properties of capital allocation under capital mobility.
Lemma 1. Capital inflow to unit \( i \), \( k_i(I_i, r) \) has the following properties:

(i) \( \partial k_i / \partial I_i > 0 \) and \( \partial k_i / \partial r < 0 \); (ii) \( \partial k_\alpha / \partial \rho > 0 \); (iii) if \( f_{kkk} f_{kl} - f_{kkl} f_{kk} \geq 0 \), then \( \partial^2 k_i / \partial r \partial I_i \leq 0 \); and (iv) if \( f_{kl} \partial f_{kk} / \partial \rho - f_{kk} \partial f_{kl} / \partial \rho \geq 0 \), then \( \partial^2 k_i / \partial I_i \partial \rho \geq 0 \).

Proof: See the Appendix.

The last two parts of Lemma 1 will be useful later. The condition \( f_{kkk} f_{kl} - f_{kkl} f_{kk} \geq 0 \) means that \( \partial k_i / \partial I_i = - f_{kl} / f_{kk} \) does not decrease in \( k \). Essentially, units do not become quickly satiated in capital. This condition is trivially satisfied if \( f \) is quadratic (since \( f_{kkk} = f_{kkl} = 0 \)), and is also satisfied by other common production functions such as Cobb-Douglas.

The condition in (iv), \( f_{kl} \partial f_{kk} / \partial \rho - f_{kk} \partial f_{kl} / \partial \rho \geq 0 \), is trivially satisfied when \( f(k, I; \rho) = g(k, I) + \rho k \), for any \( g(\cdot, \cdot) \). It is also satisfied if \( f(k, I; \rho) = A(\rho)g(k, I) \), for any \( g(\cdot, \cdot) \) and \( A(\cdot) \). We assume the two conditions in (iii) and (iv) hold.

Governments understand the capital allocation process of Equation (4) and have rational expectations about others’ investment behavior, and hence about the future economy-wide net return to capital, \( r \). Given \( r \), \( G_i \) chooses \( I_i \) to maximize its payoff \( U_i = (1 - t)F_i(k_i, I_i) + \lambda \nu(c_i) \) subject to its budget constraint and the capital allocation rule \( k_i(I_i, r) \). Substituting its budget constraint into its objective function, we obtain the following first order condition

\[ \frac{\partial U_i}{\partial k_i} = f_{kk} f_{kl} - f_{kkl} f_{kk} \geq 0. \]

If \( f = A \alpha I^{1-\alpha} \), then \( f_{kkk} f_{kl} - f_{kkl} f_{kk} = A^2 \alpha^2 (1 - \alpha)^2 (2 - \alpha)k^{2\alpha - 4} I^{1 - 2\alpha} - A^2 \alpha^2 (1 - \alpha)^2 k^{2\alpha - 4} I^{1 - 2\alpha} = A^2 \alpha^2 (1 - \alpha)^2 k^{2\alpha - 4} I^{1 - 2\alpha} \geq 0. \]

If \( f = A(\rho)g(k, I) \), then \( f_{kl} \frac{\partial f_{kk}}{\partial \rho} - f_{kk} \frac{\partial f_{kl}}{\partial \rho} = A(\rho)A(\rho)g_{uu} - A(\rho)A(\rho)g_{uu} = 0. \)

\[ \frac{\partial U_i}{\partial k_i} = f_{kkk} f_{kl} - f_{kkl} f_{kk} \geq 0. \]
\[ \frac{\partial F_i}{\partial I_i} + \frac{\partial F_i}{\partial k_i} \frac{\partial k_i}{\partial I_i} = \lambda v' \left[ 1 + (\lambda v' - 1) r \right] \]  

Equation (7) has the usual interpretation for optimality: the marginal benefit of infrastructure investment on the left-hand side must equal its marginal cost on the right-hand side. Comparing this with the first order condition for the case of capital immobility in Equation (3), there is an additional term \( \frac{\partial F_i}{\partial k_i} \frac{\partial k_i}{\partial I_i} \) on the benefit side, which represents the indirect effect of infrastructure investment \( I_i \) on unit \( i \)'s output due to the additional capital it attracts to the unit.

Previous papers have often pointed to this infrastructure- and output-increasing effect of capital competition to argue that fiscal decentralization and the liberalization of capital controls can discipline governments and increase welfare (see, e.g., Qian and Roland, 1998). An unnoticed, yet critical, assumption for that conclusion is that the capital allocation under capital mobility must be the same as that under capital immobility.\(^ {17} \) Given this, \( \frac{\partial F_i}{\partial I_i} \) is unchanged, and so the left-hand side of (7) is unambiguously greater than the left-hand side of (3). However, if a unit’s capital allocation under mobility is lower than under immobility, there will be a second offsetting effect. Because capital and infrastructure are complements, a lower capital allocation will reduce the unit’s incentive to invest in infrastructure: \( \frac{\partial F_i}{\partial I_i} \) will be lower under mobility.

For such units, the total effect will be ambiguous—the left-hand side of (7) might be either greater or smaller than that of (3). Previous papers avoid this by assuming completely identical units and focusing on symmetric equilibria so that the capital allocation is the same under mobility and immobility. As we show below, when units have different endowments, no symmetric equilibrium exists and capital allocation will generally be different in the two cases. Under mobility, as initial asymmetry increases, capital allocation becomes more and more uneven.

\(^ {17} \) We thank a referee for pointing this out and suggesting the intuition below.
Using $k_i(I_i, r)$ from Equation (4), we can solve $G_i$’s optimal infrastructure investment $I_i(r)$ from Equation (7). We first prove the following result.

**Lemma 2:** Suppose $G_i$’s optimal infrastructure investment $I_i(r)$ is interior. Then it is decreasing in $r$, and for well-endowed units, $I_n(r; \rho)$ is increasing in $\rho$.

**Proof:** See the Appendix.

Lemma 2 shows that governments reduce their infrastructure investments if they expect the economy-wide net return to capital to be high (i.e., if they expect others to invest a lot in infrastructure.) At the same time, well-endowed units invest more in infrastructure when they have a greater initial advantage over poorly-endowed units. Note that asymmetry in endowments has a much stronger impact on governments’ infrastructure choices under capital mobility than under immobility. By Proposition 1, under capital immobility $T_n(k_n, \rho)$ may increase in $\rho$ if the marginal productivity of infrastructure is directly increasing in $\rho$. This effect is also present under capital mobility. But simultaneously, under capital mobility well-endowed units expect to get more capital, which motivates their governments to invest more in infrastructure. While the direct technological effect of initial asymmetry may or may not be important, the indirect competition effect is always strictly positive and can be strong for large $\rho$.

Given governments’ investment behavior $I_i(r; \rho)$, we can derive each unit’s capital allocation as a function of $r$: $k_m(r)$ for poorly-endowed units and $k_n(r; \rho)$ for well-endowed units. In equilibrium, the following market-clearing condition must hold:

$$N k_n(r) + M k_m(r) = K$$

(8)
Proposition 2: Suppose there is an interior equilibrium where \( k_m^*(\rho) > 0 \) and \( I_m^*(\rho) > 0 \). Then \( k_n^*(\rho) > k_m^*(\rho) \) and \( I_n^*(\rho) > I_m^*(\rho) \). Furthermore, \( r^*(\rho) \) and \( k_n^*(\rho) \) increase in \( \rho \), but \( k_m^*(\rho) \) and \( I_m^*(\rho) \) both decrease in \( \rho \). \( I_n^*(\rho) \) can either increase or decrease in \( \rho \).

Proof: See the Appendix.

In words, in an interior equilibrium—if one exists—well-endowed units invest more in infrastructure and get more capital than poorly-endowed units. As the initial asymmetry, \( \rho \), increases, the equilibrium net return to capital \( r^*(\rho) \) rises. Capital becomes more concentrated in the well-endowed units, and infrastructure investment in the poorly-endowed units falls. Infrastructure investments in well-endowed units, \( I_n^*(\rho) \), may either increase or decrease.

