Neoclassical Models in Macroeconomics*

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Abstract
This chapter develops a toolkit of neoclassical macroeconomic models, and applies these models to the U.S. economy from 1929 through 2014. We first filter macroeconomic time series into business cycle and long-run components, and show that the long-run component is typically much larger than the business cycle component. We argue that this empirical feature is naturally addressed within neoclassical models with long-run changes in technologies and government policies. We construct two classes of models that we compare to raw data, and also to the filtered data: simple neoclassical models, which feature standard preferences and technologies, rational expectations, and a unique, Pareto-optimal equilibrium, and extended neoclassical models, which build in government policies and market imperfections. We focus on models with multiple sources of technological change, and models with distortions arising from regulatory, labor, and fiscal policies. The models account for much of the relatively stable postwar U.S. economy, and also for the Great Depression and World War II. The models presented in this chapter can be extended and applied more broadly to other settings. We close by identifying several avenues for future research in neoclassical macroeconomics.

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1 Introduction

This chapter analyzes the role of neoclassical models in the study of economic growth and fluctuations. Our goal is to provide macroeconomists with a toolkit of models that are of interest in their own right, and that easily can be modified to study a broad variety of macroeconomic phenomena, including the impact of economic policies on aggregate economic activity.

Since there is no generally recognized definition of neoclassical macroeconomics within the profession, we organize the development of these models around two principles. One is based on the exogenous factors driving changes in aggregate time series, and the other is based on the classes of model economies that we consider.

The primary sources of changes in macroeconomic variables that we study are long-run changes in technologies and government policies. We focus on these factors because of the observed large changes in productivity and in policies that affect the incentives and opportunities to produce and trade. Policy factors that we consider include changes affecting competition and business regulatory policies, labor policies, and fiscal policies.

We study two classes of intertemporal models that we call neoclassical macroeconomic models. The first has standard preferences and technologies, competitive markets, rational expectations, and there is a unique equilibrium that is Pareto-optimal. We call these Simple Neoclassical Models. This class of models is the foundation of neoclassical macroeconomics, and provides the most transparent description of how competitive market forces operate within a dynamic, general equilibrium environment.

In contrast to common perceptions about neoclassical macroeconomics, we acknowledge that economies are affected by policy distortions and other market imperfection that go beyond the scope of simple models. The second class of models modifies simple models as needed to incorporate changes that require departing from the model assumptions described above. We call the second class of models Extended Neoclassical Models, which are constructed by building explicit specifications of government policies or market imperfections and distortions into simple models.

This method nests simple models as special cases of the extended models. Developing complex models in this fashion provides a clear description of how market imperfections and economic policies affect what otherwise would be a laissez-faire market economy. We modify the models in very specific ways that are tailored to study episodes in U.S. economic history, and which provide researchers with frameworks that can be applied more broadly. All of the models presented in this chapter explicitly treat fluctuations and growth within the same framework.

Neoclassical frameworks are a powerful tool for analyzing market economies. An important reason is because the U.S. economy has displayed persistent and reasonably stable growth over much its history while undergoing enormous resource reallocation through the competitive market process in response to changes in technologies and government policies. These large reallocations include the shift out of agriculture into manufacturing and services, the shift of economic activity out of the Northern and Mideastern sections of the United States to the Southern and Western states, and large changes in government’s share of output, including changes in tax, social insurance, and regulatory labor policies. This also

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includes the reallocation of women’s time from home production to market production, and the increased intensity of employment of highly-skilled labor. Most recently, this has included the reallocation of resources out of the development of mature, mechanical technologies to the development of information processing and communication technologies, including the integrated circuit, fiber optics, microwave technology, laptop computers and tablets, software applications, cellular technology, and the internet.

Our focus on technologies and policies connects with considerable previous research. This ranges from Schumpeter (1927, 1942), who argued that changes in entrepreneurship and the development of new ideas are the primary drivers of a market economy, and Kydland and Prescott (1982) and Long and Plosser (1983), who focused on technology shocks and fluctuations. This also includes Lilien (1982), who argued that sectoral shifts significantly affect fluctuations and resource reallocation, Davis and Haltiwanger (1991), who established that resource reallocation across U.S. manufacturing establishments is very large and is continuously evolving, and Greenwood and Yorokoglu (1997) and Manuelli and Seshadri (2008), who analyze the diffusion of new technologies and their long-run economic effects. The analysis also connects with studies of the long-run consequences of government policies, including research by Ljungqvist and Sargent (1998), Prescott (2004), and Rogerson (2008), who analyze how public policies such as tax rate changes, and changes in social insurance programs, have affected long-run labor market outcomes.

Our principle of focusing on long-run movements in data requires a quantitative approach that differs from standard practice in macroeconomics that involves both the selection of the data frequencies that are analyzed, and how the model is compared to data. The standard approach removes a trend from the data that is constructed using the Hodrick-Prescott filter (1997), hereafter referred to as HP filter, with a smoothing parameter of 1600, and then typically compares either model moments to moments from the HP-filtered data, or compares model impulse response functions to those from an empirical vector autoregression (VAR). This analysis uses a band pass filter to quantify movements not only at the HP-business cycle frequency, but also at the lower frequencies. Our quantitative-theoretic analysis evaluates model economies by conducting equilibrium path analyses, in which model-generated variables that are driven by identified shocks are compared to actual raw data and to filtered data at different frequencies.

We report two sets of findings. We first document the empirical importance of very long-run movements in aggregate variables relative to traditional business cycle fluctuations using post-Korean War quarterly U.S. data, long-run annual U.S. data, and postwar European data. We find that low frequency movements in aggregate time series are quantitatively large, and that in some periods, they are much larger than the traditional business cycle component. Specifically, we analyze movements in periodicities ranging from two to 50 years, and we find that as much as 80 percent of the fluctuations in economic activity at these frequencies is due to the lower frequency component from 8-50 years.

The dominant low frequency nature of these data indicates that the business cycle literature has missed quantitatively important movements in aggregate activity. Moreover, the fact that much of the movement in aggregate data is occurring at low frequencies suggests that models that generate fluctuations from transient impediments to trade, such as temporarily inflexible prices and/or wages, may be of limited interest in understanding U.S. time
The importance of low frequency movements also has significant implications for the two dominant episodes of the last 35 years, the Great Moderation and the Great Recession. The Great Moderation, the period of stable economic activity that occurred between 1984 and 2008, features a sharp decline in volatility at the traditional business cycle frequency, but little volatility change at low frequencies. Similarly, the Great Recession and its aftermath feature a large, low frequency component. These data suggest that the Great Recession was not just a recession per se. Instead, much of this event appears to be a persistent decline in aggregate economic activity.

Following the decomposition of data into low and high frequency components, we report the results of quantitative-theoretic analyses that evaluate how well neoclassical models account for the U.S. historical macroeconomic record from 1929 through 2014.

Our main finding is that neoclassical models can account for much of the movement in aggregate economic activity in the U.S. economic historical record. Neoclassical models plausibly account for major economic episodes that previously were considered to be far beyond their reach, including the Great Depression and World War II. We also find that neoclassical models account for much of the post-Korean War history of the U.S.

The chapter is organized as follows. Section 2 presents the U.S. and European data that we use in this study, and provides a decomposition of the data into low frequency and business cycle frequency components. Section 3 introduces the basic neoclassical macroeconomic model that serves as the foundation for all other models developed in the chapter. Section 4 presents one, two, and three sector simple neoclassical model analyses of the post-Korean War U.S. economy. Section 5 presents extended neoclassical models to study Depressions. Section 6 presents extended neoclassical models with fiscal policies with a focus on the U.S. economy during World War II. Given the importance of productivity shocks in neoclassical models, Section 7 discusses different frameworks for understanding and interpreting TFP changes. Given the recent interest in economic inequality, Section 8 discusses neoclassical models of wage inequality. Section 9 presents a critical assessment of neoclassical models, and suggests future research avenues for neoclassical macroeconomic analysis. Section 10 presents our conclusions.

2 The Importance of Low Frequency Components in Macroeconomic Data

It is common practice in applied macroeconomics to decompose time series data into specific components that economists often refer to as cyclical components, trend components, and seasonal components, with the latter component being relevant in the event that data are not seasonally adjusted. These decompositions are performed to highlight particular features of data for analysis. The most common decomposition is to extract the cyclical component from data for the purpose of business cycle analysis, and the Hodrick-Prescott (HP) filter is the most common filtering method that is used.

Band-pass filters, which feature a number of desirable properties, and which resolve some
challenges involved with applying the HP filter, are increasingly being used to filter data.\footnote{In terms of the challenges with the HP filter, it is not clear how to adjust the HP smoothing parameter to assess data outside of the cyclical window originally studied by Hodrick and Prescott (1997). Moreover, HP-filtered data may be difficult to interpret at data endpoints.} Band-pass filtering allows researchers to choose components that correspond to periodicities over a specific data frequency. An exact band pass filter requires an infinite length of data, so Baxter and King (1999) and Christiano and Fitzgerald (2003) have constructed approximate band pass filters. These two approaches are fairly similar. The main difference is that the Baxter-King filter is symmetric, and the Christiano-Fitzgerald filter is asymmetric.

This section presents decompositions of aggregate data into different frequency components for (i) U.S. post-Korean War quarterly data, (ii) U.S. annual data that extends back to 1890, and (iii) post-World War II annual European data. We use the Baxter-King filter, given its wide use in the literature. The band pass filter isolates cyclical components in data by smoothing the data using long moving averages of the data. Baxter and King develop an approximate band pass filter that produces stationary data when applied to typical economic time series.\footnote{The Baxter-King filter yields stationary time series for a variable that is integrated of up to order two. We are unaware of any macroeconomic time series that is integrated of order three or higher.} Since the exact band pass filter is an infinite order process, Baxter and King construct a symmetric approximate band pass filter. They show that the optimal approximating filter for a given maximum lag length truncates the filter weight at lag $K$ as follows:

$$y_t^* = \sum_{k=-K}^{K} a_k y_{t-k}$$

In $y^*$ is the filtered data, $y$ is the unfiltered data, and the $a_k$ denote coefficients that produce the smoothed time series. The values of the $a_k$ coefficients depend on the filtering frequency (see Baxter and King (1999)). Following early work on business cycles by Burns and Mitchell (1946), Baxter and King study business cycles, which they define as corresponding to periodicities associated with 6 - 32 quarters. In contrast, we use the band-pass filter to consider a much broader range of frequencies up to 200 quarters. Our choice to extend the frequency of analysis to 200 quarters is motivated by Comin and Gertler (2006), who studied these lower frequencies in a model with research and development spending.

We consider much lower frequencies than in the business cycle literature since changes in technologies and government policies may have a quantitatively important effect on low frequency movements in aggregate data. Relatively little is known about the nature and size of these low frequency fluctuations, however, or how these low frequency fluctuations compare to business cycle fluctuations. We therefore band-pass filter data between 2-200 quarters, and we split these filtered data into two components: a 2-32 quarters component, which approximates the business cycle results from the standard parameterization of the HP filter ($\lambda = 1600$), and a 32-200 quarters component. This allows us to assess the relative size and characteristics of these fluctuations. To our knowledge, these comparative decompositions have not been constructed in the literature.
2.1 Band-Passed Filtered Quarterly U.S. Data

This subsection analyzes U.S. quarterly post-Korean war from 1954 through 2014, which facilitates comparison with much of the business cycle literature. We then analyze annual U.S. data extending back to 1890, followed by an analysis of postwar European data.

Figures 1 through 6 show filtered real GDP, consumption of nondurables and services, gross private domestic investment, hours worked, total factor productivity (TFP), and the relative price of capital equipment. Real GDP, consumption, and investment are from the NIPA. Hours worked is constructed by updating the hours worked data of Cociuba et al (2012), who use hours from the Current Population Survey. TFP is constructed by dividing real GDP by a Cobb-Douglas aggregate of capital, which is the sum of private and public capital stocks, and which has a share of 0.4, and hours worked, which has a share of 0.6.

We include the relative price of capital equipment in this analysis because there is a large change in this relative price over time, and because the inverse of this relative price is a measure of equipment-specific technological change in some classes of models, including Greenwood, Hercowitz and Krusell (1997), and Krusell et al (2000). We construct the relative price of equipment as the ratio of the quality-adjusted deflator for producer durable equipment, to the NIPA nondurable consumption deflator. Gordon (1990) initially constructed the quality-adjusted equipment deflator, and this time series has been continued in Cummins and Violante (2002) and in DiCecio (2009).

The figures show the 2-200 component and the 32-200 component. Since the band pass filter is a linear filter, the difference between these two lines is the 2-32 component. The most striking feature of all of these filtered data is that much of the movement in the 2-200 component is due to the 32-200 component. These filtered data indicate that business cycle variability, as typically measured, accounts for a relatively small fraction of the overall post-Korean war history of U.S. economic variability. The graphs do show that there are some periods in which the traditional business component is sizeable. This occurs in part of the 1950s, which could be interpreted as the economy readjusting to peacetime policies following World War II and the Korean War. There is also a significant 2-32 component from the 1970s until the early 1980s.

The 32-200 component of TFP has important implications for the common critique that TFP fluctuations at the standard HP frequency are affected by unmeasured cyclical factor utilization. Fernald’s (2014) TFP series is a widely used measure of TFP that is adjusted

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3The Baxter-King filter loses data at the beginning and the end of a dataset. We therefore padded all the data series at both the starting and ending dates by simulating data from ARMA models fit to each series. These simulated data extend the series before the starting date and after the end date, which allows us to construct band-passed data for the entire period length. We conducted a Monte Carlo analysis of this padding procedure by generating extremely long artiﬁcial time series, and comparing band-passed ﬁltered series using the padded data, to band-passed data that doesn’t use padding. The length of the data padding is equal to the number of moving average coefﬁcients, k. We use k = 50 for the quarterly data, and k=12 for the annual data. The results were insensitive to choosing higher values of k.

4We do not use the NIPA equipment deflator because of Gordon’s (1990) argument that the NIPA equipment price deflator does not adequately capture quality improvements in capital equipment. We use DiCecio’s (2009) updating of the Gordon-Cummins-Violante data. This data is updated by DiCecio on a real time basis in the Federal Reserve Bank of St. Louis’s FRED database. The mnemonic for this series is PERIC.
for unmeasured factor utilization. Figure 7 shows the 32-200 component of Fernald’s adjusted and unadjusted measures of business sector TFP. The long-run component of the adjusted and unadjusted series are very similar, particularly over the last forty years. This indicates that unmeasured factor utilization is not an issue for measuring TFP at these lower frequencies.

To quantify the relative contribution of the 32-200 component for these variables, we construct the following ratio, which we denote as $z_i$, in which $x_i$ is the 32-200 filtered component of variable $i$, and $y_i$ is the 2-200 filtered component of variable $i$:

$$z_i = \sum_t \left( \frac{(x_{it})^2}{(y_{it})^2} \right)$$

(2)

On average, the 32-200 component accounts for about 80 percent of the fluctuations in output, consumption, TFP, and the relative price of equipment and about 64 percent of hours. It accounts for about 56 percent of fluctuations in gross private domestic investment, which includes the highly volatile category of inventory change.

The 32-200 component is also large during the Great Moderation. Specifically, the well-known volatility decline of the Great Moderation, which is typically dated from 1984-2007, is primarily due to lower volatility of the 2-32 component. The figures show that the volatility of the 32-200 component remains quantitatively large during the Great Moderation. This latter finding may reflect the large and persistent technological advances in information processing and communications that occurred throughout this period.

This finding regarding the nature of these frequency components in the Great Moderation is consistent with the conclusions of Arias et al (2007), and Stock and Watson (2002), who report that the traditional business cycles frequency shocks that affected the economy during this period were smaller than before the Great Moderation. This finding about the Great Moderation may also reflect more stable government policies that reduced short-run variability, such as John Taylor’s (2010) argument that improved monetary policy is important for understanding the Great Moderation.

The 32-200 component is also important for the Great Recession and its aftermath. This largely reflects the fact that there has been limited economic recovery relative to long-run trend since the Great Recession.

2.2 Band-Pass Filtered Annual U.S. and European Data

This section presents band-pass filtered annual long-run U.S. data and annual European data. The output data were constructed by splicing the annual Kuznets-Kendrick data (Kendrick (1961)) beginning in 1890, with the annual NIPA data that begins in 1929. The annual Kendrick hours data, which also begins in 1890, is spliced with our update of the hours worked data from Cociuba et al. (2012). These constructions provide long annual time series that are particularly useful in measuring the low frequency components.

Figures 8 and 9 show the filtered annual U.S. data. The low frequency component, which is measured using the band pass filter from 8 - 50 years for these annual data, is also very large. Extending the data back to 1890 allows us to assess the importance of these different components around several major events, including the Panic of 1907 and World War I.
data show that both the Depression and World War II were dominated by lower frequency components, while the traditional business cycle component was significant during World War I and the Panic of 1907.

