Reserve Price Signaling

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Abstract

This paper studies an auction model in which the seller has private information about the object’s characteristics that are valued by both the seller and potential buyers. In a general valuation model in which buyers’ signals are affiliated, we identify sufficient conditions under which the seller can use reserve prices to credibly signal her private information. Working with a reserve “markup” instead of the reserve price itself, we are able to transform the original problem into a standard signaling game. We characterize the unique separating equilibrium of this signaling game. When the buyers’ signals are independent, the optimal reserve price is shown to be increasing in the number of bidders under certain conditions. In a linear valuation model with independent signals, we characterize the equilibrium reserve price schedule analytically. Using this characterization, we present several applications. Extensions of our basic model are also discussed.

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1. Introduction

In this paper we consider an auction environment in which a seller of one single object has private information about the object’s characteristics. These characteristics affect the seller’s valuation of the object and the common valuation for a group of potential buyers, each of whom also has an independent private signal about the object. For example, a seller of an artwork (e.g., an auction house) may know better than potential buyers the conditions (quality, rarity, history, etc.) and the secondary market value of the artwork. Similarly, a government agency auctioning procurement of a public project may have better information than bidding firms about certain factors (e.g., environmental impacts and regulations) that affect both its valuation of the project and project costs common to all bidding firms.

If direct verification of the seller’s information is costless, it is incentive compatible for the seller to truthfully reveal her information for the following reason. Sellers with private information indicating high common value for the buyers have an incentive to reveal their information since buyers will then be willing to bid more for the object. Since the same argument holds for any set of types, the sellers with high types within the set always have an incentive to reveal and the only Nash equilibrium is the full revelation of the seller’s private information. However, in many auction settings, a costless revelation technology (e.g., a perfectly objective evaluation by a third party) may not be available to the seller. In such cases, the seller’s announcement of her information to the potential buyers may not be credible as she faces the adverse selection problem, that is, she always wants to claim the highest possible value of $s$ to the buyers. A natural way to credibly reveal the private information is through signaling, and a natural signaling instrument in this environment is the reserve price: a high type seller has an incentive to signal this to the buyers by setting a high reserve price.

In this paper we introduce a reserve price signaling model in which the buyers’ private signals are affiliated. We reformulate the model by focusing on the seller’s reserve “markup” as the key signaling variable, rather than the reserve price itself. With this reformulation, the model fits into a standard signaling framework (Riley, 1979), and the analysis is greatly simplified. We characterize the unique separating equilibrium in which the lowest type seller sets a reserve price that is optimal under complete
information. We then show that when the buyers’ signals are independent, the equilibrium reserve price is increasing in the number of bidders under fairly general specifications of buyers’ valuations. Thus our results show that a reserve price can play a more central role than perceived by the traditional literature. In the standard private value auction model, the seller’s optimal reserve price is set to capture additional revenue when there is only one buyer who has a valuation much higher than her own. This optimal reserve price is independent of the number of bidders. Therefore, unless the number of bidders is very small, the probability that the reserve price is binding is small and hence the extra profit captured by setting a reserve price is also low. In contrast, when the reserve price plays a signaling role, our results indicate that the probability that it is binding may not decrease as the number of bidders becomes large.

After analyzing the general signaling model, we study a linear valuation model in which each buyer’s valuation is the sum of his own private signal and a common value component which is the seller’s private information. The seller’s own valuation for the object is proportional to her private signal. We solve for an analytical solution of the reserve price schedule in the separating equilibrium. Based on the equilibrium reserve price schedule characterized, three applications are presented. In the first application, we consider the situation in which outside certification of the object’s common value component is available at a cost. It turns out that the seller will go for the outside certification if and only if his type is higher than some cutoff point. In the second application, we evaluate how reserve prices change with the relative importance of the private value components. We find that when the relative importance of private value component increases, the reserve price increases in an accelerating rate.\footnote{In the first two applications, we suppose that trade is always desirable, that is, the seller’s valuation is not greater than the common value component in the buyers’ valuations. The results are ambiguous if the alternative assumption is made.} Our model can also be useful to study non-auction problems. For example, we apply our model to the well-known Lemons market (Akerlof, 1970) to study the possibility of signaling in that market. We illustrate that in the case of pair-wise matching, a separating equilibrium exists in which sellers with different qualities set different prices and their private information is fully revealed to the market.
We also discuss possible ways to further extend our basic model. Our model can be extended to a more general setting that allows for positive but not necessarily perfect correlation between the seller’s expected valuation and a common value component for the buyers’ valuations. Such a setting is very natural when the object’s characteristics are multi-dimensional and the seller and the bidders may place different weights on the relative importance of different dimensions. For example, a seller of an artwork may be mostly concerned with the artwork’s secondary market value, while potential buyers (who buy for self consumption) may care more about its conditions. We show that our analysis of the basic model carry through to this more general setting almost unchanged, thus giving our results a greater applicability. Another possible extension is to allow positive correlation between the seller’s private information and the buyers’ private signals. However, it is unclear under what conditions the single crossing property will hold. Thus, this extension presents technical difficulties not easy to overcome.

A simultaneous and independent paper by Jullien and Mariotti (2004) is closely related to ours. They consider the case with two bidders in which the seller’s valuation is greater than the buyers’ common value component. They solve for the separating equilibrium of the model and compare the equilibrium outcome with the optimal mechanism for a monopoly broker who buys from the seller and sells to the buyers. They also analyze pooling and partial pooling equilibria.

The paper is organized as follows. We introduce our basic model in Section 2, followed by the equilibrium characterization in Section 3. Section 4 studies a linear valuation model and solves for the equilibrium analytically. In Section 5 we discuss possible ways to further extend our analysis, and then offer concluding remarks in Section 6.

2. The Model

We consider a second-price sealed bid auction in which \( n \) symmetric buyers bid for a single, indivisible object.\(^2\) The seller observes a signal \( s \), which is not observed by the

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\(^2\) We consider the second-price auctions mainly for simplicity of analysis. The insights of our main results should still carry over to other auction formats. However, with affiliated valuations revenue equivalence usually does not hold among different auction formats.
buyers. The seller’s signal \( s \) is drawn from a distribution function \( G(\cdot) \) with support \( S = [\underline{s}, \overline{s}] \) and density function \( g(s) > 0 \) for all \( s \in [\underline{s}, \overline{s}] \). The seller’s own valuation for the item is \( \xi(s) \), which is strictly increasing in \( s \). Each buyer \( i \) observes a signal \( X_i \), which is his private information. \( X_i \)'s have joint distribution with density function \( f(z) \), where \( z = (x_1, x_2, \ldots, x_n) \in [\underline{x}, \overline{x}]^n \). Since buyers are symmetric, the marginal distributions of \( f(z) \) for all \( x_i \) are identical. Ex ante \( X_i \)'s are affiliated, with independence as a special case. We assume that the seller’s signal \( s \) is independent of the buyers’ signals \( X_i \)'s.\(^3\)

Given \( s \) and \( z = (x_1, x_2, \ldots, x_n) \), buyer \( i \)'s valuation for the item is given by \( V_i = u_i(s, x_1, \ldots, x_n) \). We assume that there is a function \( u \) on \( \mathbb{R}^{1+n} \) such that for all \( i \), \( u_i(s, x_1, \ldots, x_n) = u(s, x_i, x_{-i}) \). Therefore, all the buyers’ valuations depend on \( s \) in the same manner and each buyer’s valuation is a symmetric function of the other bidders’ signals. Moreover, we assume that the function \( u \) is nonnegative, and is continuous and increasing in all its arguments. It is also integrable so \( EV_i < \infty \).