Proposition 2 shows that under capital mobility, initial asymmetries in endowments lead to asymmetries in government behavior, and even larger divergences in economic performance. Poorly-endowed units, expecting to do poorly at attracting capital, invest little in infrastructure and—as expected—attract little investment. By contrast, well-endowed units expect to win the competition, invest more in infrastructure, and succeed in attracting capital. Because of competition for capital under capital mobility, exogenous heterogeneity gets magnified, resulting in greater divergence in the capital allocation and government policies. Figure 1 illustrates this logic.
In Figure 1, the curves $k_n(r; \rho)$ and $k_m(r)$ represent the capital allocations to a well-endowed unit and a poorly-endowed unit, respectively, for any given $r$. We prove that both curves are strictly downward sloping and the former is above the latter. The equilibrium condition from (8) is represented by the intersection of the $Nk_n(r) + Mk_m(r)$ curve with the horizontal $K$ curve, which determines the equilibrium $r^*(\rho)$. An increase in $\rho$ of $\Delta \rho$ shifts the $k_n(r; \rho)$ curve up, which in turn shifts the $Nk_n(r) + Mk_m(r)$ curve up. This increases $r$ from $r^*(\rho)$ to...
\( r^* (\rho + \Delta \rho) \), which lowers \( k_m^* \) and hence increases \( k_n^* \).

3.3 Polarization equilibrium under capital mobility

The previous section characterized the interior equilibrium under capital mobility. But what if no such equilibrium exists? If the asymmetry in endowments is large, the only equilibrium may be one of polarization, in which the well-endowed units invest in infrastructure and get all the capital, while their poorly-endowed counterparts make no infrastructure investments and receive no capital.

For a non-interior equilibrium to exist, the marginal productivity of capital and infrastructure cannot be infinite (the familiar Inada conditions cannot hold.) Various plausible production functions meet this requirement—for instance a quadratic function, or a modified Cobb-Douglas function \( f(k, I) = A(k + k_0)^{\alpha} (I + I_0)^{1-\alpha} \), where \( k_0 > 0 \) and \( I_0 > 0 \) are the initial (immobile) capital stock and infrastructure in the unit at the beginning of the game. For a non-interior equilibrium, the effect of \( \rho \) on capital productivity cannot be infinitely small. If these conditions are met, and \( \rho \) and \( \lambda \) are sufficiently large, there is no interior equilibrium:

**Proposition 3**: If (a) the first and second derivatives of the production function are neither infinitely large nor infinitely small, (b) \( \partial^2 f(k, I; \rho) \partial k \partial \rho \) is not too small, and (c) \( \rho \) and \( \lambda \) are sufficiently large, then the unique equilibrium is polarization: \( k_m^* = l_m^* = 0 \) and \( k_n^* = \frac{K}{N}, \) and \( l_n^* > 0 \).

**Proof**: See the Appendix.

Polarization becomes more likely as the initial asymmetry increases. This is very
intuitive. When \( \rho \) is large, poorly-endowed units see little hope of competing with their better-endowed rivals for capital, and so they quite rationally give up and invest nothing in infrastructure. As expected, they attract no capital. In contrast, for large \( \rho \), governments of well-endowed units invest aggressively to attract capital and indeed get a large amount. Returning to Figure 1, only a polarization equilibrium will exist if \( \Delta \rho \) is so large that the equilibrium \( r^*(\rho + \Delta \rho) \) is greater than the value of \( r \) at which \( k_m(r) = 0 \). Then, poorly-endowed units will anticipate getting no capital, and invest nothing in infrastructure. (The conditions in Proposition 3 ensure that the \( k_m(r) \) curve does fall to zero—as drawn in Figure 1.)

### 3.4 Comparing capital immobility and capital mobility

Comparing the interior equilibrium of Proposition 2 or the polarization equilibrium of Proposition 3 under capital mobility with that under capital immobility (Proposition 1), we have

**Proposition 4:** If the first and second derivatives of the production function are not infinitely large, then when \( \rho \) is sufficiently large, governments in poorly-endowed units invest less in infrastructure, attract less capital, and thus have lower total output under capital mobility than under immobility. In contrast, governments in well-endowed units invest more in infrastructure, attract more capital and have greater total output under capital mobility than under immobility.

**Proof:** See the Appendix.

Proposition 4 shows that under some plausible conditions, competition for capital exacerbates initial inequalities and hinders economic development in poorly-endowed units. Such competition does not discipline governments in such units, forcing them to improve their business environment. Instead, seeing little hope of winning the competition, those governments simply
give up on attracting capital: they spend little or no resources on infrastructure and focus instead on public consumption. Governments in well-endowed units invest more in infrastructure in absolute terms and get more capital under capital mobility than under immobility, resulting in higher output. As capital becomes mobile, governments of well-endowed units may also spend more in absolute terms on public consumption since their budgets expand due to the capital inflow. But whether public consumption increases or decreases as a share of the budget is indeterminate. In sum, capital mobility may discipline governments and promote growth in well-endowed units. But in their poorly-endowed counterparts, our model shows that it can make governments even more corrupt or profligate and cause these units to lose capital.

In order to prove Proposition 4 in the most transparent way, we focused on situations in which extreme polarization can occur in equilibrium. Examination of Figure 1 should convince the reader that the result of Proposition 4 will hold even if extreme polarization does not arise in equilibrium. The logic of our argument applies equally well to cases in which $k_m(r)$ does not fall below zero but approaches zero as $r$ increases. In Section 4.2 below, we present such a case with Cobb-Douglas production functions (so the Inada conditions hold) under endogenous tax competition.

4 Extensions

4.1 Direct strategic competition with small number of units

We demonstrate here that the main results from Section 3 also hold in a setting in which a few units directly compete against each other. Specifically, we study the case in which there is one well- and one poorly-endowed unit (i.e., $N = M = 1$). Denote the government of the well-endowed unit “$G_1$” and that of the poorly-endowed unit “$G_2$”. Clearly, the analysis for capital immobility is the same as before.
Under capital mobility, in an *interior equilibrium* with \( k_1 > 0 \) and \( k_2 > 0 \), the net return to capital in the two units must be equalized: \((1-t) \frac{\partial F_1}{\partial k_1} = (1-t) \frac{\partial F_2}{\partial k_2}\). This implies

\[
\frac{\partial f(k_1, I_1; \rho)}{\partial k_1} = \frac{\partial f(k_2, I_2)}{\partial k_2}
\]

With \( k_1 + k_2 = K \), solving for \((k_1, k_2)\) from Equation (9) gives \(k_1(I_1, I_2, \rho)\) and \(k_2(I_1, I_2, \rho)\). By the Implicit Function Theorem, we have

\[
\frac{\partial k_1}{\partial I_1} = -\frac{\partial k_2}{\partial I_1} = -\frac{\partial^2 f(k_1, I_1; \rho)/\partial k_1^2 + \partial^2 f(k_2, I_2)/\partial k_2^2}{\partial^2 f(k_1, I_1; \rho)/\partial k_1^2 + \partial^2 f(k_2, I_2)/\partial k_2^2} > 0
\]

\[
\frac{\partial k_1}{\partial I_2} = -\frac{\partial k_2}{\partial I_2} = \frac{\partial^2 f(k_1, I_1; \rho)/\partial k_1^2 + \partial^2 f(k_2, I_2)/\partial k_2^2}{\partial^2 f(k_1, I_1; \rho)/\partial k_1^2 + \partial^2 f(k_2, I_2)/\partial k_2^2} < 0
\]

\[
\frac{\partial k_1}{\partial \rho} = -\frac{\partial k_2}{\partial \rho} = -\frac{\partial^2 f(k_1, I_1; \rho)/\partial k_1 \partial \rho}{\partial^2 f(k_1, I_1; \rho)/\partial k_1^2 + \partial^2 f(k_2, I_2)/\partial k_2^2} > 0
\]

Equation (10) is analogous to Equations (5) and (6). The only difference is that now each unit recognizes that its infrastructure investment directly affects capital flows into the other. Before, we supposed that units were small and so their governments took the economy-wide net return to capital as given.