The large low frequency component of World War II stands in contrast to World War I, and also stands in contrast to standard theoretical models of wartime economies. These models typically specify wars as a highly transient shock to government purchases. The low frequency component is also large for the Great Depression. Sections five and six develop neoclassical models of Depressions and of wartime economies, in which both of these events are driven by persistent changes in government policies.

The decomposition ratio presented in [2], and that was used to construct the share of variation in the 2-200 quarter component due to the 32-200 quarter component, is used in a similar way to construct the share of variation in the 2-50 year component due to the 8-50 year component. This low frequency component share is also large in the annual data, ranging between 80-85 percent for real GNP and hours worked.

We also construct the decomposition using annual postwar logged real output data from several European economies: Germany, France, Italy, Spain, and Sweden. These data are from the Penn World Tables (Feenstra et al (2015). Figures 10 - 14 present the filtered data. Most of the variation in the European output data in the 2-50 year component also is accounted for by the low frequency (8-50) component. The long-run European components reflect clear patterns in these data. All of the European economies grow more rapidly than the U.S. during the 1950s and 1960s. All of these economies then experience large declines relative to trend that begin in the early 1970s and continue to the mid-1980s. The share of the 2-50 component that is accounted for by the 8 50 component is about 80 percent for Germany, France, Spain, and Sweden, and is about 71 percent for Italy.

2.3 Alternative to Band-Pass Filtering: Stochastic Trend Decomposition

This subsection presents an alternative decomposition method, known as stochastic trend decomposition, for assessing the relative importance of low frequency components. One approach to stochastic trend decompositions was developed by Beveridge and Nelson (1981), and is known as the Beveridge-Nelson decomposition. Watson (1986) describes an alternative approach, which is known as unobserved components model decomposition. In both frameworks, a time series is decomposed into two latent objects, a stochastic trend component, and a stationary component, which is often called the cyclical component.

Decomposing the time series into these latent components requires an indentifying restriction. The Beveridge-Nelson identifying restriction is that the two components are perfectly correlated. This identifying assumption is thematically consistent with our view that permanent changes in technologies and policies generate both stationary and permanent responses in macroeconomic variables.

5The unobserved components models have traditionally achieved identification of the two latent components by imposing that the trend and stationary components are orthogonal. More recently, Morley et al (2003) show how to achieve identification in unobserved components models with a non-zero correlation between the two components. Morley et al find that the decomposition for real GDP for their unobserved
The Beveridge-Nelson decomposition, which is simple and widely-used, is applied in this chapter. The Beveridge-Nelson statistical model begins with a variable that is assumed to have a stochastic trend component. The variable may also have a drift term, which drives secular growth in the variable. The Beveridge-Nelson decomposition removes the drift term, and then decomposes the variable, which we denote as $y_t$, into a stochastic trend component, $x_t$ and a stationary stochastic component, $s_t$. The stochastic trend is a random walk, and the innovation term, which is denoted as $\varepsilon_t$, is a white noise process:

\begin{align}
    y_t &= x_t + s_t & (3) \\
    x_t &= x_{t-1} + \varepsilon_t, E(\varepsilon) = 0, E(\varepsilon^2) = \sigma^2 & (4)
\end{align}

This decomposition is applied to the log of U.S. real GDP. The decomposition first requires specifying an ARIMA model for the data. We selected an IMA (0,1,1) model for the log of real GDP, given that the first three autocorrelations of the first difference of the logged data are 0.34, 0.19, and 0.06. Stock and Watson (1988) also use this ARIMA specification for the log of real GDP. The estimated statistical model for the log of real GDP using quarterly data between 1954:1 and 2013:4 is given by:

$$
\Delta \ln(GDP_t) = 0.0077 + \varepsilon_t + 0.40\varepsilon_{t-1}. \quad (5)
$$

These estimated coefficients are similar to the Stock and Watson estimates that were based on a shorter dataset. Stock and Watson estimated a slightly higher drift term of about .008, and a somewhat smaller moving average coefficient of 0.30 rather than 0.40.

Using the Wold decomposition, Beveridge and Nelson show that the permanent component for this estimated statistical model is given by:

$$
1.4 \sum_{j=1}^{t} \varepsilon_j. \quad (6)
$$

Figure 15 plots the detrended log of real GDP, which is constructed as the log of real GDP less its accumulated drift component, and the Beveridge-Nelson permanent component of these detrended data. The figure shows that almost all of the movement in detrended real GDP is due to the permanent component, rather than the transitory component. This finding is consistent with the band-passed filtered results regarding the large size of the long-run component.

The results presented in this section show that the bulk of observed fluctuations in aggregate time series are from longer-run changes than those associated with traditional business cycle frequencies. This finding motivates our focus on neoclassical models that are driven by long-run changes in technologies and policies, as opposed to models that are driven by very transient shocks, such as monetary shocks that operate in models with temporarily inflexible prices and/or wages.

components model is very similar to the Beveridge-Nelson decomposition. They also present evidence that the zero correlation identifying restriction that traditionally has been used in unobserved components models is empirically rejected.
3 Cass-Koopmans: The Foundation of Simple Models

This section summarizes the one-sector Cass-Koopmans optimal growth model with elastically supplied leisure, as it serves as the foundation for the other models that are developed in this chapter. This model features (1) standard utility maximization problems for households, and standard profit maximization problems for firms, both of whom behave competitively and who have rational expectations, (2) complete markets, (3) a unique and Pareto optimal equilibrium, (4) constant returns to scale technology.

Since the welfare theorems hold in this economy, we express this model as a social planning problem. For heuristic purposes, we assume perfect foresight. The planner’s maximization problem is given by:

$$\max_0^\infty \beta^t \sum_{t=0}^{\infty} u(c_t, l_t).$$

Maximization is subject to the economy’s resource constraint, a household time constraint, a transition equation for the capital stock, and non-negativity constraints on consumption, hours, and capital:

$$f(k_t, h_t) \geq c_t + i_t$$
$$1 \geq h_t + l_t$$
$$k_{t+1} = (1 - \delta)k_t + i_t$$
$$c_t \geq 0, h_t \geq 0, k_t \geq 0, k_0$$ given.

It is also necessary to impose the transversality condition to rule out explosive paths for the capital stock:

$$\lim_{t \to \infty} \beta^t u'(c_t) k_{t+1} = 0$$

The utility function satisfies the usual restrictions: it is concave in its arguments and twice continuously differentiable. The technology, $f$, is constant returns to scale in the two inputs capital, $k$, and labor, $h$, and is also twice continuously differentiable.

We will tailor the construction of different neoclassical models to focus on policies and technological change that we highlight for specific historical episodes. This should not be confused with the idea that fundamentally different models are needed to address different time periods in the history of the U.S. economy. Rather this means that the relative importance of different policies and different types of technological change has varied over time. Specifically, this includes the importance of biased technological change for understanding the post-Korean War U.S. history, cartelization and unionization government policies for understanding the 1930s, and changes in government fiscal policies for understanding the 1940s.
4 Neoclassical Models of the U.S. Post-Korean War Economy

In this section we present a series of neoclassical models, driven by permanent changes in technologies to study the post-Korean War U.S. economy. Our approach, which we describe in detail below, compares the equilibrium paths of the model economies in response to identified shocks, to the actual time series data. We will compare model results to unfiltered data, and also to the three different filtering frequencies described in section 2. In addition to evaluating the fit of the model for the raw data, this will allow us to assess how well the model matches data at the traditional business cycle frequencies (2-32 quarters), and also at low frequencies (32-200 quarters).

4.1 Quantitative Methodology

Neutral technological change that affects all sectors identically is the standard specification of technology in neoclassical macroeconomic models. However, there is a growing body of evidence that technological change is advancing much more quickly in the information processing sectors of the economy, particularly in capital equipment. This includes the areas of computer hardware, computer peripherals, photocopying equipment and telecommunications equipment, among others.

As described earlier in this chapter, Gordon (1990), Cummins and Violante (2002), and DiCecio (2009) construct capital equipment price data that they argue captures much more of the quality change that has occurred in these goods than is present in the NIPA equipment price data. Figure 16 shows the relationship between real GDP and the relative price of equipment at the three sets of frequencies that we consider. These figures show that the relative price of equipment is strongly countercyclical at all frequencies.

{Figure 16 goes here.}

These strong countercyclical patterns are interesting as a growing number of neoclassical studies are using these data to identify capital-equipment specific technological change. The following sections develop multisector growth models that include both neutral and equipment-specific technological change to study the evolution of the post-Korean War U.S. economy. This is a particularly interesting period for applying multisector models with biased technological change since this period features a number of major advances in information processing and telecommunications technologies, including the integrated circuit, personal computers and tablet technologies, fiber optics, software applications, cellular technologies, and the internet.

Focusing on this period also allows us to connect this analysis with the large business cycle literature, including Kydland and Prescott (1982), Hansen (1985), and the studies in Cooley (1995), which have analyzed the post-Korean War U.S. economy. Note that the post-Korean War period also includes a number of interesting sub-periods: the Vietnam War (1957-1971), the oil shock years (1974-1981), the Great Moderation (1984-2007), and the Great Recession and its aftermath (2008-present).
Our quantitative approach differs from the standard approach used in the real business cycle literature. The real business cycle approach specifies a dynamic stochastic general equilibrium model, which includes a specification of the stochastic process for the exogenous shocks that generate fluctuations in the model economy. The equilibrium decision rules and laws of motion are computed using numerical methods, and these equations plus a random number generator is used to simulate time series for the artificial economy. Summary statistics are then computed and compared with the same summary statistics computed from actual U.S. time series.

The approach we follow is similar to that employed in Hansen and Prescott (1993). We begin with a two-sector growth model in which movements in aggregate time series are the result of two factors we identify from U.S. data that we take to be the exogenous forcing processes in the model. These include technology shocks that are identified with total factor productivity and equipment specific technological change, which we identify from the relative price of equipment. We then calibrate and solve the model in manner consistent with the real business cycle literature. But, rather than drawing random realizations of the exogenous shock processes, we identify time paths for our two technology shocks from U.S. time series data. We then compute the equilibrium time paths for the endogenous variables (output, consumption, investment and hours worked) using the actual time path of the exogenous shocks. As noted above, we compare model variables to quarterly real variables for the unfiltered data over 1954-2014, as well as for frequency bands corresponding to 2-200, 2-32, and 32-200 quarters.

After comparing the time paths from the two-sector model with the corresponding time paths from U.S. data, we then compare these time paths with those of a standard one-sector neoclassical model in which neutral technology shocks are the only exogenous process hitting the economy. We then consider a three-sector model that adds a non-market home production sector to our baseline two-sector model. This extension allows us to study how equipment biased technological change may have induced movements in labor from the home production sector to the market sector.

We omit the details of numerically solving these models. Instead, we focus on the specifics of the model economies, the construction of U.S. data counterparts to the model variables, and the calibration that we use in our computational analyses.

In terms of assessing model fit, our approach differs considerably from the recent approach that is used in the New Keynesian literature. In New Keynesian models, such as Smets and Wouters (2007), as many shocks are added to the model as needed so that the model fits all of the data very closely. While this approach delivers a very good model fit, some of the shocks in the model are often difficult to interpret. Our approach to model fit follows from our theme that permanent changes in technologies are key drivers of the economy. The models analyzed in the following sections have very few shocks, which allows us to transparently evaluate the models’ successes and deviations.
4.2 A Two Sector Model with Aggregate and Investment-Biased Technological Change

This section develops a model with investment-specific technological change, as well as aggregate technological change that impacts all sectors equally. This approach was first developed in Greenwood, Hercowitz, and Krusell (1997), who document and discuss investment-specific technological change and its impact on long-run growth. Biased technological change has also been used to study wage inequality (Krusell et al (2000)) and business cycles (Fisher (2006), Justiniano et al (2010)).

The two-sector stochastic growth model we study consists of a primary sector, \( i = 1 \), producing \( C_{Mt} \), which is the sum of consumer services, nondurable consumption and government consumption, and \( I_{st} \), which is investment in structures.\(^6\) The second sector, \( i = 2 \), produces equipment \( I_{et} \) and consumer durables \( I_{dt} \). The technologies associated with each sector are as follows:

\[
C_{Mt} + I_{st} = Y_{1t} = z_t A K_{e1t}^\theta_1 K_{s1t}^\theta_2 H_{1t}^{1-\theta_1-\theta_2} \tag{13}
\]

\[
I_{dt} + I_{et} = Y_{2t} = q_t z_t A K_{e2t}^\theta_1 K_{s2t}^\theta_2 H_{2t}^{1-\theta_1-\theta_2} \tag{14}
\]

All variables are measured in per capita terms with a population growth factor \( \eta \). Here \( K_{eit}, K_{sit} \) and \( H_{it} \) are equipment, structures and hours worked, each in sector \( i \). The variables \( z_t \) and \( q_t \) are technology shocks that impact these sectors. The laws of motion for the stocks of equipment, structures, and durables is given by the following, where \( K_{e,t} = K_{e1t} + K_{e2t} \) and \( K_{s,t} = K_{s1t} + K_{s2t} \):

\[
\eta K_{e,t+1} = (1 - \delta_e) K_{e,t} + I_{et} \tag{15}
\]

\[
\eta D_{t+1} = (1 - \delta_d) D_t + I_{dt} \tag{16}
\]

\[
\eta K_{s,t+1} = (1 - \delta_s) K_{st} + I_{st} \tag{17}
\]

The logarithms of the two shocks, \( z \) and \( q \), follow random walks with drift.

\[
\log z_{t+1} = \log z_t + \varepsilon_{1,t+1}, \quad \varepsilon_1 \sim N(\mu_1, \sigma_1^2) \tag{18}
\]

\[
\log q_{t+1} = \log q_t + \varepsilon_{2,t+1}, \quad \varepsilon_2 \sim N(\mu_2, \sigma_2^2) \tag{19}
\]

The random variables \( \varepsilon_1 \) and \( \varepsilon_2 \) are i.i.d across time and are contemporaneously uncorrelated.

There is a stand-in household who maximizes the expected discounted sum of utility defined over consumption of non-durables and services, the stock of durables, and leisure:

\[
\max_{E_0} \left\{ \sum_{t=0}^{\infty} (\beta \eta)^t \left[ \alpha \log C_{Mt} + (1 - \alpha) \log D_t + \phi \log(1 - H_{1t} - H_{2t}) \right] \right\} \tag{20}
\]

\(^6\)We will also lump investment in intellectual property with investment in structures.
Optimality implies that the value marginal product of each input will be equalized across sectors. Given that identical Cobb-Douglas production functions are assumed, this implies the fraction of the total quantity of each input assigned to each sector is the same across inputs. Letting $H_{Mt} = H_{1t} + H_{2t}$, this implies that $K_{et}^1/K_{et}^2 = H_{et}/H_{Mt}$ for $i = 1, 2$. Given this result, and the fact that the technology is constant returns to scale, it is possible to aggregate over sectors to obtain the aggregate resource constraint:

$$C_{Mt} + I_{st} + \frac{1}{q_t} (I_{dt} + I_{et}) = z_t AK_e^{\theta_1} K_s^{\theta_2} H_{Mt}^{1-\theta_1-\theta_2} \equiv Y_t$$

(21)

Note that in this aggregate resource constraint, the outputs $I_d$ and $I_e$ are divided by $q$. In the decentralized version of this economy, $\frac{1}{q}$ is the price of equipment goods relative to output from sector 1. This result shows that data on the relative price of equipment can be used to measure equipment-specific technological change.

Given values for $K_{e0}$, $K_{s0}$ and $D_0$, the equilibrium stochastic process for this economy can be found by solving the planner’s problem maximizing (20) subject to (15) through (19) and (21).