Let \( X_{(i)} \) denote the highest signal among all but \( i \)'s signals. We define a function \( v_i : \mathbb{R}^2 \rightarrow \mathbb{R} \) by \( v_i(s, x) = E[V_i \mid S = s, X_i = X_{(i)} = x] \). By the symmetry of \( u_i \), \( v_i \) is identical for all \( i \), that is, \( v_i(s, x) = v(s, x) \). Because \( (s, x) \) are independent and hence affiliated, and because \( u \) is increasing in its arguments, \( v(s, x) \) is also increasing in both variables (Milgrom and Weber, 1982, Theorem 5). We add the non-degeneracy assumption so that \( v \) is strictly increasing in \( s \) and \( x \).

Our specification of buyers’ valuations is quite general. To give some examples, let \( \phi, \psi \) and \( \zeta \) be any positive increasing functions. Then the following valuation functions all fit in our framework:

1. For any signal profile \((s, z)\), \( u_i(s, z) = \phi(s) + \psi(x_i) \) or \( u_i(s, z) = \phi(s) \cdot \psi(x_i) \). In either of the two cases, buyers’ valuations are independent of other buyers’ signals. If their signals

\(^3\) This implies that even if bidders can communicate about their signals, they cannot infer the seller’s private information \( s \).
are independent and the seller’s signal \( s \) is revealed to the buyers, then the ensuing auction has the features of independent private value auctions.

2. For any signal profile \((s, z)\), \( u_i(s, z) = \phi(s) + \psi(x_i) + \zeta(x_{-i}) \) or

\[
u_i(s, z) = \phi(s) \cdot [\psi(x_i) + \zeta(x_{-i})], \text{ where, for example, } \psi(x_i) + \zeta(x_{-i}) = \sum_{j=1}^{n} x_j,
\]

\[
\psi(x_i) + \zeta(x_{-i}) = \frac{1}{n} \sum_{j=1}^{n} x_j.
\]

In these cases, buyers’ valuations depend on the seller’s signal, their own signals as well as other buyers’ signals. If their signals are independent and the seller’s signal \( s \) is revealed to the buyers, then the auction has the features of independent-signal common value auctions.

3. For any signal profile \((s, z)\), \( u_i(s, z) = \lambda \phi(s) + (1-\lambda) \max(x_i, x_2, ..., x_n) \). This is another case of a common value auction in which the valuation common to all buyers is a linear combination of the seller’s signal and the highest buyer signal.

Let \( X_{(1)} \) and \( X_{(2)} \) denote the highest and the second highest signal statistics among all the signals of the \( n \) buyers. For \( k = 1, 2 \), let \( F_{(k)}(\cdot) \) and \( f_{(k)}(\cdot) \) be the corresponding distribution and density functions, respectively. Our analysis will heavily rely on the following assumption:

**Assumption (R):** For any \( s \), \( J(s, x) = v(s, x) - \frac{\partial v(s, x)}{\partial x} \cdot \frac{F_{(2)}(x) - F_{(1)}(x)}{f_{(1)}(x)} \) is strictly increasing in \( x \).

This assumption is a generalization of the standard assumption in the independent private value auction setting that the “virtual surplus” \( x - (1 - F(x)) / f(x) \) is strictly increasing in \( x \). The following lemma identifies sufficient conditions for Assumption (R) to hold when \( X_i \)’s are independent.
Lemma 1: When $X_i$'s are independent, Assumption (R) is satisfied if

1. the hazard rate function of $X$ is increasing; and
2. $\frac{\partial^2 v(s,x)}{\partial x^2} \leq 0$.

Proof: Let $f(\cdot)$ be the density function of $x_i$ for all $i$. Let $h(x) = f(x)/(1-F(x))$ denote the hazard rate function of $X$. Suppose it is increasing (the regular distribution case). Then, by independence,

$$\frac{F_{(2)}(x) - F_{(1)}(x)}{f_{(1)}(x)} = \frac{1-F(x)}{f(x)}$$

Differentiating $J(s,x)$ with respect to $x$ we have

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x} - \frac{\partial^2 v}{\partial x^2} \frac{1}{h(x)} - \frac{\partial v}{\partial x} \frac{\partial}{\partial x} \left( \frac{1}{h(x)} \right)$$

The first and third terms above are positive by assumptions. The middle term is also positive as long as $\frac{\partial^2 v}{\partial x^2}$ is non-positive. \(Q.E.D.\)

We study the following signaling game. The seller announces a reserve price $r$ at the beginning of the auction. The buyers then submit sealed bids in the second price auction (or, equivalently, in an English ascending price auction.)\(^4\) As is typical in the signaling literature, our game has many equilibria. By the standard equilibrium refinement concepts such as the Intuitive Criterion (Cho and Kreps, 1987), pooling or partial pooling equilibria can be ruled out. In fact, by the results of Riley (1979), there is a unique separating equilibrium in our game if the lowest type seller chooses the reserve price that is optimal under complete information.\(^5\) Following this literature, we focus on such a unique separating equilibrium.

\(^4\) For the independent valuation case our results generalize immediately to other auctions (see Corollary 1.)

\(^5\) See also Mailath (1987) for his discussion of the existence and differentiability of such a separating equilibrium.
In a separating equilibrium, suppose the buyers, upon observing a reserve price $r$, believe that the value of the seller’s signal is $\hat{s}$. By Milgrom and Weber (1982), it is a Bayesian Nash equilibrium for each buyer $i$ to bid $v(\hat{s}, x_i) = E[V_i | S = \hat{s}, X_i = X_{(i)} = x_i]$, that is, his expected valuation conditional on that the seller’s signal is $\hat{s}$ and that the highest signal among all other buyers equals his own signal $x_i$. Following the literature, we focus on this symmetric equilibrium in this paper.

Define $r = v(\hat{s}, m)$, that is, $m$ is the lowest signal for a buyer to be willing to pay for the object at price $r$, given that the seller’s signal is $\hat{s}$. Later we will show that $m$ can be thought of as the “reserve markup” in the case of linear valuation models. Given the reserve price $r$, the auction has three possible outcomes. Contingent on these outcomes, the seller’s payoffs are determined as follows:

1. If $v(\hat{s}, X_{(1)}) \leq r = v(\hat{s}, m)$, then the highest bid is below the reserve and hence the good is not sold. In this case, the seller’s payoff is $\xi(s)$.
2. If $v(\hat{s}, X_{(2)}) \leq r = v(\hat{s}, m) < v(\hat{s}, X_{(1)})$, then only the highest bid is above the reserve and hence the good is sold to the buyer with the highest bid at the price of $r$. In this case, the seller’s payoff is $r = v(\hat{s}, m)$.
3. If $v(\hat{s}, X_{(2)}) > r = v(\hat{s}, m)$, then at least two buyers submit bids greater than the reserve and hence the good is sold to the buyer with the highest bid at the price of the second highest bid. In this case, the seller’s payoff is $v(\hat{s}, X_{(2)})$.