Given \( G_2 \)'s choice of \( I_2 \), \( G_1 \) sets \( I_1 \) to maximize \( U_i = (1-t)f(k_i, I_i; \rho) + \lambda \nu(c_i) \) subject to its budget constraint and to the capital allocation \( k_i(I_1, I_2, \rho) \). Solving the problem gives \( G_i \)'s best response function \( I_i(I_2, \rho) \). \( G_i \)'s best response can be found similarly. A Nash equilibrium of the investment game is a pair of strategies \((I'_1, I'_2)\) that are best responses against each other.

Solving for \( G_i \)'s best response by substituting its budget constraint into its objective function and maximizing, we derive exactly the same first order condition as before—Equation (7)—which is reproduced here:

\[
\frac{\partial F_i}{\partial I_i} + \frac{\partial F_i}{\partial k_i} \frac{\partial k_i}{\partial I_i} = \lambda \nu'/[1 + (\lambda \nu' - 1)t]
\]

However, characterizing the properties of \( G_i \)'s best response in a fully general model is difficult for technical reasons. From Equation (10), \( \frac{\partial k_i}{\partial I_i} \) depends not only on \( \partial^2 f(k_i, I_i; \rho)/\partial k_i \partial I_i \) and
\[ \frac{\partial^2 f_1(k_i, I_i; \rho)}{\partial k_i^2} \text{, but also on } \frac{\partial^2 f_2(k_2, I_2; \rho)}{\partial k_2^2} \text{ in a complicated way. The existing literature typically circumvents the technical problems by assuming identical units (} \rho = 0 \text{ ) and focusing on the symmetric equilibrium. But since we are interested in the consequences of asymmetry, this course is not open to us.}

To see whether our results hold in a fully strategic setting, it helps to place more structure on the production functions. Specifically, we will assume quadratic functions of the following form: 
\[ f(k, I) = (a + bI)k - 0.5\gamma k^2 - 0.5\delta I^2, \]
where parameters \( a, b, \gamma, \) and \( \delta \) are all positive.

The total outputs in the two units are now 
\[ F_1 = f(k_1, I_1; \rho) = f(k_1, I_1) + \rho k_1 \text{ and } F_2 = f(k_2, I_2). \]
Although the results that follow relate to this type of production function, we believe that the basic insights of our analysis are general.

With the quadratic production functions, it is easy to verify that
\[ k_i = \frac{0.5K + b(I_i - I_2)}{2\gamma} + \frac{\rho}{2\gamma}; \quad k_2 = \frac{0.5K - b(I_i - I_2)}{2\gamma} - \frac{\rho}{2\gamma}, \]
(11)
Using this capital allocation rule, we can characterize the properties of the best response investments for the two units, and prove the following result.

**Proposition 5**: Suppose production functions are quadratic. If \( \gamma \delta \geq b^2 \) and \( \rho \) is sufficiently small, there is an interior equilibrium where \( k_2^* > 0 \) and \( I_2^* > 0 \). In the equilibrium, \( k_1^* > k_2^* \) and \( I_1^* > I_2^* \). Furthermore, \( I_1^*(\rho) \) and \( k_1^*(\rho) \) increase in \( \rho \), but \( k_2^*(\rho) \) and \( I_2^*(\rho) \) decrease in \( \rho \).

**Proof**: See the Appendix.

Proposition 5 is analogous to Proposition 2. Again, as asymmetry in endowments increases, the equilibrium capital allocation becomes more and more unbalanced, and government
behavior in the two units diverges. This point can be seen most clearly when \( v(c) = c \), in which case we have

\[
I_1^* - I_2^* = \frac{b \rho}{\gamma \delta - b^2} \quad \text{and} \quad k_1^* - k_2^* = \frac{\delta \rho}{\gamma \delta - b^2}
\]

Then, endogenous divergence between units—both in their infrastructure investments and capital allocations—increases linearly in the exogenous asymmetry in their endowments, \( \rho \). When the exogenous asymmetry is sufficiently large, the endogenous inequality in capital allocations, \( k_1^* - k_2^* \), hits the extreme upper bound (it cannot be greater than \( K \))—at which point the interior equilibrium ceases to exist and the only possible equilibrium is one of polarization. From Equation (11), it is easy to see that this conclusion does not depend on the assumption that \( v(c) = c \). In particular, it is straightforward to show that, analogous to Proposition 3, when \( \rho \) is sufficiently large, the only possible equilibrium is polarization whereby \( k_1^* = K \), \( I_1^* > 0 \) and \( k_2^* = I_2^* = 0 \).\(^{18}\) Then the following result, analogous to Proposition 4, follows immediately.

**Proposition 6:** Suppose the production functions are quadratic. When \( \rho \) is sufficiently large, \( G_2 \) invests less in infrastructure, attracts less capital, and generates lower total output under capital mobility than under capital immobility; the converse is true for \( G_1 \).

### 4.2 Endogenous tax competition

So far, we have assumed that tax rates are fixed. In this subsection, we argue that the main results still hold when units compete for capital with both infrastructure and tax rates.

\(^{18}\) From (11), \( k_1 - k_2 = b(I_1 - I_2)/\gamma + \rho / \gamma \). Since from Proposition 5, \( I_1 - I_2 > 0 \), when \( \rho \) is sufficiently large, it must be that \( k_2^* = 0 \).
First consider the case of capital immobility. $G_i$ maximizes $U_i = (1 - t_i)F_i + \lambda v(c_i)$, subject to $c_i = S + t_iF_i - I_i$. The first order condition with respect to $t_i$ simplifies to:

$$\lambda v'(c_i) = 1$$

(12)

This simply says that when governments can choose tax rates under capital immobility, they set them so that the opportunity cost of fiscal revenue equals 1. In this case, equilibrium public consumption depends only on $\lambda$ and the concavity of $v$.

The first order condition with respect to $I_i$ is still given by (3), which now becomes

$$\partial F_i / \partial I_i = 1$$

(13)

Clearly, $G_i$’s optimal infrastructure investment under capital immobility $\bar{I}_i(k, \rho)$ is strictly increasing in $k$ and weakly increasing in $\rho$, just as in Proposition 1.

Consider now the case of capital mobility. From the capital allocation rule of (4):

$$\frac{\partial k_i}{\partial t_i} = \frac{r}{(1 - t_i)^2 f_{kk}} < 0$$

(14)

which says that, all else equal, lowering the tax rate will attract capital. Thus governments can use both tax rates and infrastructure investment to compete for capital. $G_i$ sets its infrastructure investment and tax rate, taking the economy-wide net return to capital, $r$, as given. The first order condition with respect to $I_i$ is still given by Equation (7), while that with respect to $t_i$ is

$$F_i = (1 - t_i)\partial F_i / \partial k_i \partial k_i / \partial t_i + \lambda v'(c_i)(F_i + t_i \partial F_i / \partial k_i \partial k_i / \partial t_i)$$

(15)

Eliminating $\lambda v'(c_i)$ from Equations (7) and (15), one can show that

$$\partial F_i / \partial I_i + \partial F_i / \partial k_i \partial k_i / \partial I_i = 1 - \frac{(1 - t_i)\partial F_i / \partial k_i \partial k_i / \partial t_i}{F_i}$$

(16)

Comparing (16) to (13), we see the two consequences of capital mobility when governments can compete with two policy instruments—infrastructure investment and the tax rate. Capital mobility adds a second term to each side of Equation (13). The second term on the left-hand side captures
the (by now familiar) additional indirect benefit of infrastructure investment under capital mobility, since such investment attracts capital. The second term on the right-hand side captures the increase in the opportunity cost of infrastructure investment under mobility because, as a result of the downward pressure on tax rates caused by capital competition, the marginal value of tax dollars is higher. (Note that this second term on the right-hand side is always positive, and so tends to reduce spending on infrastructure.)