4.2.1 Balanced Growth Path

Due to the positive drift in the random walks (18) and (19), this model exhibits stochastic growth. In a certainty version of the model in which $\sigma_1 = \sigma_2 = 0$, there is a balanced growth path where the asymptotic growth factors are given by $g_c = \frac{Y_{t+1}}{Y_t} = \frac{C_{Mt+1}}{C_{Mt}} = \frac{I_{st+1}}{I_{et}} = \frac{K_{st+1}}{K_{et}} = e^{\mu_1+\theta_1 \mu_2}$ and $g_e = \frac{I_{et+1}}{I_{et}} = \frac{I_{dt+1}}{I_{et}} = \frac{K_{et+1}}{K_{et}} = \frac{D_{t+1}}{D_t} = g_c e^{\mu_2}$. Given these growth factors, the asymptotic growth path can be written $Y_t = g_c^t Y$, $H_{Mt} = H_M$, $C_{Mt} = g_c^t C_M$, $I_{st} = g_e^t I_s$, $K_{st} = g_e^t K_s$, $I_{et} = g_e^t I_e$, $I_{dt} = g_e^t I_d$, $K_{et} = g_e^t K_e$ and $D_t = g_e^t D$, where the steady state values are the solutions to the following equations (given $\bar{q}$ and $\bar{z}$):

$$g_c = \theta_2 \frac{Y}{K_s} + 1 - \delta_s$$

(22)

$$g_e = \theta_1 \frac{Y}{K_e} + 1 - \delta_e$$

(23)

$$g_e = \frac{(1 - \alpha) C_M}{\alpha D} \bar{q} + 1 - \delta_d$$

(24)

$$\frac{\phi}{1 - H_M} = \frac{\alpha (1 - \theta_1 - \theta_2) Y}{H_M C_M}$$

(25)

$$\bar{Y} = AK_e^{\theta_1} K_s^{\theta_2} H_M^{1-\theta_1-\theta_2}$$

(26)

$$\bar{C}_M = \bar{Y} - \bar{I}_s - \frac{1}{\bar{q}} [\bar{I}_e + \bar{I}_d]$$

(27)

$$\bar{I}_s = (\delta_s + \eta g_c - 1) K_s$$

(28)

$$\bar{I}_e = (\delta_e + \eta g_c - 1) K_e$$

(29)

$$\bar{I}_d = (\delta_d + \eta g_c - 1) D$$

(30)
We use this nonstochastic asymptotic growth path to help us calibrate the model and to construct capital stock series that are consistent with the model’s balanced growth properties.

4.2.2 Calibrating the Model with U.S. Data

We proceed by connecting each endogenous variable of this model with a counterpart taken from the U.S. National Income and Product Accounts. The data we use runs from 1954Q1 to 2014Q4. On the product side, the model has one nondurable consumption good \((C_{Mt})\) which we take to be the sum of nondurable consumption, services and government consumption. There are three forms of investment: \(I_e\) is the sum of private and government investment in equipment; \(I_s\) is the sum of private investment in structures, intellectual property, residential structures, and government investment in structures and intellectual property; and \(I_d\) is purchases of consumer durables. Given that we have not allocated every component of Gross Domestic Product to one of these expenditure components, we take total output to be \(Y_t = C_M + I_s + \frac{1}{q}(I_d + I_e)\). The relative price of equipment in our model is equal to \(\frac{1}{q_t}\), so we identify \(q_t\) from the relative price of equipment calculated by Riccardo DiCecio (see DiCecio (2009)).

The capital stocks, which are the sum of both private and government fixed assets, are computed from annual quantity indexes of fixed assets obtained from the Bureau of Economic Analysis and is the stock associated with each investment series. In particular, \(K_s\) is nonresidential and residential structures along with intellectual property, \(K_e\), is the stock of equipment, and \(D\) is the stock of consumer durables. To obtain quarterly real stocks of capital, the annual quantity indexes are multiplied by the corresponding 2009 nominal value and quarterly series are obtained by iterating on the laws of motion [15]-[17] using the corresponding quarterly investment series. Per capita capital stocks and output are obtained by dividing by the civilian population (16-64) plus military personnel. Finally, the hours series we use is average weekly hours per person (including military hours) based on data from the Current Population Survey. In particular, we have updated the series created by Cociuba, Prescott and Ueberfeldt (2012).

Given these empirical counterparts, the growth factor for population is \(\eta = 1.003\) and the growth factor for per capita output is \(g_c = 1.0036\). The parameter \(\mu_2 = 0.0104\), which

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7This data series is available on the FRED database maintained by the Federal Reserve Bank of St. Louis.

8Given that the model assumes constant depreciation rates, which does not hold in our data sample, we allow the depreciation rate to vary across 10 year periods when constructing the quarterly capital stock series. That is, an initial value for the annual series in year \(t\) and a terminal value in year \(t + 10\), we find the depreciation rate such that iterations on the law of motion of the capital stock hits the terminal value in 40 quarters using the corresponding quarterly investment series. In particular we find the depreciation rate \(\delta_i\) for decade \(i\) such that \(K_{i+10} = (1 - \delta_i)^{40}K_i + \sum_{j=1}^{40}(1 - \delta_j)^{40-j}I_j\), where \(K_i\) is the capital stock at the beginning of year \(i\), \(K_{i+10}\) is capital at the beginning of year \(i + 10\), and \(\{I_j\}_{j=1}^{10}\) is investment for each quarter between those dates. Once we know \(\delta_i\) for each subperiod in our sample, it is straightforward to construct quarterly capital stocks for each quarter of year \(i\).

The capital stock obtained, however, is inconsistent with the trend introduced by our empirical measure of \(q\), which is based on different price deflators than those used in producing the NIPA capital stocks. As a result, we also adjust the trend growth of the capital stocks so that these stocks are consistent with long run growth properties of the model. That is, a trend is added to our quarterly series for \(K_s\) so that it has an average growth rate equal to \(g_c\) and \(D\) and \(K_e\) are similarly adjusted to have an average growth factor \(g_e\).
is the average of $\log q_{t+1} - \log q_t$. This implies that $g_e = g_c e^{\mu_2} = 1.014$.

We calibrate the model by setting $\beta = 0.99$, labor’s share, $1 - \theta_1 - \theta_2$, equal to 0.6 and the depreciation rates equal to the average of the depreciation rates obtained when forming the quarterly capital stock series. This gives us $\delta_e = 0.021$, $\delta_s = 0.008$, and $\delta_d = 0.05$. The individual capital shares are based on estimates in Valentinyi and Herrendorf (2008) renormalized so they sum to 0.4. In particular we set $\theta_1 = 0.21$ and $\theta_2 = 0.19$. The parameter $\alpha$ is computed from a version of equation (24) where the term $\bar{C}_M / \bar{q}$ is replaced with the average value of $C_{M,t} q_t / D_t$ from the empirical counterparts to these variables. This gives $\alpha = 0.217$.

Next, we set $Y$, $H_M$, and $\bar{q}$ equal to the initial observation in the time series for each of these variables. The seven remaining steady states ($K_s$, $K_e$, $D$, $\bar{I}_s$, $\bar{I}_e$, $\bar{I}_d$, and $\bar{C}_M$) are obtained by solving seven equations (22) - (24) and (27) - (30). So that the steady state capital stocks are equal to the first observations for these variables, we multiply all observations of $K_s$ by $K_s / K_{s,0}$, all observations of $K_e$ by $K_e / K_{e,0}$ and all observations of $D$ by $D / D_0$. These are the capital stocks used to construct the empirical counterpart to $z_t$.

We construct a quarterly time series for the exogenous shock, $z_t$, from 1954Q1 to 2014Q4 by setting $z_t = Y_{t,AK} K_{t,1}^{\mu_1} H_{M,t}^{\mu_2}$ where the parameter $A$ is chosen so that the first observation of $z_t$ is equal to one. This implies $A = 6.21$. Somewhat surprisingly, the growth rate of $z_t$ when computed in this way turns out to be zero ($\mu_1 = 0$). That is, when measured through the lens of this model, the average rate of growth in per capita income during the postwar period is accounted for entirely by equipment specific technological improvement.

We summarize the calibration of the model in Table 1 in the column labeled "Two-Sector." This table reports the calibrated parameter values for all models considered, so we will refer back to this table as we discuss these alternatives.

4.2.3 Comparison of Model with Data

Given our time series for $z_t$ and $q_t$, times series for the endogenous variables of the model are computed for the sample period 1954Q1 to 2014Q4. This is done using log-linear approximations of the decision rules that solve the planner’s problem obtained using standard numerical methods [see, for example, Uhlig (1999)]. Figure 17 shows our measures of output and hours from U.S. data along with the time series for these variables implied by our model.

Output from the data and model are quite close to each other until the mid 1980’s when model output becomes lower than in the data. By 2002, however, model output has recovered. Model hours tend to be higher than in the data during the 1960’s and 70’s, and lower from the mid 1980’s until the Great Recession. Following the Great Recession, the data shows some recovery in hours worked that the model does not.

Figure 18 consists of four panels showing output, hours, consumption and investment—from both the model and the data—that has been filtered to show only fluctuations between 2
and 32 quarters. The real business cycle literature has demonstrated that neoclassical models
of this sort generate fluctuations similar to those in postwar U.S. data at this frequency. As
the figure illustrates, this is particularly true for output and investment.

{Figure 18 goes here}

Less studied, however, are the low frequency fluctuations exhibited by models of this
sort. Figure 19 is a plot of model and U.S. data for the same four variables that has been
filtered to show fluctuations between 32 and 200 quarters. The model seems to do a pretty
good job in tracking fluctuations in output, consumption and investment in this frequency
band. For hours worked, the model captures some of the low frequency movements, but not
others. In the late 1950’s, the model shows hours falling sooner than it does in the data,
while the model and data track pretty closely during the 1960’s and early 1970’s. In the
late 1970’s, the data shows an increase in hours worked that the model does not capture,
but the model and data follow each other throughout the 1980’s and 90’s. At the time of
the Great Recession, the decline in hours—as well as other macro aggregates—is less in the
model than in the data.

{Figure 19 goes here}

Figure 20 plots the same data as the previous figure for filtered output and hours for both
the 2-32 quarter frequency and the 32-200 quarter frequency. The difference is that we have
included a third time series in each plot that shows simulated data under the assumption
that there were no fluctuations in \( z_t \) and only fluctuations in \( q_t \). That is, when computing
the simulation, the time series for \( z_t \) is replaced by the nonstochastic growth path for \( z \).
That is, \( z_t = e^{\mu t} \) for all \( t \).

{Figure 20 goes here}

This figure shows that much of the high and low frequency fluctuations in hours worked
are due to movements in \( q_t \), but this is not as true for fluctuations in output. It is also less
true for business cycle fluctuations in hours worked in more recent decades.

4.3 One-sector model

We now proceed to compare the fluctuations exhibited by the two-sector model with a stan-
dard one-sector neoclassical stochastic growth model. This one-sector economy consists of
a single production sector that produces output from capital and labor that can be con-
sumed or invested. It differs from the two-sector model in that there is only one type of
capital stock, no separate role for consumer durables, and one type of technology shock. In
particular, the resource constraint, which replaces equation (21), is

\[
C_t + I_t = Y_t = z_t AK_t^\theta H_t^{1-\theta}.
\]  

(31)

The law of motion for capital next period is given by
\eta K_{t+1} = (1 - \delta)K_t + I_t \quad (32)

where the depreciation rate is \(0 < \delta \leq 1\) and \(1 \leq \eta \leq \frac{1}{\beta}\) is the population growth factor. The logarithm of the technology shock, \(z_t\), is assumed to follow a random walk with drift \((\mu \geq 0)\). We assume that the period \(t\) realization of \(z\) is observed at the beginning of the period.

\[
\log z_{t+1} = \log z_t + \varepsilon_{t+1}, \quad \varepsilon \sim N(\mu, \sigma^2) \quad (33)
\]

The preferences of the representative infinitely-lived household are given by

\[
E \sum_{t=0}^{\infty} (\beta \eta)^t [\log C_t + \phi \log L_t] \quad (34)
\]

where \(0 < \beta < 1\) and \(\phi > 0\). The variable \(L_t\) is leisure, where

\[
L_t + H_t = 1. \quad (35)
\]

Given \(K_0\), we compute an equilibrium sequence for \(\{C_t, I_t, Y_t, H_t, L_t, K_{t+1}\}\) by maximizing (34) subject to (31) - (33) and (35).

### 4.3.1 Calibrating the one-sector model with U.S. Data

For comparison purposes, we begin by keeping the definition of output the same as in the two sector model, \(Y = C + I_s + \frac{1}{q}(I_d + I_e)\). Given that there is no separate role for consumer durables in this model, we define investment in the one-sector model to be \(I = I_s + \frac{L_q}{q}\) and consumption to be the sum of nondurable consumption plus services and \(\frac{L_q}{q}\). That is, \(C_t = C_{Mt} + \frac{L_q}{q}\), where \(C_{Mt}\) is consumption from the two-sector model. The capital stock is the sum \(K = K_e + K_s\). The quarterly capital stock series for this sum is formed using the same method as for the two-sector model and the quarterly depreciation rate turns out to be \(\delta = 0.013\). As in the two-sector model, \(\beta = 0.99\) and labor’s share is taken to be 0.6, so \(\theta = 0.4\). Given this, a quarterly time series for the exogenous shock \(z_t\), from 1954Q1 to 2014Q4, is constructed by setting \(z_t = \frac{Y_t}{AK_{t+1}H_t^\theta}, \) where the parameter \(A\) is set so that \(z_0 = 1\). This implies that \(A = 2.7\). In addition, the drift parameter, \(\mu\), turns out to be 0.0021.

As in the two-sector model, we set the steady state values for \(K, H\) and \(Y\) equal to the first observation in our data sample (for 1954Q1). Steady state consumption is then obtained from the steady state version of the resource constraint (31). We can then calibrate the parameter \(\phi\) from the steady state condition for hours worked. That is, \(\phi = \frac{(1-\theta)(1-H)}{C_H} = 2.37\).

To facilitate comparison across models, the parameter values are also reported Table 1.
4.3.2 Comparing the one and two-sector models with U.S. Data

Table 2 provides two metrics for comparing the closeness of the one and two-sector model simulations with filtered data. These measures include the ratio of the standard deviations of the model series with the standard deviation of the data series. This provides a measure of how well the model is capturing the volatility in the data. The second measure is the correlation between the model simulations and the data. We report these measures for data filtered to extract fluctuations of 2-32 quarters, 32-200 quarters and 2-200 quarters. In all cases, a number closer to one implies a better fit.\(^9\)

\{Table 2 goes here\}

The table shows that the correlation between model and data for business cycle fluctuations are higher for the two-sector model, with the exception of consumption. For low frequency fluctuations, the one-sector model does slightly better, although the correlation between hours worked from the model and data is slightly higher for the two-sector model. The volatility of the various series is generally better accounted for by the two-sector model. Hence, the main conclusion we draw from this table is that the two-sector model fits the data better than the one-sector model, with the exception of consumption fluctuations. We find it interesting that the two-sector model is able to account for volatility in spite of the fact that we have assumed random walk technology shocks and divisible labor. These are both assumptions that tend to reduce the size of fluctuations.\(^10\)

Figure 21 provides the same information as Figure 20 except that the comparison is now with the one-sector simulation for output and hours rather the "q-shock" only simulation. The figure illustrates that much of the low frequency movements in output can be accounted for by the one-sector model almost as well as the two-sector. The low frequency volatility of hours, however, is better explained by the two-sector model than the one-sector.

\{Figure 21 goes here\}

4.4 A three-sector model

This section studies a model constructed by adding a non-market home production sector to the two sector model. We develop the three sector model with two alternative home production specifications. One is the standard home production specification of Benhabib, Rogerson and Wright (1991) and much of the literature that follows from this. This formulation provides an additional margin of substitution for the household in which time can be allocated to market production, home production, or leisure. In the Benhabib, Rogerson and Wright model, there is a relatively high substitution elasticity between home-produced goods and market-produced goods, and this high elasticity generates significant movement.

\(^9\)In this table and subsequent tables, we only use data starting from 1955Q1. The reason is that there is an unusual hours observation in 1954 that can be seen in Figure 17, and we don’t want that observation distorting the statistics reported in these tables.

of labor between the home sector and market sector in response to shocks. Home goods are produced using a Cobb-Douglas technology with labor and consumer durables.

The alternative home production formulation is motivated by Greenwood et al (2005), which argues that rapid technological change in labor-saving consumer durables has secularly reallocated time from home production to market production, mainly by women moving into the labor force. In this specification, consumer durables are more substitutable with labor than in the Benhabib, Rogerson and Wright (1991) specification that assumes a Cobb-Douglas technology for the home sector.

The model presented here nests both of these specifications. In particular, we assume that a non-market consumption good, \( C_{Nt} \), is produced using labor \( (H_{Nt}) \) and the stock of consumer durables. As in Greenwood et al (2005), we allow for the possibility that durables and labor are more substitutable than implied by the standard Cobb-Douglas production function. In particular, we assume the following functional form for the home production function with \( \sigma > 0 \):

\[
C_{Nt} = A_N \left[ \varphi \left( \frac{D_t}{e^{\mu t}} \right)^{\sigma} + (1 - \varphi)(g_{Ht}^{\dagger}H_{Nt})^{\sigma} \right]^{\frac{1}{\sigma}} \tag{36}
\]

The standard version of the model can be recovered by making \( \sigma \) close to zero. Note that the terms \( e^{\mu t} \) and \( g_{Ht}^{\dagger} \) are included here to guarantee that \( C_{Nt} \) grows at the same rate as total output along the balanced growth path.