Since $v(\hat{s}, x)$ is strictly increasing in $x$, the first outcome occurs if $X_{(1)} \leq m$; the second outcome occurs if $X_{(2)} \leq m < X_{(1)}$; and the last outcome occurs if $X_{(2)} > m$. Therefore, when her signal is $s$, the buyers’ perception of $s$ is $\hat{s}$, and the seller’s expected payoff can be written as follows:

$$U(s, \hat{s}, m) = \xi(s)F_{(1)}(m) + v(\hat{s}, m)[F_{(2)}(m) - F_{(1)}(m)] + \int_m^x v(\hat{s}, x) dF_{(2)}(x)$$

(2.1)

Differentiating, we have
\[
\frac{\partial U}{\partial m} = \xi(s) f_{(1)}(m) + \nu(\hat{s}, m)[f_{(2)}(m) - f_{(1)}(m)] + \frac{\partial \nu}{\partial m}[F_{(2)}(m) - F_{(1)}(m)] - \nu(\hat{s}, m) f_{(2)}(m)
\]
\[
= (\xi(s) - \nu(\hat{s}, m)) f_{(1)}(m) + \frac{\partial \nu}{\partial m}[F_{(2)}(m) - F_{(1)}(m)]
\]
\[
= f_{(1)}(m)[\xi(s) - \hat{J}(\hat{s}, m)]
\]

(2.2)

\[
\frac{\partial U}{\partial \hat{s}} = \frac{\partial \nu}{\partial \hat{s}}[F_{(2)}(m) - F_{(1)}(m)] + \int_{m}^{\hat{s}} \frac{\partial \nu}{\partial \hat{s}} dF_{(2)}(x)
\]

(2.3)

Clearly, \( \frac{\partial U}{\partial m} \) is increasing in \( s \) and \( \frac{\partial U}{\partial \hat{s}} \) is independent of \( s \). Thus, the slope of the indifference curve in the \( m - \hat{s} \) plane is decreasing in \( s \), i.e.,

\[
\frac{d}{ds} \left[ \frac{d \hat{s}}{dm} \bigg|_{U=\hat{U}} \right] = \frac{d}{ds} \left[ -\frac{\partial U}{\partial m} \bigg/ \frac{\partial U}{\partial \hat{s}} \right] < 0
\]

(2.4)

Thus the single crossing condition holds, which opens up the possibility of signaling. By using the reserve markup \( m \) rather than the reserve price \( r \) itself as the key variable, we are able to transform our problem into a standard signaling model.\(^{6}\)

If \( s \) were directly observable to buyers, their perception \( \hat{s} = s \), so the seller would choose her markup \( m \) to maximize \( U(s, s, m) \). Let \( m^\ast(s) \) be the optimal full information reserve markup, then by Equation (2.2) and Assumption (R), we have

\[
m^\ast(s) = \begin{cases} \hat{x}, & \text{if } \xi(s) < J(s, \hat{x}); \\ J^{-1}_s(\xi(s)), & \text{if } J(s, \hat{x}) \leq \xi(s) < J(s, \hat{x}); \\ \hat{x}, & \text{if } \xi(s) \geq J(s, \hat{x}); \end{cases}
\]

(2.5)

where \( J^{-1}_s(\cdot) \) is the inverse function of \( J(s, \cdot) \).

3. Unique Separating Equilibrium

If there exists a separating equilibrium, then the inverse markup schedule \( s(m) \) must satisfy the following condition in equilibrium:

\(^{6}\) It can be verified that the single crossing property still holds if we work directly with the reserve price \( r \). But as can be seen below, working with the reserve markup will greatly simplify our analysis.
\[
\frac{ds}{dm} = -\frac{U_1(s,s,m)}{U_2(s,s,m)} \equiv b(m,s) \tag{3.1}
\]

where
\[
b(m,s) = \frac{f_{(1)}(m)[J(s,m) - \xi(s)]}{\hat{c}_S[F(2)(m) - F(0)(m)] + \int_m \hat{c}_S dF(2)(x)} \tag{3.2}
\]

That is, given any separating equilibrium schedule, type \( s \) seller will optimally choose reserve markup \( m \) according to the solution of (3.1). This condition says that the slope of the equilibrium schedule should equal the marginal rate of substitution between the reserve markup and the market perception about the type.

To understand this, consider the following figure. Buyers believe that the seller’s type is an increasing function \( \hat{s} = s(m) \). Type \( s \) then chooses her markup \( \hat{m} \) and hence her perceived value \( \hat{s} = s(\hat{m}) \) to maximize her payoff \( U(s,s(\hat{m}),\hat{m}) \). For incentive compatibility it must be the case that \( \hat{m} = s^{-1}(s) \), that is \( s = s(\hat{m}) \). The indifference map for type \( s \) must therefore be tangential to the function \( s(m) \) at \( \hat{s} = s \).

Fig. 3-1: Separating Equilibrium

**Theorem 1:** The differential equation (3.1) through the full information optimum \((s,m^*(s))\) for the lowest type \( s \) characterizes the unique separating equilibrium.
Proof: Consider the function $b(m,s)$ (See equation (3.2)). By Assumption (R), for each $s$ either $b(m,s) > 0$ or $b(m,s) = 0$ at a unique point in $[x, \bar{x}]$ and $b(m,s) > 0$ only for larger $m$. Therefore, following arguments paralleling those in Riley (1979), there is a unique solution going through $(s, m^*(s))$, the full information optimum for the lowest seller type. We next show that this unique solution is incentive compatible, that is, no type $s$ wants to deviate from $m(s)$ on the signaling schedule, hence constitutes the unique separating equilibrium.

Suppose that the buyers’ perception is given by $\hat{s} = s(m)$, which is the solution to (3.1) above. A seller of type $s$ thus chooses $m$ to maximize $U(s, s(m), m)$. Differentiating with respect to $m$,

$$
\frac{d}{dm}U(s, s(m), m) = U_2(s, s(m), m)s'(m) + U_3(s, s(m), m)
$$

$$
= U_2(s, s(m), m)\left[ s'(m) + \frac{U_3(s, s(m), m)}{U_2(s, s(m), m)} \right]
$$

$$
= U_2(s, s(m), m)\left[ -\frac{U_3(s(m), s(m), m)}{U_2(s(m), s(m), m)} + \frac{U_3(s, s(m), m)}{U_2(s, s(m), m)} \right]
$$

By the single crossing condition (2.4), the terms in the bracket above only changes signs once and $U(s, s(m), m)$ takes on its maximum at $m$ where $s(m) = s$. Therefore, incentive compatibility is satisfied. Q.E.D.

Theorem 1 characterizes the separating equilibrium in terms of the reserve markup $m$ for our general model. To recover the reserve price schedule, we have

$$
r = v(s, m(s)).$$

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7 Suppose there is only one buyer. Consider the seller’s strategy from a mechanism design perspective. If the item is sold with probability $\pi$ and the expected transfer from the buyer to seller is $t$, the expected seller payoff is $V(\pi, t, s) = (1 - \pi)\xi(s) + t$ while the expected buyer payoff is $U(\pi, t, s, x) = \pi u(s, x) - t$.

For the finite type case, the results of Maskin and Tirole (1992) imply that the separating equilibrium analyzed here is the optimal incentive compatible mechanism for the seller. We conjecture their analysis can be generalized to the $n$ buyer auction environment when signals are independent.
Using the general characterization result in Theorem 1, the rest of the paper studies the properties of the equilibrium reserve schedule in more specific valuation models. First we consider the case in which the bidders’ signals $X_i$’s are independent. Under certain conditions regarding buyers’ valuations, we can show that the equilibrium reserve price is increasing in the number of bidders $n$.

**Theorem 2:** Suppose the bidders’ signals $X_i$’s are independent. In the separating equilibrium, the markup and hence the reserve price $r(s) = v(s, m(s))$ is higher for larger $n$ for every $s > \xi$ if (i) $v(s, x)$ is non-increasing in $n$, and (ii) $\partial v(s, x)/\partial s$ is non-decreasing in $n$.