To see the total effect of capital competition, we need to derive $G_i$’s optimal investment and tax rate $I_i(r; \rho)$ and $t_i(r; \rho)$, or at least characterize how these functions change with $r$ and $\rho$ (for well-endowed units). However, as is typical with multivariate optimization problems, a fully general solution is very cumbersome. To see how the two types of competition interact in a transparent way, it is useful to place more structure on the production function. Specifically, we assume a Cobb-Douglas function of the form:

$$F_i = A(\rho)k_i^{\alpha}I_i^{1-\alpha},$$

where $A(0) = A_0 > 0$ and $A'(\rho) > 0$. To ensure that $\partial^2 f / \partial k \partial \rho$ does not vanish to zero, we assume that $A'(\rho)$ does not become infinitesimally small as $\rho$ increases. In this case, we can demonstrate that the results in Section 3 continue to hold even when governments can compete for capital with both $I_i$ and $t_i$ (see Proposition 7.) Although the technical complications make it difficult to prove this, we believe this conclusion should hold more generally.

**Proposition 7:** Suppose the production functions are Cobb-Douglas. When $\rho$ and $\lambda$ are sufficiently large, governments in poorly-endowed units invest less in infrastructure, attract less capital, and generate lower total output under capital mobility than under capital immobility; the converse is true for governments in well-endowed units.

**Proof:** See the Appendix.
In this case, the equilibrium approaches polarization when $\rho$ and $\lambda$ are sufficiently large, even though the Cobb-Douglas production function satisfies the Inada conditions. As $\rho$ increases, well-endowed units become more and more productive. As their advantage over poorly-endowed units increases, they can actually increase their tax rates without losing capital. Higher tax rates mean higher tax revenues, which allows for more infrastructure investment. Exactly the opposite is true for poorly-endowed units. In the limit, poorly-endowed units set tax rates of zero, but still cannot attract much private investment. With a small budget and no prospect of attracting capital, governments of poorly-endowed units spend almost all their fiscal revenue on non-productive public goods or their own consumption.

5 Illustrations

Does the previous analysis cast light on actual patterns of government policy and capital flows? We discuss two settings in which our model seems to fit empirical realities better than the standard thinking. These illustrations certainly do not constitute any sort of “test” of our argument; indeed, they were part of what motivated us to re-examine conventional theories of capital competition in the first place. We aim merely to show there is something to explain.

5.1 Interregional capital flows in post-communist Russia

A particularly promising place to see the effects of capital competition is within a large, decentralized country that has recently liberalized internal capital flows. Based on the conventional wisdom, one should expect all regional governments to compete for investment by enacting business-friendly policies, building infrastructure, and cutting back on corruption and waste. By contrast, our model suggests that only the better-endowed regions would compete. Capital would tend to flow out of poorer- and into better-endowed regions.
In Russia, the transition from communism after 1991 liberated private capital to flow relatively unimpeded among the federation’s 89 regions. A network of investment banks sprang up from 1990 to channel such flows. The resulting financial system is imperfect in numerous ways. Still, by the mid-1990s few restrictions remained on interregional capital flows.

Russia’s regions differed greatly as of the early 1990s. To measure such asymmetries, we constructed an index of initial “endowments”. We included indicators of: natural resources (the log of the region’s share in total Russian raw materials production as of 1993, the first year for which we had data); geographical advantages (the distance from either Berlin or Tokyo, whichever was closest, entered negatively); inherited human capital (the proportion of the population with higher education as of 1989, the number of research and development organizations as of 1992), and physical infrastructure (the percentage of roads that were paved as of 1990, the number of public buses per 1000 inhabitants as of 1992). We summed the standardized values of each variable to form our index.

Did better endowed regions increase their effort to attract outside investment more than poorly-endowed ones after capital flows were liberalized? We used several indicators to measure change in such effort. First, we examined the pace of construction of two kinds of physical infrastructure—paved roads and water mains. We compared the average annual rate of construction of each in 1995-2000 to the rate in 1990 (the only pre-transition year for which data were available.) A region with better transportation and utilities would likely be more attractive to investors seeking to locate new plants. Although construction of these types of infrastructure slowed everywhere in the 1990s, those regions with better initial endowments tended to have

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19 The log was taken because the distribution was highly skewed, with the oil-producing Tyumen region accounting for 34 percent of the total, about four times the share of the next highest region.

20 For lack of a theoretical reason to do otherwise, we attribute equal weight to each indicator.
smaller declines. The change in the rates of both types of construction correlated positively with initial endowments (for paved roads: $r = .23$, $p < .06$; for water mains: $r = .41$, $p < .01$).  

Such correlations might reflect greater fiscal capacity to finance infrastructure investments in better endowed regions rather than a stronger motivation to make them. Poorly endowed regions might have suffered sharper falls in government revenues. However, better-endowed regions seem also to have allocated a larger share of their budgets to financing growth-promoting infrastructure. Since 1996, the Finance Ministry’s reports on regional budget execution have included the category “spending on the development of markets”. We combined this with regional government spending on transport, roads, communications, and information technology to get a measure of the share of the budget allocated to enhancing business productivity. This share (averaged for 1996-98) correlated positively with initial endowments ($r = .48$, $p < .01$).

Initial endowments also correlated with the speed at which regional governments replaced the regulatory apparatus of communism with market institutions. The business magazine *Ekspert* publishes annual ratings of the “degree of development of the leading institutions of a market economy”. Since no regions had market institutions at the start of transition, this variable by definition measures change since 1990. In regions that started with better endowments, the regional governments tended to develop market institutions more effectively during the decade that followed. The correlation between the *Ekspert* institutional rating as of 2001 and initial endowments was positive and significant ($r = .41$, $p < .01$; see Figure 2).

But did the more capital-friendly policies in better-endowed regions translate into higher inflows of investment? Did capital flow out of regions with poorer policies into their more

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21 The latter correlation excludes one extreme outlier, the Republic of Sakha, which had extremely low water mains construction in the 1990 base year, which made a subsequent moderate rate of construction in later years into a giant percentage leap. While in all other regions construction fell—by from 4 to 100 percent—in Sakha it increased by 214 percent.

22 We lacked data to analyze the change in this proportion since the start of transition. But since “spending on market development” did not occur before the early 1990s, this part at least already measures reallocation of resources toward market infrastructure. The correlation is weaker, but still significantly positive, if the two largest cities, Moscow and St Petersburg, are excluded ($r = .27$, $p < .03$).
Figure 2: Degree of development of leading institutions of a market economy, 2001; rating compiled by staff of *Ekspert* magazine.

Initial endowment, early 1990s

Correlation: .41 (p < .01); without Moscow and St Petersburg, .32 (p < .01).

Source: Goskomstat. "Initial endowment" is sum of standardized values of (1) ln of share of region in RF raw materials output 1993, (2) percentage of population with higher education 1989, (3) percentage of roads that were paved as of 1990, (4) number of research and development organizations as of 1992, and (5) number of public buses per 1,000 inhabitants as of 1992, minus the distance from the nearer of Berlin and Tokyo. Natural log of (1) taken because distribution highly skewed. Index of development of market institutions from *Ekspert* (www.ekspert.ru), adjusted so that "most developed" is 89, "least developed" is 1.

business-friendly neighbors? Data available to judge this are not ideal. We calculated two alternative estimates of net regional capital inflows. First, we computed the difference between total investment in non-financial assets and total savings of the population in each region in 1998, the latest year for which we had data. Greater investment than savings in a region suggests a capital inflow. Second, we calculated the difference between total bank credits issued and total
savings in each region, also in 1998. Again, more bank credits than savings suggests an inflow of financial capital. These are both imperfect estimates—for instance, both credit issues and non-financial investment include government loans. Still, they serve as reasonable approximations for the net flows of private capital into regions.