The second modification relative to the two-sector model is to replace the objective function (20) with the following:

\[
\max E_0 \left\{ \sum_{t=0}^{\infty} (\beta \eta)^t \left[ \log C_t + \phi \log(1 - H_{Mt} - H_{Nt}) \right] \right\}, \tag{37}
\]

where consumption, \( C_t \), is a composite consumption good, standard in the home production literature, derived from market and non-market consumption goods

\[
C_t = [\alpha C_{Mt}^{\omega} + (1 - \alpha)C_{Nt}^{\omega}]^{\frac{1}{\omega}} \tag{38}
\]

Given values for \( K_{\epsilon 0} \), \( K_{\sigma 0} \) and \( D_0 \), the equilibrium stochastic process for this economy can be found by solving the planner’s problem maximizing (37) subject to (15) through (19), (21), (36), and (38).

### 4.4.1 Calibrating the three-sector model to U.S. data

The calibration strategy is exactly the same as for the two-sector case, although the model introduces four new parameters \( (A_N, \varphi, \omega, \text{ and } \sigma) \) and two other parameters \( (\alpha \text{ and } \phi) \) have different interpretations in this model. In addition, two new variables are introduced that are not directly observable in the U.S. data. These are non-market consumption \( (C_N) \) and non-market hours worked \( (H_N) \). In the absence of measured counterparts to these variables, we assume that in steady state \( \bar{C}_{Mt}^{\omega} = 0.25 \) and \( \bar{H}_N = \frac{1}{6} \), which are values consistent with the home production literature. The mapping between all other model variables and U.S. time series is the same as in the two-sector model.
The steady states values for $K_s$, $K_e$, $Y$, $C_M$, $I_s$, $I_e$, $I_d$, $D$, $H_M$, $H_N$, $C_N$, and $C$ are determined by equations (22), (23), (26)-(30), and the following five equations:

\[
\begin{align*}
\frac{g_E}{\beta} &= \frac{(1 - \alpha)A_N^\sigma \varphi q C_M^{1-\omega}}{\alpha C_N^{\sigma-\omega} D^{1-\sigma}} + 1 - \delta_D \\
\frac{\phi}{1 - H_M - H_N} &= \alpha(1 - \theta_1 - \theta_2) \frac{\overline{Y}}{H_M C^{\omega} C_M^{1-\omega}} \\
\frac{\phi}{1 - H_M - H_N} &= \frac{(1 - \alpha)A_N^\sigma(1 - \varphi)}{H_N^{1-\sigma} C_N^{\sigma-\omega}} \\
C_N &= A_N \left[ \varphi D^\omega + (1 - \varphi)H_N^\omega \right]^{\frac{1}{\sigma}} \\
C &= \left[ \alpha C_M^{\sigma} + (1 - \alpha)C_N^{\sigma} \right]^{\frac{1}{\omega}}.
\end{align*}
\]

We experiment with two different sets of values for the parameters $\sigma$ and $\omega$ to differentiate between our two home production specifications. Given values for these parameters, values for $\alpha$, $\phi$, $\varphi$ and $A_N$ can be obtained from equations (39) through (42) subject to $\frac{C_N}{C_M} = 0.25$, $H_N = \frac{1}{6}$ and $C$ is given by equation (43). \[11\]

The first calibration we consider is referred to as the "standard home production" model. In this case, $\omega = 0.6$ and $\sigma = 0$, which corresponds to values common in the home production literature. In this case, the utility function (38) allows for more substitutability between home consumption and market consumption than implied by a Cobb-Douglas specification while the home production function (36) is assumed to be Cobb-Douglas. The second calibration, which we refer to as the "alternative home production" model, is motivated by Greenwood et al (2005) and sets $\omega = 0$ and $\sigma = 0.4$. Here (38) is assumed to be Cobb-Douglas and we allow for an elasticity of substitution between durables and hours that is greater than 1 in the home production function (36). The parameter values associated with both calibrations are given in Table 1.

### 4.4.2 Fluctuations in the three-sector model

We begin by comparing the simulations produced by the two versions of the three-sector model that we consider. Figure 22 shows unfiltered output and hours from the two models as well as from U.S. time series. Both models account for output movements quite well, although the alternative calibration does a somewhat better job in the 1960’s and 70’s while the standard home production calibration fits the data better in the 1980’s and 90’s. Both models imply similar paths during the Great Recession period. The same is also true for hours worked—the alternative calibration does better during the early periods and less well during the 80’s and 90’s. Both calibrations give essentially identical results during the 2000’s.

{Figure 22 goes here}

\[11\] We also use the fact that, as in the two-sector case, we choose parameters so that $\overline{Y}$, $H_M$ and $H_N$ are the first observation in our data sample.
An interesting difference between hours worked from the two models can be seen from examining the period from about 1982 to 2000. The rise in hours worked predicted by the alternative calibration during this period is significantly larger than that predicted by the standard home production model. In the spirit of Greenwood et al (2005), this calibration does a better job of capturing the secular increase in hours worked that occurs over this period, mainly due to women entering the labor force. As one can see from Figure 23, this difference does not appear in the low frequency fluctuations that we report.

{Figure 23 goes here}

The two calibrations, however, give essentially the same results once the data is filtered. Figure 23 illustrates this by plotting filtered data for output and hours from the two versions of the model. The data for both business cycle fluctuations as well as low frequency fluctuations essentially lay on top of each other. In particular, the alternative home production model does not exhibit the significantly larger increase in hours worked relative to the standard home production model during the 1980’s and 90’s as was observed in Figure 22.

The closeness of the filtered data from these models with filtered data from U.S. time series is illustrated in Figure 24 and Table 3. Figure 24 shows filtered data from the standard home production calibration and the U.S. economy for output and hours. When one compares the panels in Figure 24 with the corresponding panels in Figures 18 and 19, the results from the home production model appear very similar to the two-sector model with slightly more volatility in hours worked at both sets of frequencies.

{Figure 24 goes here}

The same sorts of conclusions that can be drawn from Figure 24 are also apparent in Table 3. This table provides the same set of statistics as in Table 2 for comparing model data with actual data. Here we compare both calibrations of our three-sector model with the U.S. time series.

{Table 3 goes here}

The final set of tables we present in this section report the statistics for comparing model simulation and actual data for three subperiods of the postwar period. Table 4 looks only at the early post war period from 1955Q1 to 1983Q4 and Table 5 reports statistics for the Great Moderation period from 1984Q1 to 2007Q3. Finally, statistics for the Great Recession and after are reported in Table 6.

{Tables 4 - 6 go here}

Which model best explains postwar fluctuations in output, consumption, investment and hours worked? These tables show that it depends on the sample period and the frequency band of interest.

In the early postwar period (Table 4), all three models do a similar job fitting the data, but different models are better at accounting for fluctuations in different frequency bands.
Hours is explained the least well by all of the models, but the correlation between model and data hours is highest for the two-sector model at business cycle frequencies and the home production model for lower frequencies. Output fluctuations are best explained by the two-sector model in all frequency bands considered. Consumption fluctuations are best explained by the one-sector model and investment fluctuations are almost equally well explained by the two and three-sector models.

A feature seen in all three of these tables is that the volatility of model data relative to actual data rises as the number of sectors is increased. This is due to the increased substitution opportunities offered by multisector economies.

During the Great Moderation (Table 5), the one-sector model provides the highest correlations between model and actual data for output, consumption and investment, which is different from what is observed in the earlier period. Hours, however, are slightly better explained by the three-sector model. At lower frequencies, the three sector model shows the highest correlation for all variables except consumption.

In the most recent period (Table 6), which covers the Great Recession and aftermath, a striking finding emerges regarding hours fluctuations. All three models show negative correlations between model and data hours worked at business cycle frequencies. However, this correlation is quite high, especially for the two and three-sector models, at lower frequencies. At business cycle frequencies, all three models do similarly poor job in accounting for fluctuations in output and investment. Again, the one-sector model does best in explaining consumption. But, at lower frequencies, all three neoclassical models show high correlations between model and data for these three variables as well as hours worked.

It is interesting and important that the fit of the two and three sector models for the 32-200 component is no different during the Great Moderation than during the 1955-1983 period. This is important because some economists have argued that neoclassical models cannot fit data from this specific period because the business cycle correlation between labor productivity and hours worked becomes negative during the Great Moderation (see Gali and van Rens (2014)). We find that the change in this higher frequency statistic has no bearing on the ability of these models to fit the large, longer-run component in the data. We also note that these models also fit the 32-200 component of the data well during the Great Recession and its aftermath. However, it should be noted that this is a short data interval for measuring the long-run component.

5 Neoclassical Models of Depressions

This section describes neoclassical models of depressions, which are prolonged periods in which aggregate economic activity is far below trend. Kehoe and Prescott (2007) define a Great Depression as an event in which per capita real output is at least 20 percent below trend, in which trend is constructed using a 2 percent annual growth rate. They also require that real output is at least 15 percent below this trend within a decade, and that real output always grows at less than two percent per year during the episode.

Neoclassical modelling of depressions has become a very active research field in the last 15 years and is providing new insights into several episodes that have long been considered
economic pathologies. Some of the models presented here are tailored to capture features of specific episodes, but all of these models can be modified to study other episodes of depressed economic activity.

This subsection focuses on the U.S. Great Depression, which is the most widely-studied depression in the literature, and is perhaps the most striking and anomalous period of macro-economic activity in the economic history of the U.S. The Great Depression began in the Fall of 1929, and the economy did not recover to its pre-depression trend until the early 1940s.

Lucas and Rapping (1969) developed the first modern model of the U.S. Great Depression. This model represented a breakthrough by analyzing the Depression within an equilibrium framework. Previous studies of the Depression noted the coincidence of deflation and depression in the early 1930s, and viewed deflation as causing the Depression. The Lucas-Rapping model provided a very different interpretation of this relationship. In the Lucas-Rapping model, deflation depresses output through imperfect information about nominal price changes. Specifically, workers misinterpret falling nominal wages as reflecting a lower relative price for their labor services. This mistaken perception of the real wage leads to lower employment and lower output. This change in employment and production reflects intertemporal substitution, in which employment and output expand during periods in which workers perceive high real wages, and contracts during periods of perceived low real wages. The mechanism of imperfect information and nominal price changes was developed further in Lucas's (1972) seminal contribution that rationalized Phillips Curve-type relationships within an optimizing model.

Lucas and Rapping's study spawned a large neoclassical literature on fluctuations that focused on intertemporal substitution as the principal channel for understanding business cycle fluctuations. This literature includes contributions by Barro (1981), Barro and King (1984), Lucas (1973), Sargent (1973), Sargent and Wallace (1975), among others.

But many economists were skeptical of these early neoclassical interpretations of fluctuations, particularly for deep and prolonged crises such as the U.S. Great Depression. Modigliani (1977), argued that neoclassical models of the Depression implausibly portrayed individuals as exhibiting a "contagious attack of laziness". Modigliani, Rees (1970), and many other economists interpreted the substantial job loss of the Depression as involuntary unemployment, which stands in sharp contrast to the market-clearing equilibrium interpretation of Lucas and Rapping. The Modigliani quip has been repeated frequently over time, and is viewed widely as a fundamental critique of neoclassical macroeconomic modeling. This subsection presents neoclassical models of the Depression that directly confront Modigliani's criticism. The analysis shows how simple neoclassical models can be extended to assess economies with market distortions that create substantial and persistent involuntary job loss.

Recent models of the Great Depression analyze a number of policies and mechanisms in order to understand this episode. This includes the wage fixing and work-sharing policies of Herbert Hoover (Ohanian (2009), Ebel and Ritschl, (2011) and Amaral and Mcgree (2012)), the worker-industry cartels of the National Industrial Recovery Act and the National Labor Relations Act (Cole and Ohanian (1999), (2004)), changes in capital income tax rates (McGrattan (2012)), the cartel policies of Mussolini in Italy, and Hitler in Germany (Cole and Ohanian (2016)), the impact of tariffs on resource allocation and productivity (Bond et al (2013)), the impact of financial market imperfections and misallocation in the Depression (Ziebarth (2014)), and the impact of contractionary monetary policy on labor markets (Bordo, Erceg and Evans (2000)).
5.1 The Depth, Duration and Sectoral Differences of the U.S. Great Depression

The depth, duration, and sectoral differences in severity of the Depression represent a significant challenge for neoclassical models, or for any quantitative theoretic model. Tables 7 through 9 summarize these features by presenting data on output, consumption, investment, hours worked, and productivity. The data in these tables are divided by the population. In addition, all of the data except for hours worked are detrended at two percent per year. Thus, the value of 100 means that a variable is equal to its steady state growth path value.

Table 7 shows that real GDP declines by more about 35 percent between 1929 and the Depression’s trough in 1933, and remains far below trend after that. Consumption also falls considerably, and remains near its trough level after 1933. Investment declines by about 75 percent, and remains 50 percent below trend by the late 1930s. Hours worked declined about 27 percent between 1929 and 1933, and remain more than 20 percent below trend after that.

Total factor productivity (TFP) declines by about 14 percent below trend by 1933. Such a large drop in productivity raises questions about measurement, and whether this decline reflects factors other than changes in efficiency. Ohanian (2001) found that this TFP decline was not easily reconciled with capacity utilization, labor hoarding, or compositional shifts in inputs, which suggests significant efficiency loss during this period. TFP recovers quickly and ultimately rises above trend by the late 1930s. This rapid productivity growth after 1932 led Field (2013) to describe the 1930s as "the most technologically progressive decade of the 20th century."

The severity of the Depression differed considerably across sectors. Table 8 shows that manufacturing hours declined enormously, but agricultural hours remained close to trend through the mid-1930s. These two sectors account for roughly 50 percent of employment at that time.

The data summarized here challenge long-standing views of the Depression. Traditional studies omit productivity, and focus instead on monetary contraction and banking crises as the key determinants of the Depression (see Friedman and Schwartz (1963) and of Bernanke (1983)).

However, these factors cannot account for the early stages of the Depression, nor can they account for the post-1933 continuation of the Depression. In terms of the early stages of the Depression, industrial production declined by about 35 percent between the Fall of 1929 through November of 1930, but there were neither banking crises nor significant monetary contraction during this time.\(^\text{13}\)

After 1933, the money stock expanded rapidly and banking crises were quickly eliminated by the introduction of bank deposit insurance. The Lucas-Rapping model and New Keynesian models, such as Eggertsson (2012), counterfactually predict a very rapid recovery to trend as a consequence of rapid monetary expansion and the end of banking crises. In the Lucas-Rapping model, monetary expansion stops deflation, and employment expands as workers perceive that the relative price of their labor services has recovered. In New Key-

\(^{13}\)Ohanian (2010) discusses the immediate severity of the Great Depression that occurred before monetary contraction and before banking crises.
nesian models, such as Eggertsson (2012), inflation moves the economy away from the zero lower interest rate bound, and hours worked increase substantially. These models cannot account for the failure of hours to remain significantly depressed after 1933. Rees (1970) and Lucas and Rapping (1970) discuss the failure of the Lucas and Rapping model to account for hours worked after 1933, and Ohanian (2011) discusses the failure of the Eggertsson model to account for hours worked after 1933.

Moreover, the traditional view of the Depression counterfactually implies that the agricultural sector and the manufacturing were identically depressed. The large differences between these two sectors means that any successful model of the Depression must account for the enormous manufacturing depression, but only a modest agricultural decline.

5.2 Diagnosing Depressions with Simple Neoclassical Models

Cole and Ohanian (1999) advocate using simple neoclassical models to diagnose depressions. Their idea is that both the successes and the deviations between model and data, are informative for developing theories of specific episodes. Cole and Ohanian (1999) focused on the contribution of TFP for the Depression within a standard one-sector stochastic growth model for the 1930s. They fed TFP shocks from 1930-39 into the model and found that TFP drop accounts for about 60 percent of the drop in output between 1929-33, and about half of the drop in labor. However, the model generates a completely counterfactual path for the economy after 1933. The rapid recovery of TFP generates a rapid recovery in the model, with labor input recovering to trend by the mid-1930s. In contrast, the actual economy appears to have shifted onto a lower steady state growth path after 1933, with consumption and hours worked remaining near their 1932 trend-adjusted levels.

The post-1933 deviation between model and data provide valuable information about this episode. The results indicate that understanding the post-1933 data requires a large and persistent change in a state variable that substantially depressed and/or restricted the opportunities to produce and trade. The impact of the missing factor must be sufficiently large, such that it prevents recovery in hours worked, despite rapid productivity recovery and despite the low capital stock.