**Proof:** Since $X_i$’s are independent, $\frac{F_2(x) - F_1(x)}{f_1(x)} = 1 - \frac{F(x)}{f(x)}$. Therefore, $J(s, x)$ is non-increasing in $n$ as $v(s, x)$ is non-increasing in $n$ and $\partial v(s, x)/\partial s$ is non-decreasing in $n$.

Define

$$\kappa(m, s; n) = \frac{\partial v(s, m)}{\partial s} \frac{F_2(m) - F_1(m)}{f_1(m)} \int_{m}^{\bar{s}} \frac{\partial v(s, x)}{\partial s} dF_2(x)$$

$$= \frac{\partial v(s, m)}{\partial s} \frac{1 - F(m)}{f(m)} + \int_{m}^{\bar{s}} \frac{\partial v(s, x)}{\partial s} dF_2(x)$$

$$= \frac{\partial v(s, m)}{\partial s} \frac{1 - F(m)}{f(m)} + \frac{\int_{m}^{\bar{s}} \partial v(s, x)}{\partial s} dF_2(x)$$

We want to show that $\kappa(m, s; n)$ is strictly increasing in $n$. Since $J(s, m)$ is non-increasing in $n$ and $\partial v(s, x)/\partial s$ is non-decreasing in $n$, the first term above is non-decreasing in $n$. So it suffices to show that

$$\rho(s, m; n) = \frac{\int_{m}^{\bar{s}} \partial v(s, x)}{\partial s} dF_2(x) = \frac{(n-1) \int_{m}^{\bar{s}} \partial v(s, x)}{\partial s} F^{n-2}(x)(1 - F(x)) dF(x)$$

$$= \frac{F^{n-2}(m) f(m)}{F^{n-1}(m) f(m)}$$
is strictly increasing in $n$. Taking logarithms and then differentiating with respect to $n$, we have

$$
\frac{\partial \log \rho}{\partial n} = \frac{1}{n-1} - \log F(m) + \frac{\int \frac{\partial v}{\partial s} F_{n-2}(x)(1-F(x)) \log F(x) dF(x)}{\int m \frac{\partial F_{n-2}(x)(1-F(x))}{\partial s} dF(x)} + \frac{\int \frac{\partial^2 v}{\partial s^2} F_{n-2}(x)(1-F(x)) dF(x)}{\int m \frac{\partial F_{n-2}(x)(1-F(x))}{\partial s} dF(x)}
$$

$$
\geq \frac{1}{n-1} - \log F(m) + \frac{\int \frac{\partial v}{\partial s} F_{n-2}(x)(1-F(x)) \log F(x) dF(x)}{\int m \frac{\partial F_{n-2}(x)(1-F(x))}{\partial s} dF(x)}
$$

$$
\geq \frac{1}{n-1} - \log F(m) + \frac{\int \frac{\partial v}{\partial s} F_{n-2}(x)(1-F(x)) \log F(m) dF(x)}{\int m \frac{\partial F_{n-2}(x)(1-F(x))}{\partial s} dF(x)}
$$

$$
= \frac{1}{n-1} - \log F(m) + \log F(m)
$$

$$
= \frac{1}{n-1} > 0
$$

Therefore, $\partial \kappa(s, m; n) / \partial n > 0$, which implies that

$$
\frac{\partial}{\partial n} \left( \frac{\partial m}{\partial s} \right) = \frac{\partial}{\partial s} \left( \frac{\partial m}{\partial n} \right) > 0 \quad (3.3)
$$

Now we consider the sign of $\partial m^*(s) / \partial n$. By (2.5), if $m^*(s)$ takes a corner solution, it is independent of $n$; when it takes an interior solution, we have $\xi(s) = J(s, m^*(s))$. Differentiating with respect to $n$ on both sides, we have

$$
0 = \frac{\partial J(s, m^*(s))}{\partial n} + \frac{\partial J(s, m^*(s))}{\partial m} \frac{\partial m^*(s)}{\partial n}
$$
Since $J(s,m)$ is non-increasing in $n$ and strictly increasing in $m$, $m(s) = m^*(s)$ must be non-decreasing in $n$. Therefore, by (3.3) we have $\frac{\partial m(s)}{\partial n} > \frac{\partial m(s)}{\partial m} \geq 0$ for all $s > s$. 

**Q.E.D.**

Lemma 2 below shows that the conditions of Theorem 2 are easily satisfied by many specifications of buyers’ valuations.

**Lemma 2:** With independence of $X_i$’s, conditions (i) and (ii) in Theorem 2 are satisfied in the following cases:

1. $u_i(s,z) = \phi(s) + \psi(x_i)$ or $u_i(s,z) = \phi(s) \cdot \psi(x_i)$;

2. $u_i(s,z) = \phi(s) + \frac{1}{n}[\alpha x_i + \beta \sum_{j \neq i} x_j]$, where $\alpha \geq \beta \geq 0$;

3. $u_i(s,z) = \lambda \phi(s) + (1 - \lambda) \max(x_1, x_2, ..., x_n)$;

where $\phi$ and $\psi$ are any positive and increasing functions and $(s,z)$ is any signal profile.

**Proof:** Recall that $v_i(s,x) = E[V_i | S = s, X_i = X_i^{-1} = x]$. It is easily verified, corresponding to the three cases, that

1. $v_i(s,x) = \phi(s) + \psi(x)$ or $v_i(s,z) = \phi(s) \cdot \psi(x)$;

2. $v_i(s,x) = \phi(s) + \frac{\alpha + \beta}{n} x + \frac{n - 2}{n} \beta E(X | X \leq x)$;

3. $v_i(s,x) = \lambda \phi(s) + (1 - \lambda) x$.

Clearly, $\frac{\partial v_i(s,x)}{\partial s}$ is independent of $n$ in all three cases. Also, $v(s,x)$ is independent of $n$ for cases (1) and (3). For case (2), note that

$$v_i(s,x) = \phi(s) + E(X | X \leq x) + \frac{1}{n}[(\alpha + \beta)x - 2 \beta E(X | X \leq x)]$$

Since $\alpha \geq \beta \geq 0$ and $x \geq E(X | X \leq x)$, $v_i(s,x)$ is decreasing in $n$. **Q.E.D.**

With affiliation, analysis of auctions such as the sealed high bid auction is much more complicated since a change in perceptions in general affects different types of buyer
differently. However, under the assumption of independent private signals, the Revenue Equivalence Theorem applies. Hence we have the following Corollary.

Corollary 1: *If the buyers’ private signals are independent, the optimal markup function* \( m(s) \), *is the same in the sealed high bid auction, as in the sealed second price auction.*

As is well known, in the literature of optimal reserve prices (Maskin and Laffont, 1979) and optimal auctions with independent private valuations (Myerson, 1981, and Riley and Samuelson, 1981), the optimal reserve price is invariant in the number of bidders. Theorem 2 shows that when the signaling role is taken into account, the optimal reserve price increases in the number of bidders in many valuation models including independent private valuations.

4. A Linear Valuation Model

In this section we study a linear valuation model. We first solve for the equilibrium reserve price schedule analytically, then explore several applications. Specifically, buyer \( i \)’s valuation for the object is \( u_i = s + x_i \), where \( s \) is a common value component only observed by the seller, and \( x_i \) is a private value component only observed by buyer \( i \). We suppose that \( s \) and \( x_i \)’s are all independent. The seller’s own valuation \( u_0 = \gamma s \), \( \gamma > 0 \). This corresponds to the case in our general framework in which \( v(s, x_i) = s + x_i \), and \( \xi(s) = \gamma s \).