As Table 1 shows, regions that had more business-friendly policies or more developed market institutions as of the mid-1990s tended to have larger net inflows of capital in 1998, by either measure. The correlations were positive and highly significant. Such correlations might be caused by a direct relationship between better endowments and capital inflows, rather than by the business-friendly policies that better endowments induce. However, the correlations between policies and inflows remain positive and significant controlling for our measure of initial endowments. For a given regional endowment, more business-friendly policies were associated with larger capital inflows.

Table 1: Business-friendly policies and capital inflows, Russia late 1990s, correlation coefficients

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<tbody>
<tr>
<td>Share of regional budget spent on market institutions, transport, roads, communications, and information technology, 1996-8</td>
<td>.49</td>
<td>.55</td>
<td>.38</td>
</tr>
<tr>
<td>-controlling for initial endowment</td>
<td>.35</td>
<td>.33</td>
<td>.20a</td>
</tr>
<tr>
<td>Ekspert rating of “development of market institutions” 1996</td>
<td>.49</td>
<td>.51</td>
<td>.66</td>
</tr>
<tr>
<td>-controlling for initial endowment</td>
<td>.32</td>
<td>.30</td>
<td>.61</td>
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*a p < .10; all other correlations significant at p < .01.

In both cases, the savings variable is measured as the difference between total income of the population and expenditures on goods and services plus taxes and other obligatory payments; it includes growth in bank deposits, purchases of securities and foreign currency, and the surplus of incomes of the population over their recorded outlays.

If government loans and investments go disproportionately to aid less developed regions, this should reduce the positive relationship between investment climate and credits or investment that we found.
A final indicator of capital flows is the allocation of foreign investment among regions. There was a positive correlation between the two measures of business-friendly policies and the log of foreign investments made in 1998-2000; and these were still positive and significant (although one only marginally so) controlling for initial endowments.

In short, the regional patterns of government policy and capital flows in Russia since the liberalization of internal markets seem to fit the predictions of our model far better than they do the conventional wisdom. In regions with better initial endowments, governments tended to allocate a larger share of the budget to growth-promoting infrastructure, to cut back less on construction of roads and water mains, and to develop more highly-rated market institutions. These more business-friendly policies in the better-endowed regions correlated, in turn, with larger net inflows of both domestic and foreign capital. Major urban or industrial centers such as Moscow, St Petersburg, and Samara competed vigorously for capital—and got it. By contrast, more remote, resource-poor, underdeveloped regions such as the Republics of Altai, Tyva, or Kalmykia did not bother to compete. Each let their physical infrastructure run down unusually fast, spent practically nothing on market development, and had among the lowest-rated market institutions. They each suffered net outflows of domestic savings.

5.2 Capital account liberalization in the developing world

The 1980s and 1990s witnessed a worldwide trend toward capital account liberalization, during which many developing countries reduced capital controls. Did the newly open developing countries invest in infrastructure, reform their bureaucracies, and improve their business environments sufficiently to compete with their better endowed rivals for mobile capital?

In fact, the developing world’s share in global private capital flows decreased in precisely this period of capital market liberalization. It fell from 11.8 percent in 1991 to 7.6 percent in 2000, even though the developing countries’ share in global output grew from 19.8 to 22.5
percent (see Table 2). Net private inflows to developing countries were very low in most years—they peaked in 1996 at $129 billion, compared to total global capital flows that year of $2.4 trillion. After the Asian financial crisis of 1997-9, net flows even turned negative.25 The growth in capital outflows and errors and omissions in 1991-2000 was more than five times as large as the growth in long-term private capital inflows. At the end of a decade of capital market integration, private capital appeared on balance to be flowing out of, rather than into, the developing world.26

Even this paints too rosy a picture of the capital accounts of the least competitive economies. Capital inflows to the developing world were highly concentrated on a dozen or so success stories in Latin America and Asia—countries such as China, Mexico, and Brazil. Despite significant capital market liberalization in many countries and low capital saturation, Africa saw almost none of the increase. Private capital inflows to Sub-Saharan Africa fell from 3.9 percent of the region’s GNP in 1975-82 to 1.8 percent in 1990-98 (UNCTAD 2000). Inflows to North Africa fell from 7.2 to 0.8 percent of GNP in the same period. In both regions outbound profit remittances and interest payments were larger in the 1990s than private capital inflows. While developing countries’ share of worldwide foreign direct investment inflows (FDI) increased from 17.1 percent in 1988-90 to 21.4 percent in 1998-2000, Africa’s share fell from 1.8 to 0.8 percent during the same decade (UNCTAD 2001, p.256), and most of that 0.8 percent was concentrated on countries with oil and mining sectors and therefore a natural resources advantage (Mishra, Mody, and Murshid 2001). During this period, annual FDI to a number of countries—Algeria, Libya, Morocco, Benin, Liberia, Mauritania, Niger, Nigeria, Rwanda, Sierra Leone, and Swaziland—fell in absolute terms (Ibid, p.292).

25 The picture would not be changed by including short term capital flows. We do not have data on such flows broken down into private and public components, but total short term inflows came to just –$18.3 bn in 1999 and +$3.5 bn in 2000.

26 Another way of gauging total capital flows is to look at the current account, which measures the difference between domestic savings and investment. A current account surplus indicates net outflows. Developing countries’ aggregate current account was in surplus of $60.3 billion in 2000 (World Bank 2001, Ch.2).
Capital flight often increased after capital market opening. In various countries that liberalized—Egypt, Mauritius, Uganda—outflows by residents rose substantially in relative terms in the 1990s (UNCTAD 2000, p.37). Even before capital accounts were opened, huge sums left the continent as unofficial capital flight. Starting from the “errors and omissions” data in balance of payments statistics, and correcting for underreported external borrowing and trade mis-invoicing, Boyce and Ndikumana (2001) estimate that capital flight for 25 Sub-Saharan African countries in 1970-1996 exceeded their total accumulated external debt. By the end of the 1990s, Africans held a larger proportion of their wealth overseas than residents of any other continent.

Table 2: Private capital flows to developing countries in the 1990s ($ bn)

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<tr>
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</thead>
<tbody>
<tr>
<td>1. Longterm private capital in-flows to developing countries</td>
<td>62.1</td>
<td>99.3</td>
<td>279.3</td>
<td>299.8</td>
<td>280.3</td>
<td>219.2</td>
<td>257.2</td>
</tr>
<tr>
<td>2. Capital outflows plus “errors &amp; omissions”</td>
<td>16.9</td>
<td>93.1</td>
<td>150.3</td>
<td>228.3</td>
<td>188.5</td>
<td>246.9</td>
<td>306.6</td>
</tr>
<tr>
<td>3. Net private inflow (1) – (2)</td>
<td>45.2</td>
<td>6.2</td>
<td>129.0</td>
<td>71.5</td>
<td>91.8</td>
<td>-27.7</td>
<td>-49.4</td>
</tr>
<tr>
<td>4. Developing countries’ share in total global private capital flows</td>
<td>11.8</td>
<td>12.4</td>
<td>13.2</td>
<td>14.4</td>
<td>9.9</td>
<td>7.6</td>
<td>7.6</td>
</tr>
<tr>
<td>5. Developing countries’ share in total global output</td>
<td>19.8</td>
<td>19.2</td>
<td>22.1</td>
<td>23.2</td>
<td>21.6</td>
<td>21.7</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Source: World Bank (2001, Tables 2.1, 2.2, 2.3).

Except for a few mineral-rich countries such as Nigeria, Sub-Saharan Africa is poorly endowed with the human capital, infrastructure, and resources that would attract investors. African countries have only 55 kms of rural highways per thousand square kilometers, compared to more than 800 kms in India; Africa has 10 times fewer telephones per capita than Asia (Collier and Gunning 1999, pp.71-2). These countries do not seem to have done much to improve their infrastructure in order to compete for investment since reducing capital controls. The percentage of paved roads in Sub-Saharan Africa actually fell in the 1990s (World Bank 2001, p.309). Between 1980 and 1995, electricity generating capacity and the number of telephone mainlines
both grew more slowly in the average African country than in the rest of the world.\textsuperscript{27}

In short, capital account liberalization does not appear to have resulted in a significant net inflow of capital into the most underdeveloped countries. And there is little evidence that governments in these countries have been pressured by the competition for capital to institute more business-friendly policies.