Business cycle accounting (BCA) is another neoclassical diagnostic tool, and its application provides insight regarding this state variable. Cole and Ohanian (1999, 2002), Mulligan (2005), and Chari, Kehoe, and McGrattan (2007, 2016), use a standard one-sector neoclassical model to measure which of the decision margins in that model deviate from theory when actual data is substituted into the first order conditions of the model. For the Great Depression, the condition that equates the marginal rate of substitution between consumption and leisure to the marginal product of labor is significantly distorted. Specifically, the marginal product of labor is higher than the marginal rate of substitution throughout the decade. The deviation in this condition, which is typically called a labor wedge, grows further after 1933, and suggests a major factor that distorted the opportunities and/or the incentives to trade

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14The idea of large productivity declines during depressions was initially met with skepticism by some economists. This skepticism is based on the narrow interpretation that lower TFP implies that society lost substantial knowledge over a short period of time. More recently, however, economists are interpreting aggregate productivity changes from alternative perspectives. Section 7 discusses this in detail.
labor services.

Ohanian (2009) identified economic policies that significantly distorted the opportunities to trade labor services by depressing labor market competition and by preventing wages from adjusting. Simon (2001) analyzed "situation wanted" advertisements from the late 1920s and the early 1930s. These situation wanted advertisements are analogous to help wanted advertisements, but from the supply side of the labor market. In these ads, workers would describe their experience and qualifications, and the wage that they were seeking. Simon shows that the supply price of labor - the desired wage posted in the situation wanted ads - was much lower than the wages that were actually paid in the 1930s. This large gap between the supply price of labor and the wage was not present in the late 1920s, however, when the supply price and actual wages paid were very similar. This evidence suggests the wages were above their market-clearing level, which in turn created an excess supply of labor.

Table 9 provides further evidence of a significantly distorted labor market. The table presents wages from manufacturing and from the farm sector. These data are measured relative to trend, which is the average growth rate of productivity in these sectors (see Cole and Ohanian (1999)). These data show that wages in manufacturing are well above trend, which suggests that they are also above their market clearing level. In contrast, real wages in the farm sector are well below trend.

Given this backdrop, a new neoclassical literature on the Depression has emerged that studies how government policy changes distorted labor markets. Ohanian (2009) studied the downturn phase of the U.S. Great Depression, and Cole and Ohanian (2004) studied the delayed recovery from the Depression. Both papers use neoclassical frameworks that build on the facts described above. Given the large differences in hours worked and wages in the manufacturing and agricultural sectors, these models begin by modifying the standard one-sector growth model to incorporate multiple sectors, and then build in government policies.

5.3 A Neoclassical Model with Wage Fixing and Work-Sharing Policies

There were large shifts in government policies throughout the 1930s that distorted labor and product markets by significantly restricting competition in industrial labor and product markets, but not in agricultural markets. Ohanian (2009) describes how these policies began in November, 1929, following the October stock market decline. President Herbert Hoover met with the leaders of the largest industrial firms, including General Motors, Ford, General Electric, U.S. Steel, and Dupont. Hoover lobbied these firms to either raise wages, or at a minimum, to keep wages at their current levels. He also asked industry to share work among employees, rather than follow the typical practice of laying off workers and keeping retained workers on a full-time shift.

In return for maintaining nominal wages and sharing work, organized labor pledged to maintain industrial peace by not striking or engaging in any efforts that would disrupt production. The Hoover bargain was perceived by firms to be in their interest. Specifically, it is widely acknowledged that the major manufacturing firms had substantial market power at this time, with considerable industry rents. Kovavich and Shapiro (2000) note that this period represents the zenith of collusion and cartels among major industry, and capital’s
share of income was at an all time high. Industry agreed to keep wages fixed, and Ford Motor in fact raised wages following the meeting with Hoover. However, as the price level declined, and as productivity declined, these fixed nominal industrial wages led to rising real wages and rising unit labor costs. Ohanian (2010) documents that industry asked Hoover several times for permission to reduce nominal wages, but Hoover declined these requests. Nominal wages among the biggest employers did not begin to fall until late 1931, after hours worked in industry had decline by almost 50 percent.

Ohanian (2009) develops a neoclassical model with a policy of nominal wage fixing and work-sharing that affected the industrial sector. This requires a model with multiple sectors, and also requires a distinction between hours per worker and employment in order to model work-sharing.

There is a representative family, and family members work in many industries. The population grows at rate $n$. Preferences over consumption and leisure, and the disutility of joining the workforce, are given by:

$$\max_{t=0}^{\infty} \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) + e_{at} \ln(1 - h_{at}) + e_{mt} \ln(1 - h_{mt}) - v(e_{at} + e_{mt}) \right\} (1 + n)^t. \quad (44)$$

Preferences are scaled by the population, which grows at rate $n$. Consumption is denoted as $c, e_a$ denotes the number of workers in the agricultural sector, $e_m$ denotes the number of workers in the manufacturing sector, $h_a$ and $h_m$ denote the length of the workweek in agriculture, and manufacturing, respectively. The function $v(e_a + e_m)$ is increasing and weakly convex, and specifies the utility cost of sending different household members to work in the market. Rank-ordering family members by their position in the distribution of this utility cost, and assuming that these costs rise linearly across family members, yields:

$$-v(e_{at} + e_{mt}) = -\int_{t=0}^{e_t} (\xi_0 + 2\xi_1 x)dx = \xi_0 e_t + \xi_1 e_t^2. \quad (45)$$

Note that there will be an optimal number of family members working, as well as an optimal number of hours per worker.

There are two production sectors, agriculture and manufacturing, and there is a continuum of industries within each sector. Industry output is given by:

$$y_i = h_t e_s(i)^\gamma k_s(i)^{1-\gamma}, \quad (46)$$

in which the length of the workweek is given by $h$, employment is given by $e$, and capital is given by $k$. Kydland and Prescott (1988), Cole and Ohanian (2002), Hayashi and Prescott (2002), Osuna and Rios-Rull (2003), and McGrattan and Ohanian (2010) use similar production technologies to study problems that require differentiating between employment and hours per worker.

The industry-level outputs are aggregated to produce sectoral output:

$$Y_s = \left( \int_0^1 y_s(i)^\theta di \right)^{\frac{1}{\theta}}. \quad (47)$$
Final output, which is divided between consumption and investment, is a CES aggregate over the two sectoral outputs:

\[ Y = [\alpha Y_m^{\phi} + (1 - \alpha) Y_a^{\phi}]^{\frac{1}{\phi}} \]  

(48)

The production of final goods is competitive, and the maximization problem is given by:

\[
\max \left\{ Y - \int p_m y_m(i) di - \int p_a y_a(i) di \right\}
\]

(49)

subject to:

\[ Y = \alpha \left( \int_0^1 y_m(i) \delta di \right)^{\phi} + (1 - \alpha) \left( \int_0^1 y_a(i) \delta di \right)^{\phi} \]

(50)

\[ \frac{1}{\phi}. \]

(52)

The solution to the final good producer’s profit maximization problem is standard, and is characterized by equating the marginal product of each intermediate input to the input price.

The parameter values for the household discount factor, the depreciation rate, and the capital and labor production share parameters are standard, with \( \beta = .95 \), \( \delta = .06 \), and \( \gamma = .67 \). The values for the three parameters that govern the disutility of hours per worker (the length of the workweek), and the utility cost of employment, are jointly set to target an average employment to population ratio of 0.7, (ii) the average workweek length at that time, which was about 45 hours per week, and (iii) that employment change accounts for about 80 percent of cyclical fluctuations in hours worked.

Ohanian (2009) discusses the fraction of the economy affected by the Hoover program, and sets the production share parameter \( \phi \) so that about 40 percent of employment was produced in industries impacted by this program. The parameter \( \phi \) governs the substitution elasticity between agriculture and manufacturing. This elasticity is set to 1/2, which is consistent with the fact that both the manufacturing share of value added and its relative price have declined over time.

To analyze the impact of the Hoover nominal wage-fixing and work-sharing policy, the observed real manufacturing wage sequence is exogenously fed into the model. This sequence of wages is interpreted as the result of Hoover’s fixed nominal wage program in conjunction with exogenous deflation. Note that the analysis is simplified considerably by abstracting from an explicit role of money in the model, such as a cash-in-advance constraint. It is unlikely that the inclusion of explicit monetary exchange in the model would change the results in any significant way, provided that a more complicated model with monetary exchange generated the same real wage path for manufacturing.

We now discuss modeling the workweek for analyzing the Hoover program. First, recall that almost all of the cyclical change in labor input prior to the Depression was due to
employment, rather than changes in hours per worker. However, about 40 percent of the decline in labor input between 1929 and 1931 was due to a shorter workweek. This suggests that the large decline in the workweek length was due to the Hoover work-sharing policy, rather than reflecting an optimizing choice.

The Hoover workweek is also exogenously fed into the model. The evidence that indicates that the workweek was not optimally chosen suggests that the Hoover work-sharing policy was inefficient. In this model, the inefficiency of forced work-sharing results in lower productivity, since reducing the length of the workweek operates just like a negative productivity shock. To see this, note that the Cobb-Douglas composite of employment and the capital stock in the production function is scaled by the length of the workweek.

The analysis is conducted between 1929:4 and 1931:4. The wage-fixing and work-sharing policies significantly depress economic activity by raising the cost of labor, which reflects both a rising real wage and declining labor productivity. The inflexible manufacturing wage means that the manufacturing labor market does not clear, and that the amount of labor hired is solely determined by labor demand. Table 10 shows the perfect foresight model predictions and data\textsuperscript{15}. The model generates about a 16 percent output decline, which accounts for over 60 percent of the actual decline\textsuperscript{16}. The model also is consistent with the fact that there is a much larger decline in manufacturing than in agriculture. Manufacturing hours fall by about 30 percent in the model and by about 44 percent in the data, and agricultural hours fall by about 12 percent in the model and by about 4 percent in the data.

The agricultural sector declines much less because it is not subject to the Hoover wage and work-sharing policies. However, the agricultural sector declines because of the general equilibrium effects of the Hoover policy. This reflects the fact that manufacturing output is a complement to agricultural output in final goods production. Thus, depressed manufacturing output depresses the agricultural wage, which in turn depresses agricultural hours.

Note that the model is consistent with Simon’s (2001) finding of excess labor supply in manufacturing, and that job seekers in manufacturing were willing to work for much less that the manufacturing wage. The model also provides a theory for why deflation was particularly depressing in the 1930s compared to the early 1920s, when a very similar deflation coincided with a much milder downturn.

While this model was tailored to study the U.S. Great Depression, it can be used more broadly to study nominal wage maintenance policies and/or work-sharing policies.

5.4 A Neoclassical Model with Cartels and Insider-Outsider Unions

The model economy with nominal wage-fixing, deflation, and work sharing accounts for a considerable fraction of the early years of the Depression. After 1933, however, deflation ended. Moreover, productivity grew rapidly, and real interest rates declined. These factors should have promoted a strong recovery, but the economy remained far below trend for the balance of the decade. The failure of the economy to return to trend is puzzling from

\textsuperscript{15}The annual NIPA data are linearly interpolated to a quarterly frequency.

\textsuperscript{16}The deterministic path solution is the reason for the immediate increase in economic activity. This reflects the fact that producers see higher future labor costs, and thus produce before these costs rise. Future research should assess the impact of these policies in a stochastic environment.
a neoclassical perspective, given productivity growth, and it is puzzling from a Keynesian perspective, given the end of deflation and banking crises, and given much lower real interest rates.

The empirical key to understanding the post-1933 depression is a growing labor wedge, as the marginal product of labor was far above the marginal rate of substitution between consumption and leisure. Cole and Ohanian (2004) develop a theory of the labor wedge that is based on changes in government competition and labor market policies. One policy was the 1933 National Industrial Recovery Act, which allowed a number of non-agricultural industries to explicitly cartelize by limiting production and raising prices. The government typically approved these cartels provided that industry raised wages of their workers. Another policy was the 1935 National Labor Relations Act (NLRA), which provided for unionization and collective bargaining. The use of the "sit-down" strike under the NLRA, in which striking workers forcibly prevented production by taking over factories, gave workers considerable bargaining power. Cole and Ohanian describe how both of these policies created an insider-outsider friction, in which insiders received higher wages than workers in sectors that were not covered by these policies.

Cole and Ohanian present industrial wage and relative price data from individual industries covered by these policies. Industry relative prices and wages jumped around the time that the industry codes were passed, and continued to rise after that. Table 9 shows that real wages rise and that ultimately are about 17 percent above trend by the late 1930s.

Cole and Ohanian (2004) develop a multisector growth model in which the industries in the manufacturing sectors are able to cartelize provided that they reach a wage agreement with their workers. They begin with a simple neoclassical environment, and then add in cartelization policies and a dynamic, insider-outsider model of a union, in which incumbent workers (insiders) choose the size of the insider group, and bargain over the wage. The objective of the insiders is to maximize the per-worker expected, present discounted value of the union wage premium.

While this model was developed to capture specific features of U.S. policy, it easily can be modified to analyze a variety of dynamic bargaining games in which a firm and a union repeatedly negotiate over wages, and in which the insiders choose their size by maximizing the expected, discounted payoff to union membership. The choice of the size of the union is central in any insider-outsider environment, but is typically missing from earlier insider-outsider models.

We begin with a neoclassical, multisector growth model, and then build in these policies. Preferences are given by:

$$\max \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \mu \ln(1 - n_t)\}. \quad (53)$$

Consumption is denoted as $c$, and the size of the household is normalized to 1. The model is simplified by assuming that work is full-time. The term $1 - n$ is the number of household members who are engaged in non-market activities (leisure). The household faces a present value budget constraint:
\[
\sum_{t=0}^{\infty} Q_t \left[ w_f n_f + w_m n_m + \Pi_0 - c_t - \sum_s r_s k_s - x_s \right] \geq 0,
\]

in which \(Q_t\) is the date-t price of output, \(w_f\) is the competitive (non-cartel) wage, \(n_f\) is the number of workers in the competitive sector, \(w_m\) is the cartel wage, \(n_m\) is the number of workers in the cartel sector, \(\Pi_0\) are date zero profits, \(r_s\) is the rental price of sector \(s\) capital, which in turn is denoted as \(k_s\), and \(x_s\) is investment in sector \(s\) capital. Time allocated to market activities is given by:

\[
n_t = n_f t + n_m t + n_u t.
\]

This indicates that total non-market time, \(n\), is the sum of household time spent working in the agricultural (non-cartel) sector, \(n_f\), the time spent working in the manufacturing (cartel) sector, \(n_m\), and the time spent searching for a job in the manufacturing sector, \(n_u\).

There is also a law of motion for the number of workers in the cartel sector. This transition equation is given by:

\[
n_{mt} \leq \pi n_{mt-1} + v_{t-1} n_{ut-1}
\]

The transition equation for the number of workers in the manufacturing sector indicates that the number of these manufacturing workers at date \(t\) consists of two components. One is the number who worked last period, less exogenous worker attrition, in which \((1 - \pi)\) is the probability of a manufacturing worker exogenously losing their manufacturing job. The other component is \(v_{t-1} n_{ut-1}\), and this is the number of new workers hired into manufacturing jobs. This is equal to the number of family members who searched for a manufacturing job in the previous period, \(n_{ut-1}\), multiplied by the probability of finding a manufacturing job, which is denoted as \(v_{t-1}\).

Note that job search is required for an outsider to be newly hired into manufacturing. This search process captures competition by the outsiders in the model for the scarce insider jobs. The insider attrition probability, \(1 - \pi\) captures features that generate job loss, but that are not explicitly modelled, such as retirement, disability, and relocation. Note that if \(\pi = 1\), then there is no insider attrition, and there will be no hiring (or job loss) in the cartel sector in the steady state of the model.

The law of motion for industry capital stocks is standard, and is given by:

\[
k_{st+1} = (1 - \delta) k_{st} + x_{st}
\]

Industry output in sector \(i\) is given by:

\[
y(i)_t = z_t k^\gamma_t (i) n_t^{1-\gamma} (i)
\]

Sector output is given by:

\[
Y_s = \left[ \int_{\varphi_{s-1}}^{\varphi_s} y(i)^\theta (i) d(i) \right]^{1/\theta}, s = \{f, m\}
\]
Final output is given as a CES aggregate of the two sectoral outputs:

\[ Y = \left[ \alpha Y_f^\phi + (1 - \alpha)Y_m^\phi \right]^{\frac{1}{\phi}} \]  

(60)

Producers in the cartel sector have a profit maximization problem that features their market power, and which depends on the elasticity parameters \( \phi \) and \( \theta \). Using the fact that industry price is given by \( p = Y^{1-\phi}Y_{m}^{\phi-\theta} \), the industry profit function is given by:

\[ \Pi = \max_{n,k} \left\{ Y^{1-\phi}Y_{m}^{\phi-\theta} \left( \left( z_t n_t \right)^{1-\gamma} k_t^\gamma \right)^\theta - wn - rk \right\} \]  

(61)

In the insider-outsider union model, the objective for an incumbent worker (insider) is to maximize the expected present discounted value of industry wage premia. The value of being an insider, in which there are currently \( n \) insiders, is given by:

\[ V_t(n) = \max \{ \min \left[ \frac{\bar{n}}{n} \right] \left( \left[ \bar{w}_t - w_f \right] + \pi \left( \frac{Q_{t+1}}{Q_t} \right) V_{t+1}(\pi \bar{n}) \right) \} \]  

(62)

The insiders propose to the firm to hire \( \bar{n} \) number of workers at the wage rate \( \bar{w}_t \). If the offer is accepted, the current period payoff to each insider is the wage premium, which is the cartel wage less the competitive wage: \( (\bar{w}_t - w_f) \). The insider’s continuation value is the expected discounted value of being an insider next period, which is \( \pi \left( \frac{Q_{t+1}}{Q_t} \right) V_{t+1}(\pi \bar{n}) \).