Assumption (R) is now equivalent to the following:

**Assumption (R’):** \( J(x) = x - (1 - F(x))/f(x) \) is strictly increasing in \( x \).

This assumption holds as long as the hazard rate of \( X \) is increasing, which is satisfied for many common distributions including uniform, normal, and exponential.

Given reserve price \( r \), suppose the buyers’ perception about \( s \) is \( \hat{s} \). Define \( m = r - \hat{s} \), which has the natural interpretation of the seller’s reserve markup since it is the difference between the reserve price and the buyers’ common value component.
Following exactly the same analysis as in previous section, we can rewrite the seller’s expected payoff as.

\[ U(s, \hat{s}, m) = \gamma s F_{(1)}(m) + \hat{s}(1 - F_{(1)}(m)) + B(m) \]  

(4.1)

where

\[ B(m) = m(F_{(2)}(m) - F_{(1)}(m)) + \int_{m}^{\hat{x}} x dF_{(2)}(x) \]  

(4.2)

Since it will be useful below we note that

\[ B'(m) = F_{(2)}(m) - F_{(1)}(m) - mf_{(1)}(m) = -f_{(1)}(m)J(m) \]  

(4.3)

Differentiating the seller’s expected payoff, we have

\[ U_2(s, \hat{s}, m) = 1 - F_{(1)}(m) \]

\[ U_3(s, \hat{s}, m) = (\gamma s - \hat{s})f_{(1)}(m) + B'(m) = (\gamma s - \hat{s} - J(m))f_{(1)}(m) \]  

(4.4)

Given Assumption (R′), the single crossing condition is obviously satisfied.

If \( s \) is directly observable to buyers, their perception \( \hat{s} = s \), then the seller will choose her markup \( m \) to maximize \( U(s, s, m) \). Let \( m^*(s) \) be the optimal full information reserve markup, then by Assumption (R′) and equation (4.4), we have

\[ m^*(s) = \begin{cases} \hat{x}, & \text{if } (\gamma - 1)s < J(\hat{x}), \\ J^{-1}((\gamma - 1)s), & \text{if } J(\hat{x}) \leq (\gamma - 1)s < J(\bar{x}) = \bar{x}, \\ \bar{x}, & \text{if } (\gamma - 1)s \geq \bar{x}. \end{cases} \]  

(4.5)

When \( \gamma = 1 \), the optimal markup is independent of \( s \). When \( 0 < \gamma < 1 \), the optimal markup is strictly decreasing in \( s \). This is because the opportunity cost of no sale, \( (1 - \gamma)s \), is increasing in \( s \). When \( \gamma > 1 \), the optimal markup is strictly increasing in \( s \), because the seller’s valuation increases faster than those of the buyers’ as \( s \) becomes larger.

In the uniform case with support \([\bar{x}, \bar{x}]\), \( J(m) = 2m - (\bar{x} - \bar{x}) \), we have

\[ m^*(s) = \text{Max} \{ \hat{x}, \frac{1}{2}(\bar{x} - \bar{x} + (\gamma - 1)s) \} \]
If there exists a separating equilibrium, then the inverse markup schedule \( s(m) \) must satisfy the following differential equation in equilibrium:

\[
 s'(m) = -\frac{U_z(s,s,m)}{U_z(s,s,m)} = \frac{(J(m) - (\gamma - 1)s) f_{(1)}(m)}{1 - F_{(1)}(m)}
\] 

(4.6)

Assuming an interior solution for the full information optimum, (4.6) can be rewritten as

\[
 s'(m) = \frac{(J(m) - J(m^*(s)) f_{(1)}(m))}{1 - F_{(1)}(m)}
\]

This implies that \( m(s) > m^*(s) \) for all \( s > \underline{s} \).

Equation (4.6) can be rewritten as:

\[
 (1 - F_{(1)}(m)) \frac{ds}{dm} + (\gamma - 1)f_{(1)}(m)s = f_{(1)}(m)J(m)
\]

Multiplying both sides by \( (1 - F_{(1)}(m))^{-\gamma} \), we have

\[
 \frac{d}{dm} [(1 - F_{(1)}(m))^{-\gamma} s(m)] = f_{(1)}(m)(1 - F_{(1)}(m))^{-\gamma} J(m)
\]

Integrating we obtain:

\[
 (1 - F_{(1)}(m))^{-\gamma} s(m) - (1 - F_{(1)}(m))^{-\gamma} \underline{s} = \int_\underline{s}^m f_{(1)}(t)(1 - F_{(1)}(t))^{-\gamma} J(t)dt
\]

Therefore, the inverse markup schedule in equilibrium can be written as

\[
 s(m) = (1 - F_{(1)}(m))^{-\gamma-1} \left[ \int_\underline{s}^m f_{(1)}(t)(1 - F_{(1)}(t))^{-\gamma} J(t)dt + (1 - F_{(1)}(m))^{-\gamma} \underline{s} \right]
\] 

(4.7)

When \( 0 < \gamma \leq 1 \), (4.7) completely characterizes the solution for the separating equilibrium.

As an example, when (i) \( X \) is uniform with support \([0,1]\); (ii) \( n = 2 \); (iii) \( \gamma = 1 \); and (iv) \( \underline{s} = 0 \); we can integrate (4.7) analytically to obtain

\[
 s(m) = -4(m - \underline{m}) + 3\log\left(\frac{1 + m}{1 + \underline{m}}\right) - \log\left(\frac{1 - m}{1 - \underline{m}}\right)
\]

where \( \underline{m} = 1/2 \).
When \( \gamma > 1 \), the equilibrium reserve price schedule may be truncated at some critical type, because the seller can be better off holding the item unsold as her own valuation becomes sufficiently large to exceed the equilibrium reserve price. With full information the seller is better off not selling if and only if his valuation \( \gamma s \) exceeds the maximum buyer valuation \( s + \bar{x} \). Thus the item is sold if and only if \( s \leq s^c \), where
\[
s^c + \bar{x} = \gamma s^c. \quad (4.8)
\]
We will show that with asymmetric information, the critical type is again \( s^c \).

**Proposition 1:** Under Assumption (R'), Equation (4.7) completely characterizes the solution for the unique separating equilibrium when \( 0 < \gamma < 1 \), or when \( \gamma > 1 \) and \( \bar{x} > (\gamma - 1)\bar{s} \). When \( \gamma > 1 \) and \( \bar{x} \leq (\gamma - 1)\bar{s} \), the equilibrium schedule determined by (4.7) is truncated at \( (m^c = \bar{x}, s^c = \bar{x}/(\gamma - 1)) \); those types of \( s \in [\bar{x}/(\gamma - 1), \bar{x}] \) will withdraw from the market.

**Proof:** We have proved the case for \( 0 < \gamma < 1 \) since there does not involve truncation. When \( \gamma > 1 \) and \( \bar{x} > (\gamma - 1)\bar{s} \), clearly the constraint of (4.8) does not bind since \( (\gamma - 1)s \leq (\gamma - 1)\bar{x} < \bar{x} \). It thus remains to show the case in which \( \gamma > 1 \) and \( \bar{x} \leq (\gamma - 1)\bar{s} \).

In this case, the constraint of (4.8) implies that there exists a truncated type \( s^c = \bar{x}/(\gamma - 1) \in [\bar{s}, \bar{x}] \) so that \( m^c = m(s^c) = \bar{x} \).