6 Conclusions

The free flow of capital is viewed by many as a powerful disciplining force, pressuring governments to enact business-friendly policies, reduce welfare programs, and cut waste and corruption. Although scholars differ over whether the benefits of such discipline outweigh the costs, few question that it exists. This view informs policy debates on the desirability of both political decentralization and the liberalization of international capital flows.

We showed, however, that when regions or countries differ markedly in natural resources, human capital, or infrastructure, the disciplining effect is likely to be one-sided. Better-endowed units, knowing they will win the competition, compete aggressively and drain most or all the capital from their poorly-endowed counterparts. Poorly-endowed units, knowing they will lose, simply give up. At the same time, capital mobility removes a second source of discipline in poorly-endowed units—the desire to make domestic capital more productive. Since all domestic capital flees such units, their governments cannot make it more productive through business-friendly policies. As a result, governments in poorly-endowed units may be even less business-friendly when capital is mobile than when it is not. When runners start so far behind that they cannot win a race, they do not run very fast.

Our argument should not be taken as a blanket endorsement of capital controls. There are well-known efficiency reasons for favoring the free flow of capital. However, it suggests that

\textsuperscript{27} Calculated from database for Canning (1998).
some more thinking may be in order about how to organize capital competition. Although we hesitate to draw policy conclusions from such a simple model, three ideas might be worth at least exploring. First, when endowment differences are not too large, external aid—if used to finance market infrastructure, improve human capital, or insure against exogenous risks—might reduce the initial productivity gap to the point at which capital competition does discipline all governments. In a decentralized state, centrally funded public investment in infrastructure in poorly-endowed regions might enable such regions to compete, giving their governments an incentive to reform themselves.28 Second, if the disciplining effect of capital competition is considered desirable, there may be a role for liberalizing capital flows first within clubs of countries (or regions) that have similar endowments. Freeing up capital flows within the European Union or within a group of African states may achieve the benefits of competition better than if poorer African countries were to integrate directly into world capital markets. Third, there are many reasons why political decentralization might be considered desirable. But if the goal is, at least in part, to impose discipline on local governments, the analysis here suggests an important qualification. Decentralization may achieve this aim in countries where local units have similar endowments. However, within geographically diverse countries, decentralization may at times actually weaken market discipline on the worst-endowed regions.

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28 The challenge, of course, is how to ensure that such central investments or international aid are spent on infrastructure rather than being misused by existing corrupt governments. To this, we do not have any simple answers.
Appendix: Proofs

Proof of Proposition 1: The first order condition for $G_i$'s optimization problem is
\[
\frac{dU_i}{dI_i} = (1-t) \frac{\partial F_i}{\partial I_i} + \lambda v'(c)(t \frac{\partial F_i}{\partial I_i} - 1) = 0
\]

To check the second order condition, note that
\[
\frac{d^2 U_i}{dI_i^2} = (1-t)f_{II} + \lambda v''(c)(tf_i - 1)^2 + \lambda tv'(c)f_{II}
\]
All three terms are negative by the concavity of $F_i$ and $v$, so $U_i$ is globally concave in $I_i$.

Hence the solution to the first order condition, if it exists, is $G_i$'s optimal choice of infrastructure.

The existence of an interior solution requires a relatively large $\frac{\partial F_i}{\partial I_i}$ at $I_i = 0$ and a relatively small $\lambda$. Because of the complementarity between capital and infrastructure, a large capital endowment helps ensure an interior solution of $I_i$.

By the Implicit Function Theorem, we have
\[
\frac{\partial T_i}{\partial k_i} = -\frac{\partial^2 U_i/\partial I_i \partial k_i}{\partial^2 U_i/\partial I_i^2} = -\frac{(1-t)f_{II} + \lambda v''(c)(tf_i - 1)f_{II} + \lambda tv'(c)f_{II}}{\partial^2 U_i/\partial I_i^2}
\]
Since $f_{II} > 0$, $v''(c) < 0$, and $(tf_i - 1) < 0$ (from the first order condition), $\frac{\partial T_i}{\partial k_i} > 0$.

Replacing $k_i$ with $\rho$ in the above expression and noting that $\partial^2 F_i/\partial I_i \partial \rho \geq 0$ and $\partial F_i/\partial \rho \geq 0$, we have $\frac{\partial T_i}{\partial \rho} \geq 0$. The public consumption of $G_i$ is $\bar{c}_i = S + tf_i - T_i$. We assume that $S$ is sufficiently large so that $\bar{c}_i \geq 0$ is satisfied for all $i$.

Q.E.D.

Proof of Lemma 1: Parts (i) and (ii) are covered in the text. For (iii), note that
\[
\frac{\partial^2 k}{\partial r \partial I} = -\frac{1}{(1-t)(f_{kk})^2}[f_{kkk} \frac{\partial k}{\partial I} + f_{kl}] = \frac{1}{(1-t)(f_{kk})^3}[f_{kkk}f_{kl} - f_{kl}f_{kk}] \leq 0
\]
For (iv), we have
\[
\frac{\partial^2 k}{\partial \rho \partial I} = -\frac{1}{(f_{kk})^2}[(\frac{\partial f_{kl}}{\partial \rho} + f_{kkk} \frac{\partial k}{\partial \rho})f_{kk} - (\frac{\partial f_{kk}}{\partial \rho} + f_{kkk} \frac{\partial k}{\partial \rho})f_{kl}] = \frac{1}{(f_{kk})^3}[(\frac{\partial f_{kl}}{\partial \rho} - \frac{\partial f_{kl}}{\partial \rho}f_{kk}) + (f_{kkk}f_{kl} - f_{kl}f_{kk}) \frac{\partial k}{\partial \rho}]
\]
Both terms in the square brackets are nonnegative, so the result follows. Q.E.D.

Proof of Lemma 2: Differentiating the left-hand side of Equation (7) with respect to $r$ gives

$$f_{kl} \frac{\partial k_i}{\partial r} + f_{kk} \frac{\partial k_i}{\partial r} \frac{\partial I_i}{\partial I_i} + f_k \frac{\partial^2 k_i}{\partial I_i \partial r}$$

$$= f_{kl} \frac{\partial k_i}{\partial r} - f_{kl} \frac{\partial k_i}{\partial r} + f_k \frac{\partial^2 k_i}{\partial I_i \partial r}$$

$$= f_k \frac{\partial^2 k_i}{\partial I_i \partial r} \leq 0$$

where the second equation above follows from Equation (5) that $\partial k_i / \partial I_i = -f_{kl} / f_{kk}$ and the last inequality follows from Lemma 1 (iii). The right-hand side of Equation (7), $\tau$, is increasing in $v'(c_i)$, and hence is decreasing in $c_i$. Since $c_i = S + tf_i(I_i, k_i(I_i, r)) - I_i$ is decreasing in $r$, the opportunity cost of infrastructure investment, $\tau$, is increasing in $r$. Therefore, $\partial I_i / \partial r \leq 0$.

Similarly, for well-endowed units, the derivative of the left-hand side of (7) with respect to $\rho$ is

$$\frac{\partial^2 f}{\partial I_n \partial \rho} + f_{kl} \frac{\partial k_n}{\partial \rho} + \left(f_{kk} \frac{\partial k_n}{\partial \rho} + \frac{\partial^2 f}{\partial k_n \partial \rho} \frac{\partial I_n}{\partial I_n} + f_k \frac{\partial^2 k_n}{\partial I_n \partial \rho} \right) \frac{\partial k_n}{\partial \rho} + f_k \frac{\partial^2 k_n}{\partial I_n \partial \rho}$$

$$= \frac{\partial^2 f}{\partial I_n \partial \rho} + \frac{\partial^2 f}{\partial k_n \partial \rho} \frac{\partial k_n}{\partial \rho} + f_k \frac{\partial^2 k_n}{\partial I_n \partial \rho} > 0$$

It is easy to see that the opportunity cost of infrastructure investment, $\tau$, is decreasing in $\rho$.