Note that the number of insiders at the start of period \( t + 1 \) is given by \( \pi \bar{n} \). Note that the attrition probability, \( \pi \), affects the continuation value of union membership in two different ways. First, the probability that any individual insider at date \( t \) will remain in the cartel at date \( t + 1 \) is \( \pi \), which scales the the date \( t + 1 \) value function. Second, the total number of date \( t \) insiders who will remain in the cartel in date \( t + 1 \) is \( \pi \bar{n} \).

The insiders bargain with the firm at the start of each period. If a wage agreement is reached, then the firm hires \( \bar{n} \) number of workers at wage \( \bar{w} \). Note that the union’s offer is efficient in the sense that given the wage offer, the number of workers hired, \( \bar{n} \), is consistent with the firm’s labor demand schedule. The bargaining protocol is that the union makes a take-it-or-leave-it offer to the firm.

In equilibrium, the union makes an offer that the firm weakly prefers to its outside option of declining the offer. The firm’s outside option is given as follows. If the offer is declined, then the firm can hire labor at the competitive wage, \( w_f \). With probability \( \omega \) the firm will be able to continue to act as a monopolist. With probability \( 1 - \omega \), the government will discover that the firm did not bargain in good faith with the union, and the government will force the firm to behave competitively and thus the firm earns no monopoly profits.

This feature of the model empirically captures the fact that some firms did fail to reach wage agreements, or violated wage agreements, and that government did enforce the wage bargaining provisions of the policy. The firm’s outside option therefore is the expected level of monopoly profits earned by declining the insider’s offer, and the firm will only accept the insider’s offer of \((\bar{n}, \bar{w})\) if it delivers at least that level of profit. It is therefore optimal for the union to make an offer that does provide the firm with its outside option.

A key parameter in this model is the share of employment in the cartelized sector. While the cartel policy was intended to cover about 80 percent of the non-farm economy, there is
debate regarding how much of the economy was effectively cartelized. Therefore, the model conservatively specifies that only manufacturing and mining were cartelized, which is about 1/3 of the economy. Another key parameter is $\omega$, which governs the probability that the government will identify a firm that breaks their wage agreement. This value was chosen so that the steady state cartel wage premium is about 20 percent above trend. This implies that $\omega$ is around 0.10. The attrition parameter, $\pi$, is set to 0.95, which yields an average job tenure in the cartel of 20 years.

Other parameters include the substitution elasticity across industries and across sectors. For these parameters, the industry substitution elasticity is picked so that the industry markup would be 10 percent in the absence of wage bargaining. The sectoral substitution elasticity, which refers to the substitution possibility between manufacturing and the farm sector, is picked to be 1/2. Other parameter values, including the household discount factor, the household leisure parameter, the income shares of capital and labor, and depreciation rates, are standard, and are described in Cole and Ohanian (2004).

The quantitative analysis begins in 1934. To generate model variables, the 1933 capital stocks from the manufacturing and farm sectors from this are specified, and the sequence of TFP from 1934 - 1939 is fed into the model. The model variables then transit to their steady state values. For comparative purposes, we show the results from the cartel model to those from the perfectly competitive version of this model. Table 11, which is taken from Cole and Ohanian (2004), shows the response of the competitive version of this model. Note that the rapid return of productivity to trend fosters a rapid recovery under competition, with hours worked rising above trend, to rebuild the capital stock to its steady state level. Moreover, the wage is well below trend in 1933, and then recovers quickly after that, as both productivity and the capital stocks rise.

Table 12 shows the transition of the cartel model. This transition stands in sharp contrast to the transition in the competitive economy from Table 11. The cartel economy transits to a steady state that is well below the competitive economy. Despite rising productivity, the cartel economy remains depressed through the 1930s, as cartel policies create rents that raise wage rates far above trend, despite the fact that both consumption and time allocated to market activities are below trend. These results indicate that the cartel policy accounts for about 60 percent of the post-1933 depression in output, consumption, and hours worked.

5.5 Neoclassical Models of Taxes and Depressions

This subsection describes how tax rate changes contributed to the U.S. Great Depression and also for more recent episodes of depressed economic activity.

Tax rates rose in the United States during the Great Depression. McGrattan (2012) studies how changes in tax rates on dividends and corporate profits affected economic activity after 1933. Specifically, a new tax rate was applied to undistributed corporate profits in 1936. The goal of this new tax was to increase corporate payments to shareholders, which in turn was expected to stimulate spending.

McGrattan analyzes a representative household economy with log preferences over consumption and leisure, and with a standard constant returns to scale Cobb-Douglas production function with capital and labor inputs. She considers two formulation for taxes. In the
traditional formulation, tax rates are applied to labor income \((\tau_h)\) and to capital income net of depreciation \((\tau_k)\). Tax revenue is the sum of labor income tax revenue and capital income tax revenue:

\[ \tau_h w h + \tau_k (r - \delta) k \] (63)

The alternative formulation includes a finer decomposition of taxes across revenue sources, and distinguishes between business and non-business capital. Tax revenue in this alternative formulation is given by:

\[ \tau_h w h + \tau_p (r - \tau_k - \delta) k_b + \tau_c c + \tau_k k_b + \tau_u (k'_b - k_b) + \tau_d \left\{ (r k - x_b) - \tau_p (r - \tau_k - \delta) k_b - \tau_k k_b - \tau_u (k'_b - k_b) \right\} \] (64)

In 64, \(\tau_p\) is the tax rate on profits, \(\tau_k\) is now the tax rate on business property, \(\tau_c\) is the consumption tax rate, \(\tau_u\) is the tax rate on undistributed profits, \(\tau_d\) is the dividend tax rate, and primed variables refer to period \(t + 1\) values.

The intertemporal first order condition that governs efficient investment shows how changes in expected taxation affects investment:

\[ \frac{(1 + \tau_{ut})(1 - \tau_{dt})}{(1 + \tau_{ct}) c_t} = \beta \mathbb{E}_t \left[ \frac{(1 - \tau_{dt+1})}{(1 + \tau_{ct+1}) c_{t+1}} \left\{ (1 - \tau_{pt+1})(r_{t+1} - \tau_{kt+1} - \delta) + 1 + \tau_{ut+1} \right\} \right] \] (65)

Note that dividend taxes and consumption taxes in 65 do not distort investment incentives at the margin in the deterministic version of this model when these tax rates are constant over time. However, expected changes in tax rates will affect investment decisions. An expected increase in these tax rates reduce the expected returns to investment, and leads firms to increase current distributions. Tax rates rose considerably in the mid-1930s, with the dividend tax rate rising from about 14 percent to about 25 percent, the corporate profit tax rate rising from about 14 percent to about 19 percent, and the newly implemented undistributed tax rate of five percent. McGrattan shows that plausible expectations of these tax rate changes can help account for the fact that business investment remained 50 percent or more below trend after 1933.

McGrattan’s analysis of the U.S. Great Depression focused on changes in capital income tax rates. Prescott (2004), and Ohanian, Raffo, and Rogerson (2008) analyze how long-run changes in labor income tax rates have affected hours worked more recently. Ohanian et al (2008) document that hours worked per adult in the OECD vary enormously over time and across countries. Hours worked in many Northern and Western European countries declined by about 1/3 between the 1950s and 2000, including a nearly 40 percent decline in Germany.

Ohanian et al use a standard neoclassical growth model with log preferences over consumption, log preferences over leisure, a flat rate labor income tax, and a flat rate consumption tax rate. The economy’s technology is a constant returns to scale Cobb-Douglas production function that uses capital and labor, which is given by \(Y_t = A_t K_t^\alpha H_t^{1-\theta}\). Preferences for the representative family are given by:

\[ \max \sum \beta^t \{ \alpha \ln(c_t - \bar{c} + \lambda g_t) + (1 - \alpha) \ln(\bar{h} - h_t) \}. \] (66)
Households value private consumption, \(c\), and public consumption, \(g\). The term \(\bar{c}\) is a subsistence consumption term to account for possible non-homotheticities in preferences that may affect trend changes in hours worked. The parameter \(\lambda, 0 < \lambda \leq 1\), governs the relative value that households place on public spending. The specification that government consumption (scaled by the parameter \(\lambda\)) is a perfect substitute for private consumption follows from the fact that much government spending (net of military spending) is on close substitutes for private spending, such as health care.

The first order condition governing time allocation in this economy is standard, and equates the marginal rate of substitution between consumption and leisure to the wage rate, adjusted for consumption and labor income taxes. This first order condition is presented below. Note that the marginal product of labor, \((1 - \theta) \frac{Y_t}{H_t}\) is substituted into the equation for the wage rate in 67

\[
\frac{(1 - \alpha)}{h - h_t} = \frac{(1 - \tau_h t)}{(1 + \tau_c)} \frac{\alpha}{(c_t + \lambda g_t)} (1 - \theta) \frac{Y_t}{H_t}.
\]

In the first order condition, \(\tau_h\) is the labor income tax rate, and \(\tau_c\) is the consumption tax rate. Ohanian et al feed McDaniel’s (2011) panel data construction of consumption and income tax rates into this first order condition, along with actual labor productivity and consumption data. They choose the value of \(\alpha\) by country so that model hours in the first year of the dataset are equal to actual hours for each country. They set \(\lambda = 1\), and labor’s share of income is set to 0.67. The subsistence consumption term is set to five percent of U.S. consumption in 1956, which represents a small departure from the standard model of homothetic preferences. Ohanian et al describe the sensitivity of results to alternative values for these parameters.

With these parameter values and data, Ohanian et al use this equation to construct a predicted measure of hours worked from the model economy, and compare it to actual hours worked by country and over time. Figure 25 shows actual hours worked and predicted hours worked from the model for 21 OECD countries\(^{17}\). Panel (a) of the graph shows results for countries which experienced at least a 25 percent decline in hours worked per capita. Panel (b) shows results for countries which experienced a decline in hours per capita that range between 10 percent to 25 percent. Panel (c) shows results for countries that experienced a decline in hours per capita of less than 10 percent, or alternatively experienced higher hours.

The figures show that the model economy accounts for much of the secular decline in hours worked, particularly for the countries which experienced the largest hours declines. Ohanian et al also report that the contribution of tax rate changes to changes in hours worked is not sensitive to other labor market factors that may have affected hours, such as changes in employment protection policies, changes in union density, and changes in unemployment benefits.

These findings indicate that the observed increases in labor and consumption tax rates can account for the large observed declines in hours worked per adult across these countries.

\(^{17}\)Ohanian et al (2008) describe the data sources and data construction in detail. The Group 1 countries are Austria, Belgium, Denmark, France, Finland, Germany, Italy, and Ireland. The Group 2 countries are Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the U.K. The Group 3 countries are Australia, Canada, Greece, New Zealand, and the U.S.
These neoclassical findings regarding the impact of tax rates on hours worked stand in contrast to other explanations of the decline in European hours. Other explanations include a preference shift for more leisure, or a preference shift in conjunction with policies that restrict work, and that may have been chosen in order for society to coordinate on a low-work equilibrium (see Blanchard (2004) and Alesina et al (2005)).

5.6 Summary

Depressions, which are protracted periods of substantial economic decline relative to trend, have been difficult to understand and are often presumed to extend beyond the scope of neoclassical economics. The models developed here show that government policies that depress competition can account for a considerable amount of the Great Depression, and can also account for much of the failure of economic activity to return to trend. More broadly, these models of the U.S. Great Depression successfully confront the frequently cited view of Modigliani (1977) that neoclassical models cannot plausibly account for the behavior of labor markets during Depressions.

Modigliani interpreted the Great Depression as the failure of the market economy to right itself. This view, and associated Keynesian views of the Depression, are based on the idea that business organizations did not expand investment in the 1930s, which in turn kept employment low. The studies discussed here turn that interpretation on its head. Specifically, these new neoclassical studies indicate that the depth and persistence of the Depression was the consequence of government policies that depressed the steady state allocation of time to market work. A lower steady state level of market hours reduced the return to capital, which in turn depressed capital accumulation.

Neoclassical models can also account for more recent periods of depressed economic activity. This includes not only the secular decline in market hours worked in much of Northern and Western Europe through higher tax rates, but also the Finish Depression of the early 1990s that reflects the trade impact of the breakup of the U.S.S.R. (Gordnichenk et al (2012)), and tax changes and productivity changes (Conesa et al (2007)). Other studies of recent Depressions include the Korean Crisis of 1998 (Ohtsu (2008)), and several case studies in Kehoe and Prescott (2007).

The Depression methodology presented in this section has also been used to study the flip side of Depression, which are Growth Miracles. This includes studies of Ireland’s Growth Miracle (see Ahearne et al (2016), who analyze a standard growth model with TFP, and Klein and Ventura, who study a small open economy model with taxes, labor wedges, and TFP), and Lu (2012) who analyzes the development of some East Asian countries in a neoclassical framework.

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6 Neoclassical Modeling of Large Fiscal Shocks: The U.S. World War II Economy

Wartime economies are interesting and important macroeconomic episodes because they feature very large, exogenous changes in government policies, particular fiscal policies, as well as large changes in macroeconomic activity. The World War II economy in the United States represents perhaps the largest fiscal policy shift of any advanced economy. This includes a nearly 400 percent increased in federal government spending, large increases in income tax rates, and a large increase in the number of men drafted into military service. Moreover, there was a very large resource reallocation from private use to military use that occurred in a very short period of time.

This striking period of policy changes provides information on how large aggregate and sectoral disruptions quantitatively affect a market economy, which provides a powerful test of neoclassical theory. These episodes are also informative about what a number of economists call the government spending multiplier, which refers to the change in output as a consequence of a change in government spending. This research area has received considerable attention since the Great Recession, when the U.S. and other countries increased government spending to expand economic activity (see Barro and Redlick (2011), Mountford and Uhlig (2009), Ramey (2011), and Taylor (2011)).

Neoclassical analysis of fiscal policies and wars has become an active research area. These studies analyze a range of issues, including the welfare costs of different wartime fiscal policies (Ohanian (1997)), the impact of the draft on economic activity (Siu (2008)), the behavior of labor productivity and investment (Braun and McGrattan (1993)), and the extent that a neoclassical model can account for aggregate time series, particularly the impact of wars on the incentives to work (Mulligan (2005) and McGrattan and Ohanian (2010)).

This section develops a neoclassical model of the World War II U.S. economy to study how well a neoclassical model can fit the wartime U.S. data. The model easily can be applied to other episodes with changes in government spending, transfers, and tax rates. The model is from McGrattan and Ohanian (2010), which in turn draws on Braun and McGrattan (1993), Ohanian (1997), and Siu (2008).

There is a representative family, with two types of family members, civilians and draftees. The size of the family is denoted as $N$. Both types of family members have identical preferences. At date $t$, $a_t$ is the number of family members in the military, and $(1 - a_t)$ is the number who are civilians. The family optimally chooses consumption of both types, which is denoted as $c_{ct}$ for civilians, and $c_{dt}$, for draftees. The family also optimally chooses investment in physical capital, $i_{pt}$, civilian labor input, $l_{ct}$, and the accumulation of government bonds, $b_{t+1}$. The inclusion of public debt follows from the fact that there was considerable debt issue during the war. The labor input of draftees is not a choice variable for the family, but rather is set exogenously by the government, and is denoted by $l_d$.

The maximization problem for the representative family is:

---

$\max E_0 \sum_{t=0}^{\infty} \{(1 - a_t)U(c_{ct}, l_{ct}) + a_t U(c_{dt}, l_d)\} N_t \quad (68)$

Maximization is subject to the following constraints:

$$E_t = (1 - \tau_{kt})(r_p - \delta)k_{pt} + (1 - \tau_{lt})w_t(1 - a_t)l_{ct} + R_t b_t + (1 - \tau_{lt})w_t a_t l_d + T_t \quad (69)$$

$$E_t = (1 - a_t)c_{ct} + a_t c_{dt} + i_{pt} + b_{t+1} \quad (70)$$

$$k_{pt+1} = [(1 - \delta)k_{pt} + i_{pt}]/(1 + \gamma_n) \quad (71)$$

$$N_t = (1 + \gamma_n)^t \quad (72)$$

$$c_{ct}, c_{dt}, i_p \geq 0 \quad (73)$$

Note that $k_p$ is the beginning-of-period capital stock, $r_p$ is the rental price of capital, $w$ is the wage rate, $\tau_k$ and $\tau_l$ are flat rate tax rates on capital income and labor income, respectively, $Rb$ is the value of matured government debt, and $T$ is government transfers. The depreciation rate is $\delta$. The population grows at the constant rate $\gamma_n$.