It remains to verify that this new endpoint condition is also implied in (4.7). First, as \( m \rightarrow \bar{x} \),
\[
(1 - F_{(i)}(m))^{\gamma-1} \left[ (1 - F_{(i)}(m))^{1-\gamma} s \right] \rightarrow 0
\]
Second, applying L’Hopital’s rule, we have
Therefore, taking limits on both sides of equation (4.7), we have $s(\bar{x}) = \lim_{m \to \pi} s(m) = s^c = \bar{x}/(\gamma-1)$, which confirms that for $\gamma > 1$, the separating equilibrium is given by (4.7) with a truncation at the second endpoint $(m^c = \bar{x}, s^c = \bar{x}/(\gamma-1))$.

Q.E.D.

Proposition 1 thus gives a complete characterization of the unique separating equilibrium for the linear valuation model. Jullien and Mariotti (2003) study a reserve price signaling model in which the seller’s valuation is $\tilde{\theta}$ and the buyers’ valuations are $\lambda \tilde{\theta} + (1 - \lambda) \tilde{c}$, where $\lambda \in [0,1]$. Their setup thus corresponds to the case of $\gamma \geq 1$ in our linear valuation model.

Based on the equilibrium characterized in Proposition 1, we now present three simple applications of the linear valuation model, the first to outside certification, the second to an analysis of relative importance of private values, and the last to a Lemons market analysis.

**Outside Certification**

We consider the situation where in addition to signaling through reserve prices, the seller can credibly reveal $s$ to the bidders through an outside certification agency at a fixed cost of $c > 0$. The question is when the seller is willing to pay for such a service.
Let $u^*(s) = U(s, s, m^*(s))$ be the type $s$ seller’s expected revenue in the full information equilibrium, and $u(s) = U(s, s, m(s))$ be the type $s$ seller’s expected revenue in the separating equilibrium with reserve price signaling. Let $W(s) = u^*(s) - u(s)$. Then $W(s)$ represents the value of certification to the type $s$ seller. Immediately, $W(s) = 0$ and $W(s) \geq 0$ for all $s$. The seller is willing to pay for the certification service if and only if $W(s) \geq c$.

To further simplify notation, let $m^* = m^*(s)$ and $m = m(s)$. By the Envelope Theorem, we have $\frac{du^*}{ds} = \gamma F_{(1)}(m^*) + 1 - F_{(1)}(m^*)$. Since $\frac{dm}{ds} = -U_2/U_3$, we have $\frac{du}{ds} = U_1 + U_2 + U_3 \frac{dm}{ds} = U_1 = \gamma F_{(1)}(m)$. Therefore,

$$\frac{dW}{ds} = \frac{du^*}{ds} - \frac{du}{ds} = \gamma F_{(1)}(m^*) + 1 - F_{(1)}(m^*) - \gamma F_{(1)}(m)$$

$$= 1 - F_{(1)}(m) + (1 - \gamma)(F_{(1)}(m) - F_{(1)}(m^*))$$

Thus, $dW/ds > 0$ when $\gamma \leq 1$.\,

Proposition 2: Suppose $\gamma < 1$. For any $c > 0$, there exists a cutoff type $s^* \leq \overline{s}$ such that for all $s \in [s^*, \overline{s}]$, the seller hires the outside certification agency; for all $s \in [\underline{s}, s^*)$, the seller signals through reserve price $r(s) = m(s) + s$.

Relative importance of private value component on reserve prices

Using the equilibrium characterization of the linear valuation model, we study how reserve prices change with the relative importance of private value component in buyers’ valuations. Suppose buyer $i$’s valuation is now given by $V_i = s + \beta X_i$,

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8 If $\gamma > 1$, then for some distribution functions it is possible that the value of certification $W(s)$ is not always monotonic; it can increase for small $s$ and then decrease for large $s$. If that is the case, it may happen that the seller buys the certification service for some intermediate range of $s$ but signals through reserve prices for very small and very large $s$.\,

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where $\beta \in (0, \infty)$ measures the relative importance of the private value component. Clearly $\beta = 1$ corresponds to the valuation structure studied before. Again let $m = r - \hat{s}$ be the reserve markup, but define $\tilde{m} = m / \beta$ as the relative markup. Under full information, the optimal relative markup only depends on the distribution of $X_i$’s and is independent of $\beta$. The interesting question in the case of incomplete information is how the relative markup $\tilde{m}$ changes as $\beta$ changes.

**Proposition 3:** Suppose $0 < \gamma < 1$. For any $s$, the relative markup $\tilde{m}$ is increasing in $\beta$. Consequently, the reserve markup $m$ and hence the reserve price $r(s) = s + m(s)$ are increasing in $\beta$ at an accelerating rate.

**Proof:** Define $\tilde{s} = s / \beta$ as the relative type, and $\tilde{t} = \hat{s} / \beta$ as the relative perception. The seller’s expected revenue can be expressed as follows:

$$
\tilde{u}(\tilde{s}, \tilde{t}, \tilde{m}) = \gamma s F_i(m / \beta) + (m + \hat{s})(F_2(m / \beta) - F_1(m / \beta)) + \int_{m / \beta}^{\infty} (\hat{s} + \beta x) dF_2(x)
$$

$$
= \beta \left( \gamma \tilde{s} F_i(\tilde{m}) + (\tilde{m} + \tilde{t})(F_2(\tilde{m}) - F_1(\tilde{m})) + \int_{\tilde{m}}^{\infty} (\tilde{t} + x) dF_2(x) \right)
$$

$$
= \beta u(\tilde{s}, \tilde{t}, \tilde{m})
$$

where $u(\cdot, \cdot, \cdot)$ is defined in (4.1)

So the problem can be viewed as a normalization from our basic model, and all the analysis follows as before immediately. In particular, the differential equation (4.7) (with the normalized variables) characterizes the unique separating equilibrium.

The only issue remaining is how the initial condition for the differential equation is affected. Notice that the lowest normalized type is now $\tilde{x} = \bar{x} / \beta$. Under full information, by Equation (4.5), we have

$$
\tilde{m}^*(\tilde{x}) = \begin{cases} 
\bar{x}, & (\gamma - 1)\tilde{x} < J(\bar{x}) \\
J^{-1}((\gamma - 1)\tilde{x}), & (\gamma - 1)\tilde{x} \geq J(\bar{x})
\end{cases}
$$
Clearly, the full information relative markup $\tilde{m}^* (\bar{s})$ is increasing in $(\gamma - 1)\bar{s}$. Since $\gamma < 1$, $\tilde{m}^* (\bar{s})$ is decreasing in $\bar{s}$ and hence increasing in $\beta$. Therefore, when $\beta$ is larger, $\bar{s}$ is smaller but $\tilde{m}^* (\bar{s})$ is greater. As a result, the equilibrium relative reserve schedule $\tilde{m}(\cdot)$ is higher everywhere. Since the reserve markup is $m = \beta \tilde{m}$, it is increasing in $\beta$ at an accelerating rate.

Q.E.D.