Thus we have $\partial I_i / \partial \rho > 0$. Q.E.D.

Proof of Proposition 2: First we show that $dk_i / dr < 0$. Note that $k_i(r) = k_i(I_i(r), r)$, where $k_i(I_i, r)$ is derived from the capital allocation equation (4). Then,

$$\frac{dk_i(r)}{dr} = \frac{dk_i}{dl_i} \frac{dl_i}{dr} + \frac{dk_i}{\partial r} < 0$$

since $\partial k_i / \partial I_i > 0$ and $\partial k_i / \partial r < 0$ by Lemma 1, and $\partial I_i / \partial r \leq 0$ by Lemma 2. It follows that the total “demand function” for capital, $Nk_n(r; \rho) + Mk_m(r)$, is strictly downward sloping in $r$. As long as the total supply of capital in the economy, $K$, is not too small (more precisely, $K < Nk_n(0; \rho) + Mk_m(0)$) or too large (more precisely, $K > Nk_n(\infty; \rho) + Mk_m(\infty)$), the
model has a unique equilibrium with a net return to capital of \( r^*(\rho) \).

For well-endowed units, let \( k_n(r; \rho) = k_n(I_n(r; \rho), r; \rho) \) be their capital “demand” curve. This curve moves up as \( \rho \) increases, because

\[
\frac{\partial k_n(I_n(r; \rho), r; \rho)}{\partial \rho} = \frac{\partial k_n}{\partial \rho} \frac{\partial I_n}{\partial \rho} + \frac{\partial k_n}{\partial \rho} > 0
\]

This holds since \( \partial k_n / \partial I_n > 0 \) and \( \partial k_n / \partial \rho > 0 \) by Lemma 1, and \( \partial I_n / \partial \rho > 0 \) by Lemma 2. Since poorly-endowed units have smaller \( \rho \) than well-endowed units (actually by our normalization, \( \rho = 0 \), their capital “demand” curve is lower than that of well-endowed units. Hence in equilibrium it must be that \( k_m^* < k_n^* \). Furthermore, the equilibrium net return to capital \( r^*(\rho) \) must be strictly increasing in \( \rho \) since greater \( \rho \) means greater total capital demand. Consequently, \( k_m^* \) will be decreasing in \( \rho \) and \( k_n^* \) will be increasing in \( \rho \).

By Lemma 2, \( I_n(r; \rho) > I(r; \rho = 0) = I_m(r) \) for all \( r \). So in equilibrium \( I_n^* > I_m^* \).

Moreover, we have

\[
\frac{d l_m}{d \rho} = \frac{d l_m}{d r} \frac{d r}{d \rho} \leq 0
\]

so \( l_m \) is decreasing in \( \rho \). We also have

\[
\frac{d l_n(r; \rho)}{d \rho} = \frac{d l_n}{d r} \frac{d r}{d \rho} + \frac{\partial l_n}{\partial \rho}
\]

Since the first term is negative while the second is positive, the sign of \( d l_n / d \rho \) is indeterminate.

Q.E.D.

**Proof of Proposition 3:** First note that for sufficiently large \( r \), the “demand” for capital by poorly-endowed units, \( k_m(r) \), is zero. This is because

\[
\frac{d k_m(r)}{d r} = \frac{d k_m}{d l_m} \frac{d l_m}{d r} + \frac{\partial k_m}{\partial r} < \frac{\partial k_m}{\partial r} = \frac{1}{(1-t)f_{kk}}
\]

is bounded away from zero by assumption. Next we show that as \( \rho \) increases, \( r^*(\rho) \) increases unboundedly. For the capital “demand” curve of well-endowed units, note that

\[
\frac{\partial k_n(I_n(r; \rho), r; \rho)}{\partial \rho} = \frac{\partial k_n}{\partial I_n} \frac{\partial I_n}{\partial \rho} + \frac{\partial k_n}{\partial \rho} > \frac{\partial^2 f(k, I; \rho) / \partial k \partial \rho}{f_{kk}}
\]
is bounded away from zero by assumption. Since $Nk_n^*(r; \rho) + Mk_n^*(r) = K$, we have

$$\frac{dr^*}{d\rho} = -\frac{N\partial k_n^*(r; \rho)/\partial \rho}{N\partial k_n^*(r; \rho)/\partial r + M\partial k_n^*(r)/\partial r}$$

which is bounded away from zero. Thus, $r^*(\rho)$ increases unboundedly as $\rho$ increases.

It follows that there will be some $\rho$ such that $r^*(\rho)$ is high enough that $k_m^* = 0$.

Immediately we have $k_n^* = K/N$. As for infrastructure investment, at $k_m^* = 0$, $\partial f / \partial I$ is small because of the complementarity between capital and infrastructure, and $\partial k / \partial I$ is small by Lemma 1. Thus, from Equation (7), the marginal benefit of infrastructure investment for poorly-endowed units is small. For sufficiently large $\lambda$, we have $I_m^* = 0$. For well-endowed units, at $k_n^* = K/N$, $\partial f / \partial I$ is large and $\partial k / \partial I$ is large. Moreover, by assumption, $\partial f (k, I; \rho) / \partial k$ is sufficiently large for large $\rho$. Therefore, $I_n^* > 0$.

Q.E.D.

Proof of Proposition 4: By Propositions 2 and 3, as $\rho$ increases, the equilibrium of the model under capital mobility continuously shows more and more divergence between the two kinds of units, all the way to polarization. By Proposition 1, $\bar{k}_m > 0$ and $\bar{T}_m > 0$, and they are non-decreasing in $\rho$. Therefore, for sufficiently large $\rho$, $\bar{k}_m > k_m^*$ and $\bar{T}_m > I_m^*$. Immediately we have $\bar{F}_m = f(\bar{k}_m, \bar{T}_m) > F_n^* = f(k_m^*, I_m^*)$. For well-endowed units, since $\bar{k}_m > k_m^*$, it must be that $\bar{k}_n < k_m^*$. Comparing Equations (3) and (7), we can see that $I_n^*(k) > \bar{T}_n(k)$ for the same $k$. Hence, $I_n^*(k_n^*) > I_n^*(\bar{k}_n) > \bar{T}_n(\bar{k}_n)$. So the total output $\bar{F}_n = f(\bar{k}_n, \bar{T}_n) < F_n^* = f(k_n^*, I_n^*)$.

Q.E.D.

Proof of Proposition 5: From $G_1$’s first order condition, we have

$$(4\gamma\delta - 3b^2)I_1 + b^2I_2 = \gamma(bK - 4\tau_1) + b(2a + 3\rho)$$

where $\tau_1 = \lambda v'(c_I) / [1 + (\lambda v'(c_I) - 1)t]$. Since $\gamma\delta \geq b^2$ and $v$ is concave, the second order condition is satisfied. From the above equation, it is easy to verify that $G_1$’s best response function, $I_1(I_2, \rho)$, is strictly decreasing in $I_2$ and strictly increasing in $\rho$. Similarly, $G_2$’s first order condition can be written as

$$(4\gamma\delta - 3b^2)I_1 + b^2I_2 = \gamma(bK - 4\tau_2) + b(2a - \rho)$$
where \( \tau_2 = \lambda v'(c_2)/[1 + (\lambda v'(c_2) - 1)t] \). Hence \( G_2 \)'s best response function \( I_2^*(I_1, \rho) \) is strictly decreasing in \( I_1 \) and strictly decreasing in \( \rho \).

If \( \rho = 0 \), then there exists a symmetric equilibrium \( I_1^* = I_2^* = I^* > 0 \), where \( I^* \) is the solution to \((4\gamma \delta - 2b^2)I = \gamma (bK - 4\tau) + 2ab\). Such a solution always exists because the right-hand side is decreasing in \( I \) (at least for sufficiently large \( I \)): \( \tau \) is increasing in \( I \) since it is increasing in \( v'(c) \), but \( v'(c) \) is decreasing in \( c \) and \( c \) is decreasing in \( I \) (at least for large \( I \)). Since both best response functions, \( I_1^*(I_2, \rho) \) and \( I_2^*(I_1, \rho) \), change smoothly as \( \rho \) increases, it follows that an interior equilibrium exists for sufficiently small \( \rho \).