The production technology is given by:

$$Y_t = F(K_{pt}, K_{gt}, Z_t L_t). \quad (74)$$

The production inputs include private capital, labor, and public capital, $K_g$. Labor-augmenting productivity is denoted as $Z$, and is given by:

$$Z_t = z_t(1 + \gamma_z)^t \quad (75)$$

Note that $z_t$ is a transient productivity term and $\gamma_z$ is the long-run growth rate of technology.

Government purchases consist of 3 components. This is a richer specification of government spending than is typically modeled in fiscal policy studies. Government consumption, $C_g$ is the first component, and this is the standard approach to modeling government purchases. It is common to assume that these wartime purchases of goods do not affect marginal utility or private production possibilities. The second component is government investment, $I_g$ which enhances production possibilities by expanding the capital stock that can be used to produce output. This is typically not modeled in the fiscal policy literature, but is modeled here because of the very large government-funded investments in plant and equipment that occurred in World War II. The government made large investments in the aircraft, automotive, and aluminum industries that raised the manufacturing capital stock by 30 percent between 1940 and 1945. The third component of government purchases is wage payments to military personnel. Government spending is therefore given by:

$$G_t = C_{gt} + + I_{gt} + N_t w_t a_t l \quad (76)$$

The evolution of the stock of government capital, which is assumed to have the same depreciation rate as physical capital, is given by:
The period government budget constraint is given by:

$$K_{gt+1} = (1 - \delta)K_{gt} + I_{gt}$$  \hfill (77)

The functional form for preferences is given by:

$$\ln(c) + \frac{\psi}{\xi}(1 - l)^\xi$$  \hfill (79)

This specification yields a compensated labor supply elasticity of $\frac{1 - l}{(1 - l)^{1+\xi}}$. McGrattan and Ohanian choose $\xi = 0$ (log preferences) as the benchmark specification. The parameter $\psi$ governs the steady state allocation of time for the household, and is chosen so that model steady state hours is equal to the average time devoted to market work between 1946 and 1960. For military time allocation, they choose $l$ such that it matches 50 hours per week, which is the average hours for soldiers in basic training (see Siu (2008)). Population growth is 1.5 percent per year, and the growth-rate of technological progress is 2 percent per year.

Government capital and private capital are modeled as perfect substitutes. This reflects the fact that much of government investment at this time was in the area of manufacturing plant and equipment:

$$Y_t = F(K_{pt}, K_{gt}, Z_tL_t) = (K_{pt} + K_{gt})^\theta(Z_tL_t)^{1-\theta}$$  \hfill (80)

It is straightforward, however, to modify the aggregator between government and private capital to accommodate government capital that is not a perfect substitute for private capital.

There are six exogenous variables in the model: conscription (the draft) ($a_t$), the tax rate on capital income ($\tau_{kt}$), the tax rate on labor income ($\tau_{lt}$), government consumption ($C_{gt}$), government investment ($I_{gt}$), and productivity ($z_t$). The evolution of the six exogenous variables is governed by a state vector, $S_t$, which specifies a particular set of values for these exogenous variables. For 1939-1946, these exogenous variables are equal to their data counterparts. The model is solved under different assumptions regarding household expectations about the post-1946 evolution of the exogenous variables. The discussion here focuses on the perfect foresight solution to the model that begins in 1939, and McGrattan and Ohanian discuss the other cases in detail.

While the model described here is based on the World War II U.S. economy, it can be tailored to study other episodes, as it includes a number of features that are relevant for wartime economies, including changes in tax rates on factor incomes, changes in conscripted
labor, changes in productivity, government debt issue to help pay for the war, government payments to military personnel, and government investment.

Figure 26 shows the model’s exogenous variables. Government consumption, which includes state and local spending, as well as federal spending, rises from about 14 percent of steady state output in 1940 to 50 percent of steady state output by 1944. Government investment rises from about 4 percent of steady state output in 1940 to about 9 percent by 1942. The tax rates on labor and capital income, which are average marginal tax rates taken from Joines (1981), also rise considerably, with the labor income tax rate rise from about 8 percent to about 20 percent, and with the capital income tax rates rising from about 43 percent to about 63 percent. The draft reduces potential labor supply significantly, as almost 12 percent of the working age population is in the military by 1944.

There is a considerable increase in TFP, and there are a number of good reasons why this change actually reflects higher efficiency. This includes the development of federally-funded scientific teams, the development of management science and operations research practices, and a number of technological advances during the 1940s including innovations directly or indirectly fostered by federal R & D expenditures. These include the development of modern airframes, radar, microwave technology, fertilizer, oxygen steel, synthetic rubber, nylon, sulfa drugs and chemotherapy, insecticides, and Teflon and related industrial coatings. Moreover, Herman (2012) describes how business leaders worked together in World War II to mobilize resources and to raise military output through significantly higher efficiency.

These size and diversity of these changes will affect economic activity in a variety of ways. Higher TFP will promote high labor input and output, as will public investment. In contrast, since public investment substitutes for private investment, higher public investment in plant and equipment will tend to reduce private investment. Moreover, rising tax rates and conscription of labor will tend to reduce the incentive to work.

Figure 27 shows real GNP, real consumption, and real investment, all measured as a percent of trend output. The model output series is very close to actual output, as both increase by more than 50 percent over the course of the war, and then decline after the war back to near trend. Model consumption is very flat during the war, and is close to actual consumption. Model investment has a very similar pattern as actual investment. The model investment is somewhat higher than actual investment through 1942, which reflects the perfect foresight solution. Specifically, investment rises considerably in order to build the capital stock by the time that government consumption is high. By 1944, the high level of government investment in plant and equipment, coupled with the enormous resource drain of the war, leads to investment declining significantly. Figure 28 show the behavior of total hours worked, and nonmilitary hours, which is the choice variable for the family. Both hours series rise significantly in the data and in the model. The nonmilitary hours in the model rises earlier than in the data, and this again partially reflects the perfect foresight assumption. Figure 29 shows the after-tax returns to private capital and labor. These are also quite similar to the data.

The dominant factor in driving these results is the enormous expansion of government consumption that occurred during the War. This resource drain of wartime government consumption creates a sizeable wealth effect within the model that leads to higher labor input and output, and this effect is much larger than that of any of the other shocks.
McGrattan and Ohanian (2010) analyzed the impact of each of the six shocks in the model on hours worked. The impact of just government consumption in the absence of any other shocks raises non-military labor input by about 27 percent on average between 1943-45. Adding productivity shocks raises this to about a 29 percent increase. Adding in the draft to these two preceding shocks results in about a 25 percent increase. Adding in the labor and capital income tax increases has a sizeable depressing effect, and results in an increase in non-military hours of about 10 percent. Overall, the negative wealth effect arising from government consumption is the dominant factor, followed by the impact of tax increases.

These results shed light on a number of issues that are analyzed in the literature on the macroeconomics of fiscal policy. One issue is regarding the government spending multiplier. A difficulty facing many studies of government spending multipliers is that they are primarily based on peacetime episodes, and episodes even with relatively large peace-time shifts in fiscal policy still involve small changes in fiscal policy compared to policy changes during wartime episodes. Moreover, many of these studies require exogenous changes in fiscal policy, and this can be problematic during peacetime. Consequently, it is challenging to draw sharp conclusions about the size of the multiplier based on peacetime policy changes.

The results from this World War II analysis indicate a multiplier that is considerably less than one. This is informative, not only because the wartime fiscal policy shock is so large, but also because the model explicitly distinguishes between different types of government spending. The analysis conducted here makes it possible to isolate the impacts of different types of spending and taxes on economic activity.

To see that the multiplier from this episode is fairly small, consider the following case in which we account for the impact of all government expenditures, but omit the negative impact of the tax increases and the draft. By omitting these latter two items, we construct the maximum possible effect of fiscal policy, even though tax increases, which depress labor supply, are certainly part of fiscal policy. In this experiment, the World War II episode shows that the multiplier would be about 0.6, reflecting a hypothetical 30 percent increase in output resulting from government purchases of goods. This multiplier is very similar to Barro and Redlick’s (2011) estimates and Mountford and Uhlig’s (2009) short-run estimates and is in the lower end of the range of estimates discussed in Ramey (2011).

The results have broader implications regarding neoclassical analyses of large shocks. They indicate that the U.S. economy responded to the enormous wartime economic dislocations, as well as the peacetime reversal of these dislocations, very much along the lines of a simple neoclassical growth model augmented with several large policy changes. These policy shifts include the massive reallocation of economic activity from peacetime to wartime production, the enormous drain of resources resulting from government purchases, the reduction of the labor endowment through the draft, higher taxes, and government-funded investment. This also includes the rapid unwinding of these unique factors after the war. While this represents just a single episode, this analysis provides a strong test of the neoclassical model in response to large fiscal policy changes.
7 Neoclassical Models of Productivity Shocks

Productivity change is an important feature of the models and that data that we have used to analyze the U.S. historical macroeconomic record in this chapter. This includes a large TFP decline in the Great Depression, a large TFP increase in World War II, and large TFP and equipment-specific productivity fluctuations in the post-Korean War U.S. economy.

There are long-standing questions about the nature and sources of these productivity changes. Much of the profession has viewed TFP declines during downturns, and particularly during depressions, with skepticism, and naturally so. But economists are now analyzing TFP deviations during short-run and longer-run episodes from alternative perspectives than the narrow interpretation that TFP declines reflect a loss of technological know-how and knowledge.

7.1 Resource Misallocation and TFP

Restuccia and Rogerson (2008) analyze the impact of resource misallocation on TFP in a competitive economy. The idea is to assess how the misallocation of production inputs across locations affects measured TFP. Their model is related to Hopenhayn and Rogerson (1993), in which there is a representative family and there are different producers, or alternatively, different production locations, each with a decreasing returns to scale technology with potentially different TFP levels, and which are indexed by i. The simplest case of production heterogeneity is the case of a single final good produced at multiple locations, $y_i$, that is produced with a single production input, labor $(h_i)$. The production relationship at location $i$ is given by:

$$y_i = z_i f(h_i)$$

(81)

In this economy, the technology $f$ is twice continuously differentiable, with $f' > 0$, $f'' < 0$. The term $z_i$ denotes exogenous productivity. Assume that $z_i$ is drawn from the set \{ $z_1, z_2, \ldots, z_I$ \}, and let $\mu(i)$ be the distribution of productivity across these locations.

The efficient allocation of labor requires equating the marginal product of labor across production locations. For the isoelastic technology, $z_i h_i^\theta, 0 < \theta < 1$, the efficient allocation of labor between any two locations depends on the differences in productivities at those locations, and the amount of curvature in the production technology:

$$\frac{h_i}{h_j} = \left( \frac{z_i}{z_j} \right)^{\frac{1}{1-\theta}}.$$  

(82)

We construct an economy-wide measure of TFP by aggregating TFP across all locations. Aggregate TFP in this economy is given by:

$$z = \sum_i z_i^{1-\theta} \mu(i)^{1-\theta}.$$  

(83)

The efficient allocation of labor at any specific location depends on the location’s productivity relative to aggregate productivity, as well as the amount of curvature in the technology, and is given by:
Note that as $\theta \to 1$, even small differences in productivity generate very large differences in the efficient allocation of production inputs across locations.

Atkeson, Khan, and Ohanian (1996) use data on differences in worker firing costs and job reallocation rates between the U.S. and Europe to argue that $\theta$ is around 0.85. Restuccia and Rogerson use this value for specifying the level of decreasing returns in their economy, and they study how misallocation of production inputs across locations affects aggregate productivity, $z$. Resource misallocation means that the marginal product of labor is not equated across production locations, which implies that $f^2$ and $\eta$ are not satisfied.

Restuccia and Rogerson (2008) analyze various government policies that tax the output of some producers, and that subsidize the output of other producers, and they calculate the aggregate productivity and welfare losses from these policies. There is a large literature that has built on Restuccia and Rogerson along many dimensions. This includes the application of misallocation to specific Depressions and Crises (see Oberfeld (2013) and Chen and Irarrazabal (2013) on the Chilean Depression of the early 1980s, and Sandleris and Wright (2014) on the Argentinian Depression of 2001), the connection between financial market imperfections and misallocation (see Moll (2014), Buera and Moll (2015), and Midrigan and Xu (2014), the connection between trade barriers and productivity during the U.S. Great Depression (see Bond et al (2013)). Other studies of misallocation focus on longer-run issues, including studies of the role of misallocation in the development experiences of China and India (Hsieh and Klenow (2009)), entry regulation and productivity (Poschke (2010)), size-dependent policies and productivity (Guner et al (2008)), imperfect information and productivity (David et al (2016)), the misallocation of managerial talent and productivity (Alder (2016)), and the magnification of misallocation on productivity in economies with production chains (Jones (2013)).

7.2 Intangible Investments and TFP

Neoclassical models with intangible capital are being developed to construct new measures of TFP. These studies focus on intangible investments that traditionally have not been counted as part of national product. Prior to 2013, the Bureau of Economic Analysis (BEA) counted only software as investment among the intangible categories. In 2013, the BEA implemented a comprehensive revision of the National Income and Product Accounts to include other business purchases that previously were counted as business expenses as investment, including research and development, artistic products, mineral exploration, and intellectual property. The shift of these purchases from an expensed item to business investment increases output. This BEA revision improves the measurement of real output, but the BEA does not currently count other intangible investments in the national accounts, such as marketing, advertising, and organization capital investments. These investment omissions indicate that output is mismeasured, which implies that productivity is also mismeasured.

McGrattan and Prescott (2012, 2014), and McGrattan (2016), go beyond the new NIPA measures of GDP by constructing real output measures that include other expensed items,
including advertising, marketing, computer design, management consulting, public relations, and engineering expenses as intangible investment. McGrattan (2016) develops a model of the U.S. economy that includes both tangible and intangible production, with a focus on intersectoral linkages.

McGrattan develops a model with tangible output and intangible output. Intangibles are a non-rival good. There are $s$ sectors that use both tangibles and intangibles. There is a Cobb-Douglas aggregate over consumption goods from the $S$ sectors. The technologies differ in terms of a sector-specific technology shock, and technology share parameters. The outputs for tangibles and intangibles is given by:

$$
Y_{st} = (K_{Tst}^1)^{\theta_S}(K_{Ist}^1)^{\phi_S}(\Pi_l(M_{lst}^1)^{\gamma_S})(Z_t Z_{st}^1 H_{st}^1)^{1-\theta_S-\phi_S-\gamma_S}
$$

(85)

$$
I_{st} = (K_{Tst}^2)^{\theta_S}(K_{Ist}^2)^{\phi_S}(\Pi_l(M_{lst}^2)^{\gamma_S})(Z_t Z_{st}^2 H_{st}^2)^{1-\theta_S-\phi_S-\gamma_S}
$$

(86)

$Y_s$ denotes the output of the tangible sector, $K_{Tst}^1$ is tangible capital that is used to produce tangible output in sector $S$, $K_{Ist}^1$ is tangible capital used to produce intangible output in sector $S$, $K_{Ist}^2$ is intangible capital, which is assumed to be non-rival, $M_{lst}^1$ and $M_{lst}^2$ are intermediate inputs used to produce tangibles in sector $S$, and intangibles in sector $S$, respectively. $Z$ is the aggregate productivity shock and $Z_s$ is a sector-specific productivity shocks. $H_{st}^1$ and $H_{st}^2$ are labor input for tangibles in sector $S$, and intangibles in sector $S$, respectively.

McGrattan (2016) uses maximum likelihood to estimate the parameters of the stochastic processes for $Z_t$ and for $Z_{st}$, and compares two economies, one with intangibles, and another without intangibles. The mismeasurement of productivity in the economy without intangibles generates a large labor wedge, and McGrattan argues that this may account for the empirical labor wedge measured from NIPA data. McGrattan also shows that the economy with intangibles closely accounts for the 2008-2014 U.S. economy, despite the fact that the standard measure of TFP based on NIPA data is not highly correlated with hours worked during this period.

Another literature that relates intangible investments to productivity is in the area of organization capital. As noted above, these investments are not counted in the NIPA. Atkeson and Kehoe (2005) study a neoclassical model in which an organization stochastically accumulates intangible knowledge over time. They find that the payments from these intangibles are about one-third as large as the payment from tangible capital, which suggests that organization capital is very large.

### 7.3 Neoclassical Models of Network Linkages and TFP

The impact of industry and/or sectoral shocks on the aggregate economy motivates a significant component of the real business cycle literature, including the seminal contribution of Long and Plosser (1983), and subsequent research by Dupor (1999) and Horvath (2000). One theme of this research is to provide a theory for aggregate productivity shocks that hit the economy.