The intuition for this result is the following. When $\beta$ increases, the private value component becomes more important while the common value component becomes less so. When $\gamma < 1$ and the common value component becomes less important, the relative markup for the lowest type actually increases, because the signaling cost from no sale is relatively small. It can be shown that the relative reserve schedule follows the same differential equation as before. As a result, a larger $\beta$ implies a higher initial relative markup, thus implies a higher relative markup schedule everywhere.⁹

Lemons Market

Even though our analysis so far has focused on auctions, our results can be readily applied to studying signaling in the Lemons Market. Consider the following market situation for a good (e.g., used cars). To keep things simple, suppose there is a unit mass of buyers each with unit demand, and there is also a unit mass of sellers each with one item to sell. Each seller knows $s$, the “quality” of the item for sale, and her own valuation is $\gamma s$. Buyers do not observe the quality of the good, but know that the population distribution of $s$ is given by c.d.f $G(\cdot)$ with support $S = [\underline{s}, \bar{s}]$. Buyer $i$’s valuation for a good with quality $s$ is $V_i = s + X_i$, where $X_i$ is a private value component. Each buyer $i$ only observes his own $X_i$. Ex ante, the distribution of $X_i$ is given by c.d.f $F(\cdot)$ with support $[\underline{x}, \bar{x}]$.

What is just described is a continuous type version of the Lemons Market model (Akerlof 1970). The fundamental idea of Akerlof’s analysis is that the price-taking

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⁹ When $\gamma > 1$, no definite conclusion can be made about how the relative markup schedule will change with $\beta$. 

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Walrasian equilibrium cannot achieve efficient resource allocation in the presence of the adverse selection problem. In the above model, absent the adverse selection problem (i.e., if quality is known to the buyers), the first best allocation is easily achieved by setting a price of $s$. When $s$ is not known to the buyers, for any fixed price $p$ chosen by the Walrasian auctioneer, only those sellers with valuation $s \leq p$ are willing to sell their goods, resulting in a total supply of $G(p/\gamma)$. Accordingly, the expected common value of the goods in the market is $\tau = E[s \mid s \leq p] = E[s \mid s \leq p/\gamma]$. Since only those buyers with valuation $V = \tau + x \geq p$ are willing to buy, the total demand is $1 - F(p - \tau)$. To find the market-clearing price, we set $1 - F(p - \tau) = G(p/\gamma)$. In general, the equilibrium price that clears the market leads to less than efficient level of trade. For example, when both $F(.)$ and $G(.)$ are uniform on $[0,1]$, it can be verified that the market-clearing price is $p = 2\gamma/(2\gamma + 1)$ if $\gamma \geq 1/2$, and $0.5$ if $\gamma < 1/2$, which implies a trade volume of $\min\{2/(2\gamma + 1),1\}$. Trade is efficient only when $\gamma \leq 1/2$. When $\gamma > 1/2$, equilibrium trade is less than the efficient level and is decreasing in $\gamma$.

The concept of Walrasian equilibrium assumes price-taking behavior on both sides of the market and that price is public information. In many real life situations such as in the used car market, neither of these assumptions fits: sellers set prices for their goods and buyers search for what they want. To model these features in the simplest way, we consider a situation with pair-wise random matching in which the sellers set prices and each buyer randomly samples one seller without knowing the prices in the market. What is the equilibrium outcome in this market?

Observe that in our analysis of reserve price signaling in the auction context, we can reinterpret the single seller with a type $s$ drawn from the distribution $G(.)$ as a unit mass of sellers with unit supply whose types have a population distribution of $G(.)$. Then it should be clear that our previous characterization result Proposition 1 applies to the current pair-wise matching market with $n = 1$. When $n = 1$, Equation (4.7) becomes

$$s(m) = (1 - F(m))^{-1} \left[ \int_{m}^{1} f(t)(1 - F(t))^{-\gamma} J(t)dt + (1 - F(m))^{1-\gamma}s \right] \tag{4.10}$$
where \( m = m^*(s) \). Therefore, this characterizes a separating pricing equilibrium in which a seller with type \( s \) chooses a posted price \( p = s + m(s) \) and the buyer correctly infers the true type \( s \) from this price schedule and decides whether to buy at this price. When \( F(.) \) is uniform on \([0,1]\) and \( s = 0 \) (hence \( m = 1/2 \)), the equilibrium markup is given by

\[
s(m) = \begin{cases} 
\frac{1}{(1-\gamma)(2-\gamma)}[2(1-m)]^{\gamma-1} + \frac{2(1-m)}{2-\gamma} - \frac{1}{1-\gamma}, & \gamma > 0, \gamma \neq 1, 2 \\
1 - 2m - \log(1-m) - \log 2, & \gamma = 1 \\
2(1-m)[\log 2(1-m) - 1] + 1, & \gamma = 2 
\end{cases}
\]

Again, for the case \( \gamma > 1 \), the equilibrium schedule is given by the above solution with the understanding that it is truncated at \( s^c = 1/(\gamma - 1) \) if \( 1/(\gamma - 1) < s \).

In this specific example, it can be verified that both the social welfare and volume of trade are greater in the Walrasian equilibrium than in the signaling equilibrium.\(^{10}\) However, this difference mainly results from the assumption that the searching technology is extremely primitive and costly in the signaling equilibrium --- only one round pair-wise matching is allowed --- while on the other hand, the searching cost is zero in the Walrasian equilibrium.

In the application to the Lemons Market, several extensions are desirable and worth further research. One straightforward extension is to consider markets in which each seller faces multiple buyers, e.g., as in the housing market. In this case our previous results directly apply. Another extension is to consider heterogeneous buyer preferences over quality. For example, it may be reasonable to suppose that buyer \( i \)'s valuation for a good with quality \( s \) is \( V_i = t(s)Z_i + X_i \) where \( Z_i \) is buyer \( i \)'s preference for quality and is only known to himself. In the preceding example we oversimplified situations by assuming that each buyer can only sample one seller. A more realistic model would have buyers searching more than one period and with heterogeneous preferences for quality.

\(^{10}\) Our computation results show that the seller’s expected revenue can be higher in the signaling equilibrium.
5. Extensions

**Multidimensional Characteristics and Imperfect Signaling**

In the analysis of our basic model, we assume that the seller’s own value \( u_0 \) and the “common value component” which affects all the buyers’ valuations \( s \) are perfectly correlated, that is, \( u_0 = \xi(s) \). In some situations, it may be reasonable to assume that they are not perfectly correlated. Suppose that the seller observes the object’s characteristics represented by \( \theta \in \Theta \), where \( \Theta \) can be any finite dimensional space. The object’s characteristics \( \theta \) affects the seller’s valuation for the object \( \xi(\theta) \) and the buyers’ common value component \( s(\theta) \). For example, a seller of an artwork (e.g., an auction house) may know better than potential buyers about the conditions (quality, rarity, history, etc.) and the secondary market value of the artwork. Similarly, a government agency auctioning procurement of a public project may have better information than bidding firms about certain factors (e.g., environmental impacts and regulations) that affect both its own valuation of the project and project costs common to all bidding firms.

Our analysis can carry through even when \( \theta \) is multidimensional and \( \xi \) and \( s \) are imperfectly correlated. The case of imperfect correlation can arise naturally from the following environment. The seller and the buyers may have different preferences over the object’s characteristics \( \theta \). For example, potential buyers of an artwork (for self-consumption) may care about its secondary market value quite differently from the seller. Suppose the seller’s valuation is \( W(\theta, \eta) \) and the buyers’ common value component is \( V_0(\theta, \omega) \), where \( \eta \) and \( \omega \) are random variables. Since the seller and the buyers are assumed to be risk neutral, only their expected valuations matter in the analysis. Let the seller’s expected valuation be \( \xi(\theta) = E_\eta[W(\theta, \eta)] \), and the buyers’ expected common value be \( s(\theta) = E_\omega[V_0(\theta, \omega)] \). However, since buyers do not observe \( \theta \), they have to infer the expected common value component from the seller’s signal. Upon observing the seller’s signal and forming a belief that \( \xi(\theta) = \hat{\xi} \), the buyers’ expected common value is given by \( \hat{s} = E_{[\theta, \omega]}[V_0(\theta, \omega) | \xi(\theta) = \hat{\xi}] = E_\omega[s(\theta) | \xi(\theta) = \hat{\xi}] =: \tau(\hat{\xi}) \).