Totally differentiating the best response functions gives
\[
\begin{align*}
(4\gamma \delta - 3b^2 + 4\gamma \partial \tau_1 / \partial \tau_1)(dI_1) + b^2(dI_2) &= 3b(d\rho) \\
b^2(dI_1) + (4\gamma \delta - 3b^2 + 4\gamma \partial \tau_2 / \partial \tau_2)(dI_2) &= -b(d\rho)
\end{align*}
\]
We can solve \( dI_1^* / d\rho \) and \( dI_2^* / d\rho \) from these equations. Since \( \partial \tau_1 / \partial \tau_1 > 0 \), it can be readily shown that \( dI_1^* / d\rho > 0 \) and \( dI_2^* / d\rho < 0 \). Since \( I_1^* = I_2^* \) when \( \rho = 0 \), then for \( \rho > 0 \) it must be that \( I_1^* > I_2^* \). It immediately follows from Equation (10) that \( k_1^* > k_2^* \) and \( dk_1^* / d\rho > 0 \) and \( dk_2^* / d\rho < 0 \). Q.E.D.

**Proof of Proposition 7:** Since \( \partial k_i / \partial I_i = -f_{ki} / f_{kk} = k_i / I_i \), it is easy to obtain that
\[
\partial F_i / \partial I_i + \partial F_i / \partial k_i \partial k_i / \partial I_i = F_i / I_i.
\]
Using the capital allocation rule \( r = (1-t)f_k \), we have
\[
\partial k_i / \partial t_i = r/(1-t_i)^2 f_{kk} = f_i/(1-t_i)_i f_{kk} = -k_i /[(1-t_i)(1-\alpha)].
\]
Thus, Equation (16) becomes
\[
F_i / I_i = A(k_i / I_i)^\alpha = 1 + \alpha / (1-\alpha) = 1/(1-\alpha)
\]
(17)
From the capital allocation equation \( r = (1-t)f_k \), we have
\[
(1-t_i)\alpha A(k_i / I_i)^{\alpha-1} = r
\]
(18)
Dividing Equation (17) by (18) yields
\[
k_i / I_i = (1-t_i)\alpha / [r(1-\alpha)]
\]
(19)
Plugging this back into Equation (17), we obtain
\[
\frac{1-t_i}{r} = A \frac{1}{\alpha} \alpha^{-1}(1-\alpha) \frac{1-\alpha}{\alpha}
\]
(20)
Therefore, \( t_i(r; \rho) \) is decreasing in \( r \) and increasing in \( \rho \) (since \( A(\rho) \) is increasing in \( \rho \)).
means that when \( r \) is high (because of high infrastructure investments and low tax rates in the rest of the economy), governments lower their tax rates to stay competitive in attracting capital. Moreover, all else equal, well-endowed units will have higher tax rates than poorly-endowed units. Note that for poorly-endowed units, \( A = A_o \) on the right hand side of Equation (20) is independent of \( \rho \).

From Equation (15), we can solve for \( \lambda v'(c_i) \) and get

\[
\lambda v'(c_i) = \frac{1-t_i}{1-t_i-\alpha} = 1 + \frac{\alpha}{1-t_i-\alpha} \tag{21}
\]

Clearly \( t_i \) must be smaller than \( 1 - \alpha \). From Equation (17), we have \( F_i = I_i/(1-\alpha) \). Thus,

\[
c_i = S + t_iF_i - I_i = S - \frac{1-\alpha-t_i}{1-\alpha}I_i.
\]

Since \( v' \) is strictly decreasing, its inverse function, which we denote by \( \chi \), is also strictly decreasing. From Equation (21), we get

\[
\frac{1-\alpha-t_i}{1-\alpha}I_i = S - \chi(\theta), \quad \text{where} \quad \theta = \frac{1}{\lambda} + \frac{\alpha}{\lambda(1-t_i-\alpha)} \tag{22}
\]

Since \( \theta \) is increasing in \( t_i \), the right hand side of Equation (22) is increasing in \( t_i \). Therefore, \( I_i \) is increasing in \( t_i \). As a result, \( I_i(r; \rho) \) is decreasing in \( r \) and increasing in \( \rho \).

Using Equation (20), we can rewrite Equation (21) as

\[
k_i = I_i \frac{\alpha}{1-\alpha} \frac{1-t_i}{r} = I_i \frac{1}{\alpha} B \tag{23}
\]

where \( B \) is a constant depending only on \( \alpha \). It follows immediately that \( k_i(r; \rho) \) is decreasing in \( r \). To find out how \( k_i(r; \rho) \) changes with \( \rho \), we use Equation (22) to rewrite Equation (19) as

\[
k_i = I_i(1-t_i) \frac{\alpha}{1-\alpha} \frac{1-t_i}{r} = \frac{1-t_i}{1-\alpha-t_i} \frac{\alpha}{r} [S - \chi(\theta)] = \left[1 + \frac{\alpha}{1-\alpha-t_i} \right] \frac{\alpha}{r} [S - \chi(\theta)] \tag{24}
\]

Clearly the right hand side is increasing in \( t_i \). Since \( t_i \) is increasing in \( \rho \), \( k_i(r; \rho) \) is increasing in \( \rho \).

As in the proof of Proposition 2, since \( k_n(r; \rho) \) is increasing in \( \rho \), in equilibrium \( r^*(\rho) \) must be increasing in \( \rho \). Therefore, \( k^*_m \), \( I^*_m \), and \( t^*_m \) should all decrease in \( \rho \). It follows that \( k_n^* \) must be increasing in \( \rho \). \( I^*_n \) and \( t^*_n \) can either increase or decrease in \( \rho \).

The equilibrium \( r^*(\rho) \) is solved from the market clearing condition.
\[ N k^*_n(r; \rho) + M k^*_m(r) = K \]

Consider what will happen when \( \rho \) grows larger and larger. For any fixed \( r \), as \( \rho \) becomes very large, \( A(\rho) \) becomes very large, so the right-hand side of Equation (20) becomes very small. Hence \( \tau \) will reach its upper bound of \( 1 - \alpha \). Then, from Equation (24), \( k^*_n \) will become unboundedly large. Therefore, to satisfy the market clearing condition, \( r^* \) must go to infinity as \( \rho \) goes to infinity. As \( r^* \) goes to infinity, for poorly-endowed units the first order condition of Equation (20) cannot be satisfied. Hence it must be that \( \tau^* = 0 \). From Equation (24), with \( \tau^*_m = 0 \) and \( r^* \) going to infinity, \( k^*_m \) must go to zero as \( \rho \) goes to infinity. From Equation (21), with \( \tau^*_m = 0 \), \( c^*_m \) depends only on \( \alpha \) and \( \lambda \). When \( \lambda \) is sufficiently large, \( c^*_m \) will be very large. From Equation (22), with \( \tau^*_m = 0 \), equilibrium infrastructure investments for poorly-endowed units become \( I^*_m = S - c^*_m \), so \( I^*_m \) will go to zero.

On the other hand, as \( \rho \) goes to infinity, \( k^*_n \) will converge to \( K/N \). Since \( r^* \) goes to infinity, from Equation (24), it must be that \( \tau^*_n \) goes to \( 1 - \alpha \). From Equation (21), \( c^*_n \) must go to zero. From Equation (22), \( I^*_n \) becomes very large as \( \rho \) goes to infinity.

Since the equilibrium outcome varies smoothly with \( \rho \) and becomes polarized as \( \rho \) goes to infinity, it follows that for sufficiently high \( \rho \), the equilibrium outcome will be sufficiently unbalanced that the results of Proposition 7 hold. \( \text{Q.E.D.} \)
References