This idea is now being developed further in network models, which focus on the idea that production is organized through networks of supply chains, and that small disruptions in
networks can have significant aggregate consequences, particularly if there are only a small number of suppliers of a particular input, and if there are no particularly close substitutes for that input. Carvalho (2014) describes much of the recent literature on networks and macroeconomics.

Carvalho describes a simple model of production networks in which individual sectors produce a specialized output. This output is produced using homogeneous labor and intermediate inputs from other sectors. The output of sector $i$ is given by:

$$y_i = (z_i h_i)^{1-\theta} \left( \prod_{j=1}^{n} y_{ij}^{\omega_{ij}} \right)^{\theta}. \quad (87)$$

In this technology, $y_i$ denotes sectoral output, $z_i$ is a sectoral productivity shock, $h_i$ is labor employed in sector $i$, and the exponents $\omega_{ij}$ denote the share of intermediate input $j$ used in producing good $i$. Note that labor is supplied inelastically by a representative household, so aggregate labor is in fixed supply. For simplicity, preferences are symmetric over the $i$ goods in the household utility function.

The empirical importance of network linkages can be identified from a standard input-output matrix. Since aggregate labor is in fixed supply, aggregate output is a weighted average of the sectoral productivity shocks:

$$\ln(y) = \sum_{i=1}^{n} \nu_i \ln(z_i). \quad (88)$$

In this expression, $y$ is aggregate output and the $\nu_i$ are weights that are constructed from the input-output table. Note that measured aggregate productivity in this economy, which is $\frac{y}{h}$, will fluctuate even though there is no aggregate productivity shock. This simple model shows how a single shock to an important sector can have significant aggregate affects that will be observationally equivalent to a one-sector model with an aggregate productivity shock.

8 Neoclassical Models of Inequality

Neoclassical modeling is also making considerable progress in characterizing and quantifying how technological change has affected income distribution and wage inequality. Neoclassical studies of inequality analyze how biased technological change differentially affects the demand for different types of workers.

Early empirical studies by Katz and Murphy (1992), among others, concluded that skill-biased technological change was responsible for the widening wage gap between highly-educated workers and workers with less education. This conclusion reflects the fact that the relative supply of highly-skilled workers rose considerably, and the relative wage of these workers also rose.

Krusell et al (2000) develop a neoclassical model to analyze how technological change has affected the relative wage of skilled to less-skilled workers. This relative wage is often called the *skill premium*. Krusell et al provide an explicit theory of skilled biased technological change, they show how to measure this change, and they develop a neoclassical model to quantify its effect on inequality through observable variables.
The model features two different types of labor: high-skilled labor, who are workers with 16 or more years of education, and unskilled labor, who have fewer than 16 years of education\(^{20}\). Skill-biased technological change in this model is the combination of capital equipment-specific technological change, coupled with different substitution elasticities between the two types of labor. Krusell et al construct a four factor production function that allows for different types of labor, and for different types of capital goods. There are two technology is given by:

\[
y_t = A_t k_{st}^\alpha [\mu u_t^\sigma + (1 - \mu)(\lambda k_{et}^\rho + (1 - \lambda)s_t^\omega)]^{\frac{1}{\alpha - \sigma}}
\]  

(89)

The term \(A_t\) is a neutral technology parameter. The inputs are capital structures \(k_{st}\), unskilled labor input \(u_t\), which is the product of unskilled hours and unskilled labor efficiency \(\psi_{ut} h_{ut}\), capital equipment \(k_{et}\), and skilled labor input \(s_t\), which is the product of skilled labor hours and skilled labor efficiency \(\psi_{st} h_{st}\). These inputs are specified within a nested CES technology in which the curvature parameters \(\sigma\) and \(\rho\) govern the substitution elasticities among the inputs. In this technology, rapid growth of capital equipment raises the wage of skilled workers relative to the wage of unskilled workers only if capital equipment is more complementary with skilled labor than with unskilled labor. This requires that \(\sigma > \rho\), which Krusell et al call capital-skill complementarity.

It is straightforward to see this requirement of \(\sigma > \rho\) by assuming that \(\psi_{st}\) and \(\psi_{ut}\) are constant, log-linearizing the ratio of the marginal productivities of the two types of labor, and expressing variables in terms of growth rates between periods \(t\) and \(t + 1\):

\[
g_{pt} \simeq (1 - \sigma)(g_{h_{ut}} - g_{h_{st}}) + (\sigma - \rho)\lambda \left( \frac{k_{et}}{s_t} \right)^\rho (g_{k_{et}} - g_{h_{st}})
\]  

(90)

In (90), \(g_{pt}\) is the growth rate of the skill premium, \(g_{h_{ut}}\) and \(g_{h_{st}}\) are the growth rates of unskilled and skilled hours, and \(g_{k_{et}}\) is the growth rate of capital equipment. Since the parameter \(\sigma\) is less than one, the first term on the right hand side of (90) shows that the skill premium declines if the growth rate of skilled hours exceeds the growth rate of unskilled hours. Krusell et al call this first term the relative quantity effect. The second term is called the capital-skill complementarity effect. This second term shows that the skill premium rises if the growth rate of capital equipment exceeds the growth rate of skilled hours, and if there is relatively more complementarity between skilled labor and equipment \((\sigma > \rho)\).

Krusell et al construct a dataset of skilled and unskilled labor input using data from the Current Population Survey. They use Gordon’s (1990) data on equipment prices to construct a measure of the stock of capital equipment, and they use the NIPA measure of capital structures.

They estimate the parameters of the nonlinear production function with data from 1963-1992 using two-step simulated pseudo-maximum likelihood. They fit the model using the equations that measure the deviation between model and data for total labor’s share of income, and the ratio of skilled labor income to unskilled labor income. The third equation in the criterion function measures the deviation between the rate of return to investment

\(^{20}\)Note that the term unskilled is used here not as a literal description of worker skill, but rather to clearly differentiate the two types of labor from each other.
in structures to equipment. They estimate substitution elasticities of about 1.67 between unskilled labor and equipment, and of about 0.67 between skilled labor and equipment, which provides strong support for capital-skill complementarity. They find that the model accounts for much of the movements in the skill premium over the 1963-1992 period.

Given that the Krusell et al data end in 1992, Ohanian and Orak (2016) analyze this same model, but extend the dataset through 2013 to assess the contribution of capital-skill complementarity to wage inequality for the last 20 years. Figure 30 shows the skill premium in the model and in the data from 1963 through 2013. To compare the analysis to Krusell et al, Ohanian and Orak also estimate the model from 1963 to 1992. The dashed line in Figure 30 corresponds to the end of the estimation period for the parameters (1992). Although Ohanian and Orak use the same sample period to estimate the parameters, they use revised data in the estimation. They find very similar elasticities to those in Krusell et al. Ohanian and Orak estimate an elasticity of about 1.78 between unskilled labor and equipment, and about 0.69 between skilled labor and equipment. The figure shows that the model accounts for the major changes in the skill premium, including the very large rise that has occurred in the last 30 years.21

The Krusell et al model also fits aggregate labor share very well up until the mid-2000s. After that, the model overpredicts labor’s share. This finding led Orak (2016) to analyze the same type of production function with different substitution possibilities between capital equipment and different types of skills, but with three types of labor, as opposed to two types of labor. The labor types in Orak are classified based on occupational tasks, as in Autor et al (2003), rather than on education levels, as in Krusell et al.

Orak specifies the three types of labor based on whether an occupation primarily performs cognitive tasks, manual tasks, or routine tasks. He estimates a relatively high elasticity of substitution between capital equipment and workers who perform routine tasks, and he estimates lower substitution elasticities between equipment and cognitive workers, and between equipment and manual workers. He finds that this augmented neoclassical model can account for much of the recent and significant decline in labor’s share of income.

9 Neoclassical Macroeconomics: Critical Assessments and Future Directions

This section discusses the open questions in the area of neoclassical macroeconomics, and presents our views on interesting future avenues for research that will address these questions. Perhaps the major open question for neoclassical models - and which is also a major question for other classes of macroeconomic models - is accounting for fluctuations in hours worked. The multisector models developed in this chapter account for considerably more of the fluctuations in hours worked than the standard one sector neoclassical model, but there are also changes in hours that these models do not capture. Below, we describe the research areas that we view as important and promising in addressing this issue and others.

\[\text{Krusell et al normalize the skill premium to 1 in 1963, and report fluctuations relative to the normalized value. To show the actual level of the skill premium, Ohanian and Orak estimate the model with normalized data as in Krusell et al, and then reconstruct the levels data. See Ohanian and Orak for details.}\]
9.1 Biased Technological Change and the Labor Market

Analysis of biased technological change, and its impact on both aggregate variables and on labor market outcomes of workers with different skill levels, is an interesting avenue for future research. The home production results from the model motivated by Greenwood et al (2005) indicate interesting trend changes in hours worked from the early 1980s through the 1990s, which coincide with the increase in women’s hours worked. Important future research will further connect this demographic increase in hours worked with general equilibrium models of home production.

More broadly, it will be important to further develop models in the area of directed technological change and the shape of the production function, as in Acemoglu (2002) and Jones (2005), the relationship between technologies and secular sectoral shifts, as in Lee and Wolpin (2005), human capital accumulation and technological change, as in Heckman et al (1998), and demographic shifts, technological change, and wage shifts as in Jeong et al (2015). A related area is studying movements in factor income shares, as in Karabarbounis and Nieman (2014), and Orak (2016), and the impact of factor endowments on how societies choose among biased technologies, as in Caselli and Coleman (2006).

All of these research areas are in relatively early stages of development, and merit additional analysis. Research in this area can also be combined with broader empirical studies of time allocation, including the analysis and documentation of home and market time allocation, as in Aguiar and Hurst (1997), and Aguiar, Hurst, and Karabarbounis (2013), and studies of the allocation of time across rich and poor countries, as in Bick et al (2015).

9.2 Neoclassical Analyses of the Great Recession and its Aftermath

Several open questions remain about the Great Recession and its aftermath. This includes accounting for macroeconomic aggregates from 2008 and onwards, particularly for hours worked. The results presented in this chapter indicate that neoclassical models with standard measures of equipment-specific productivity shocks, and TFP shocks, and without any policy components, miss some features of the Great Recession. McGrattan (2016) argues that output mismeasurement resulting from the omission of intangible investments in GDP has important implications for measured TFP and labor wedge measures during the Great Recession. Further research in this important area is needed.

There are also interesting aspects of economic policies during this period that merit additional analysis. Mulligan (2012, 2013) argues that changes in social insurance programs and the Affordable Care Act depressed labor by implicitly raising tax rates on labor. Kydland and Zarazaga (2016) study how expectations of different types of tax policies may have contributed to the weak recovery from the Great Recession. Baker, Bloom and Davis (2014) measure the evolution of economic policy uncertainty during the Great Recession. These uncertainty measures can be used in models in which uncertainty can depress an economy, as in Bloom (2009) and Fernandez-Villaverde et al (2011). These factors may have implications for understanding changes in hours worked in recent years.
9.3 The Effect of Policy Changes and Institutions on Macroeconomic Performance

An important area for future research is quantifying the impact of observed departures from competitive markets on economies. Cole and Ohanian (2004) developed and applied a particular methodology in their study of cartelization and unionization in the U.S. Great Depression. This approach was also applied by Lahiri and Yi (2009) in evaluating the affect of non-competitive policies in West Bengal Indian development. A similar approach has been used by Cheremuhkin et al (2013, 2015) to study the impact of Lenin’s policies and institutions on economic development in the U.S.S.R at that time, and to study the impact of Mao’s policies and institutions on Chinese development in the 1940s and 1950s. Alder et al (2016) use a related approach to analyze the contribution of labor union hold-up and imperfect competition on the decline of America’s Rust Belt region in the postwar U.S. Similar methods also can be used to study the recent evolution of the post-Soviet Union economies, to study recent Indian and Chinese development patterns (see Dekle and Vandenbroucke (2012) for a neoclassical study of recent trends in China’s economy), and to study long-run Latin American development (see Cole et al (2005) for a long-run analysis of Latin America). As better data becomes available, these methods can also be used to study how policies and institutions have affected the stagnation and development of very poor countries. Future research along these lines will allow us to understanding the relative importance of various non-competitive policies across countries, and will be an important input in developing growth-enhancing policies in poor countries.

9.4 Analyses of TFP

Since productivity is central in neoclassical growth models, advancing our understanding of changes in TFP is another important area for future research. In the last 10 years, progress in evaluating TFP has been made along three different research lines: resource misallocation, intangible investments, and network economies. Advancements in misallocation analysis of TFP will be facilitated by the assessment of how actual economic policies have affected resource allocation and productivity loss. Continued advances in computing power will facilitate the analysis of network economies and intersectoral linkages in the study of TFP. The continued expansion of intangible investments into NIPA data will advance our understanding of intangibles investment and TFP.

An area that to our knowledge that has not been studied in detail is to link changes in what Decker et al (2013) call "business dynamism" to aggregate measures of TFP. Specifically, Decker et al document lower rates of resource reallocation in the U.S., and also a lower rate of successful start-ups that have occurred over time. This decline has coincided with a secular decline in productivity growth. Analyzing theoretical and empirical connections between these observations has the potential to advance our understanding of secular movements in productivity.
9.5 Taxes and Macroeconomic Activity

The impact of tax and fiscal policies on economic activity in neoclassical models is another interesting area for future work, and may advance our understanding of changes in hours worked. Research in this area has been constrained by the availability of data on tax rates and hours worked. Constructing tax rates along the lines of McDaniel’s (2011) tax measurements for the OECD can in principal be extended to other countries. In terms of hours worked, Ohanian and Raffo (2011) construct panel data on hours in the OECD, and similar data constructions can be made for other countries.

10 Conclusions

This chapter presented aggregate data and a series of neoclassical models to show how the historical evolution of the U.S. economy reflects much longer-run changes in economic activity than previously recognized, and that much of this evolution is plausibly interpreted as the consequences of long-run shifts in technologies and government policies.

This chapter shows that neoclassical models can shed light on relatively stable periods of aggregate economic activity, such as the post-Korean War U.S. economy, but also on very turbulent periods that are typically considered to be far beyond the purview of neoclassical economics, including the Great Depression and World War II. Moreover, neoclassical analysis provides insights into not only purely aggregate issues, but also sheds light on how technological change has affected individual labor market outcomes.

Future macroeconomic analyses of fluctuations should shift from the standard practice of narrowly studying business cycle frequencies, and to include the quantitatively important lower frequency component of fluctuations that dominates much of the U.S. historical economic record. We anticipate that neoclassical research along these lines will continue to advance the profession’s knowledge in a number of areas reflecting both longer-run events, as well as business cycle fluctuations. This includes Depressions, Growth Miracles, the macroeconomic effects of various types of government regulatory and fiscal policies, the sources and nature of productivity shocks, the effects of biased technological change on the macroeconomy and on individual labor market outcomes, and understanding cyclical and longer-run fluctuations in hours worked.

References


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Notes:
(1) Government spending series are real and detrended by dividing by the population over 16 and by the growth trend in technology (scaled so the 1946 real detrended level of GNP less military compensation equals 1).
(2) Total factor productivity is defined to be $Y/(K^{\theta}L_p^{1-\theta})$, where $Y$ is real, detrended GNP less military compensation, $K$ is real detrended nonmilitary capital stock, $L_p$ is nonmilitary hours worked, and $\theta = .38$. 
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Legend
U.S. Data

Model

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Table 1 - Calibrated Parameter Values

<table>
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<tr>
<th>Parameter Description</th>
<th>Two-Sector</th>
<th>One-Sector</th>
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<th>Three-Sector (2)</th>
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Three-Sector (1) - Standard Home Production
Three-Sector (2) - Calibration inspired by Greenwood et al (2005)
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Table 3 - Comparing Models with Data (1955Q1 - 2014Q4)

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Table 4 - Comparing Models with Data (1955Q1 - 1983Q4)

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Table 6 - Comparing Models with Data (2007Q4 - 2014Q4)
Table 7 - U.S. Great Depression Levels of Real Output and its Components (Index, 1929=100)

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<th>Business Investment</th>
<th>Government Purchases</th>
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Data are measured in per capita terms and detrended.
Table 8 - Five Measures of Labor Input During U.S. Great Depression (Index, 1929=100)

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Data are measured in per capita terms.
Table 9 - Productivity and Real Wage Rates During U.S. Great Depression

(Index, 1929=100)

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Data are detrended.

*Labor Productivity is defined as output per hour.
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Table 11 - Equilibrium Path of Recovery from Depression in Competitive Model

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Table 12 - Equilibrium Path of Recovery from Depression in Cartel Policy Model

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