We assume that
\( \xi(\theta) \) and \( s(\theta) \) are positively, but not necessarily perfectly correlated, hence \( \tau(\cdot) \) is increasing.

Following exactly the same steps as we did in the basic model, we can show that
\[
 u(\xi, \hat{s}, m) = \xi F_1(m) + \tau(\hat{s})(1 - F_1(m)) + B(m)
\]
where \( B(m) \) is defined in (4.2). It is clear that even though the seller’s primitive type is \( \theta \), all that matters is her valuation (her effective true type), \( \hat{\xi} = \xi(\theta) \), and the buyers’ belief about the common value component \( \hat{s} \). In any equilibrium, sellers of different primitive types but a same “effective type” \( \hat{\xi} \) must choose the same signal and hence lead to the same \( \hat{s} \); otherwise those sellers obtaining smaller expected payoff would switch to the signal that leads to higher expected payoff. In equilibrium, for a signaling strategy \( r(\xi(\theta)) \), the buyers’ perceived common value component is
\[
\hat{s} = E[s(\theta) | \theta : r(\xi(\theta)) = r].
\]
With this reformulation, the seller’s payoff function is in the standard form of signaling models, and is expressed without direct reference to her private information about the object’s characteristics \( \theta \). As in the analysis of our basic model, we can characterize the separating equilibrium by allowing for imperfect correlation between \( \xi \) and \( s \).

**Correlation Between the Seller’s and Buyers’ Signals**

In our basic model we allow buyers’ private signals to be affiliated with each other in any form, but assume that the seller’s private signal and the buyers’ signals are independent. Can we extend our analysis so that the seller’s signal is correlated with the buyers’ signals?

To explore the possibility of such extensions, we consider a common value auction with conditional independent signals. More specifically, each buyer has a signal \( x_i \) before bidding. Ex ante signals are correlated. But conditional on the seller’s private information \( s \), they are i.i.d. with distribution function \( F(\cdot | s) \) and density \( f(\cdot | s) \).

Suppose the joint density function of \( (s; x_1, ..., x_n) \) is
\[
f(s; x_1, ..., x_n) = g(s)f(x_1 | s) \cdots f(x_n | s).
\]
As in our basic model, we assume that there is a function \( u \) on \( \mathbb{R}^{1+n} \) such that each buyer’s valuation is given by
\[ V_i = u_i(s;x_1,\ldots,x_n) = u(s;x_i,x_{-i}) \], where \( u \) is increasing in all its arguments. The seller’s own valuation for the item is given by \( \xi(s) \), which is increasing in \( s \).

We look for a separating equilibrium with reserve price signaling. Given a reserve price \( r \), suppose the buyers’ perception about \( s \) is \( \hat{s} \), then under a second-price auction, each buyer’s symmetric equilibrium bid function is given by

\[ b(x_i | \hat{s}) = E(V_i' | x_i = X_{(i)}^{-i}, s = \hat{s}) =: \nu(\hat{s}, x_i) \]

We have \( \frac{\partial \nu}{\partial s} > 0 \) and \( \frac{\partial \nu}{\partial x_i} > 0 \) by the affiliation of the signals and the monotonicity of function \( u \). A modified version of Assumption (R) is needed:

**Assumption (R'')**: For any \( s \), \( J(s,x) = \nu(s,x) - \frac{\partial \nu(s,x)}{\partial x} \frac{1-F(x | s)}{f(x | s)} \) is strictly increasing in \( x \).

Similar to Lemma 1, this assumption will be satisfied if \( \frac{\partial^2 \nu}{\partial x_i^2} \leq 0 \) and the hazard rate function of the conditional distribution \( F(x | s) \) is strictly increasing in \( x \).

As before, we can write the seller’s expected payoff as follows:

\[ U(s,\hat{s},m) = \xi(s)F_{(1)}(m | s) + \nu(\hat{s},m)[F_{(2)}(m | s) - F_{(1)}(m | s)] + \int_m \nu(\hat{s},x)dF_{(2)}(x | s) \]

Differentiating, we have

\[ \frac{\partial U}{\partial m} = f_{(1)}(m | s) \left[ (\xi(s) - \nu(\hat{s},m)) + \frac{\partial \nu}{\partial m} \frac{1-F(m | s)}{f(m | s)} \right] \quad (5.1) \]

\[ \frac{\partial U}{\partial s} = \frac{\partial \nu}{\partial s}[F_{(2)}(m | s) - F_{(1)}(m | s)] + \int_m \frac{\partial \nu}{\partial s} dF_{(2)}(x | s) \quad (5.2) \]

Since the distribution function depends on \( s \), it is now not clear whether the single crossing condition (2.4) holds. Even for simple distributions such as uniform distribution, the conditions for the single crossing property to hold are very involved. Without the single crossing condition, we cannot be certain about the incentive comparability of the solution to the differential equation analogous to (3.1), though such
a solution going through the full information optimum for the lowest type exists and is unique given Assumption \( R'' \).

By inspecting (5.1) and (5.2), we know that the problem in verifying the single crossing condition is really due to the fact that the distribution of the buyers’ signals depends on the seller’s private information. This makes it difficult to analyze the possibility of reserve price signaling when the seller’s private information and the buyers’ signals are correlated.

6. Conclusion

In this paper we study an auction model in which the seller has private information about the object’s characteristics that are valued by both the seller and the buyers. In a fairly general framework in which buyers’ signals are affiliated, we identify conditions under which reserve prices can serve as an effective signaling instrument to reveal the seller’s private information. Under such conditions we characterize the unique separating equilibrium with reserve price signaling. When the buyers’ signals are independent, it is shown that the optimal reserve price is increasing in the number of bidders under certain conditions about the valuations. This is in contrast with the optimal auction literature with independent private values. Our analysis thus suggests a more central role for reserve prices than perceived by the standard auction model.

An important step in our analysis is to work with the reserve markup instead of the reserve price itself as the key variable, which allows us to transform our model into a standard signaling game and greatly simplify the analysis. Following the signaling literature, we appeal to standard refinements and focus on the unique separating equilibrium of the signaling game in which the lowest type seller sets the reserve price that is optimal under complete information (the “Riley outcome”). As we argued earlier, when the object’s characteristics are multi-dimensional, it is very natural that the seller’s valuation and the buyers’ common value component are positively, but not perfectly correlated. In that case, standard equilibrium refinements such as the Cho-Kreps Intuitive Criterion (Cho and Kreps, 1987) no longer reduce the number of equilibria. In a separate paper (Cai, Riley and Ye, 2004), we argue that in a general signaling model for which our reserve price signaling model is a special case, it is sensible to consider a refinement
criterion which we call the “Local Credibility Test (LCT).” This is somewhat stronger than Cho and Kreps Intuitive criterion but weaker than the “strong Intuitive Criterion” of Grossman and Perry (1986a,b). There we explore necessary and sufficient conditions under which the separating equilibrium survives the LCT.

References


Cai, Hongbin, Riley, John and Ye, Lixin (2004), “Imperfect Signaling and the Local Credibility Test”, mimeo, UCLA.


