Appendix to: Quantitative Asset Pricing Implications of Housing Collateral Constraints

Hanno Lustig
UCLA and NBER

Stijn Van Nieuwerburgh

December 5, 2005

1 Additional Figures and Tables

Calibration of Expenditure Ratio Table 1 shows the estimation results of the autoregressive specification for the non-housing expenditure ratio \( r \):

\[
\log r_{t+1} = \bar{r} + \rho_r \log r_t + b_r \lambda_{t+1} + \sigma_r \nu_{t+1},
\]

where \( \nu_{t+1} \) is an i.i.d. standard normal process with mean zero, orthogonal to \( \lambda_{t+1} \). In our benchmark calibration we set \( \rho_r = .96 \), \( b_r = .93 \) and \( \sigma_r = .03 \). The parameter values are close to the estimates of (1) we find using US National Income and Products Accounts Data. Panel A of table 1 shows regression estimates for \( \rho_r \) and \( b_r \) that are consistent across samples and data sources. In periods of high aggregate consumption growth, the expenditure ratio increases. Alternatively, we could have calibrated a persistent process for the rental price \( \log(\rho_t) \). Panel B shows that rental prices increase in response to a positive aggregate consumption growth in the post-war sample.

Long-Horizon Sharpe Ratios Figure 1 plots the Sharpe ratio in the model on 5-year and 10-year cumulative returns for the collateral model. Figure 2 plots the estimated Sharpe ratio for US stocks at a 5-year and a 10-year horizon against the collateral scarcity measure, \( \tilde{m}_t = \max(m_{t+h}) - m_t \). The collateral scarcity measure \( \tilde{m}_t \) is constructed to lie between 0 and 1 for all \( t \). We see a positive correlation between the Sharpe ratio and the collateral scarcity.

[Table 1 about here.]

[Figure 1 about here.]

[Figure 2 about here.]
**Book-To-Market Betas**  Table 2 shows results from OLS regressions of excess returns of the 10 book-to-market deciles on a constant, a scaled version of the collateral measure $\tilde{m}y_t$, the aggregate consumption growth rate $\Delta(\log(c_{t+1}^a))$, the interaction term $\tilde{m}y_t\Delta(\log(c_{t+1}^a))$, the aggregate expenditure ratio growth rate $\Delta \log(r_{t+1})$, and the interaction term $\tilde{m}y_t\Delta(\log(r_{t+1}))$. These are the five risk factors in the collateral model.

[Table 2 about here.]

## 2  Model Details

We show under which conditions the sequence of budget constraints and collateral constraints in the sequential market setup can be rewritten as one time-zero budget constraint and a collection of solvency constraints, one for each node $s^t$. We then spell out the household problem with time zero trade, which is the one we actually solve numerically. Most of this second part can also be found in appendix A of Lustig and Van Nieuwerburgh (2005), but the setup there is somewhat different. There are two levels of heterogeneity in that paper: households and regions. The regions have segmented rental markets. Here, there is a single rental market and therefore no regional dimension.

### 2.1 From Sequential to Time Zero Household Problem

First, we show how the Arrow-Debreu budget constraint obtains from aggregating successive sequential budget constraints. The proof strategy follows Sargent (1984) (Chapter 8). Then we show the equivalence between the collateral constraints in sequential markets (from the main text), and solvency constraints in time-zero markets.

#### Budget Constraint

Let $\Pi_{s^t}$ be the value of a dividend stream $\{d\}$ starting in history $s^t$ priced using the market state prices $\{p\}$:

$$\Pi_{s^t} [\{d\}] = \sum_{j \geq 0} \sum_{s^{t+j} | s^t} p_{t+j} (s^{t+j}) d_{t+j} (s^{t+j}),$$

where for a given path $s^{t+j}$ following history $s^t$, $p$ is defined as

$$p_{t+j} (s^{t+j} | s^t) = q_{t+j} (s^{t+j} | s^{t+j-1}) q_{t+2} (s^{t+2} | s^{t+1}) \ldots q_{t+1} (s^{t+1} | s^t).$$

Let $\{\tilde{\eta}\}$ be the largest possible labor income stream.

**Assumption 1.** Interest rates are sufficiently high: The value of a claim to the largest possible labor income stream at time 0 is finite: $\Pi_{s^0} [\{\tilde{\eta}\}] < \infty$, 2
The sequential budget constraint is:

\[ c_t(\ell, s^t) + \rho_t(z^t)h^*_t(\ell, s^t) + \sum_{s'} q_t(s', s')a_t(\ell, s^t, s') + p_t^h(z^t)h^*_{t+1}(\ell, s^t) \leq W_t(\ell, s^t). \]

Next period wealth is:

\[ W_{t+1}(\ell, s^t, s') = \eta_{t+1}(s^t, s') + a_t(\ell, s^t, s') + h^*_t(\ell, s^t) \left[ p_{t+1}^h(z^t, s') + \rho_{t+1}(z^t, s') \right]. \]

Multiply the second equation by \( q_{t+1}(s') \) and sum over states. Then substitute the expression for \( \sum q_{t+1}(s')a_{t+1}(s') \) into the first equation.

\[ c_t + \rho_t h^*_t + \sum_{s'} q_{t+1}(s')W_{t+1}(s') \leq W_t + \sum_{s'} q_{t+1}(s')\eta_{t+1}(s') + h^*_t \left( \sum_{s'} q_{t+1}(s') \left[ p_{t+1}^h(s') + \rho_{t+1}(s') \right] - p_t^h \right). \]

Similarly, for period \( t + 1 \):

\[ c_{t+1} + \rho_{t+1} h^*_t + \sum_{s''} q_{t+2}(s'')W_{t+2}(s'') \leq W_{t+1} + \sum_{s''} q_{t+2}(s'')\eta_{t+2}(s'') + h_{t+2}^* \left( \sum_{s''} q_{t+2}(s'') \left[ p_{t+2}^h(s'') + \rho_{t+2}(s'') \right] - p_{t+1}^h \right). \]

Substituting the expression for \( t + 1 \) into the expression for \( t \) by substituting out \( W_{t+1} \), we get:

\[ c_t + \rho_t h^*_t + \sum_{s'} q_{t+1}(s') \left[ c_{t+1} + \rho_{t+1} h^*_t \right] + \sum_{s'} \sum_{s''} q_{t+1}(s')q_{t+2}(s'')W_{t+2}(s'') \leq \]

\[ W_t + \sum_{s'} q_{t+1}(s')\eta_{t+1}(s') + \sum_{s'} \sum_{s''} q_{t+1}(s')q_{t+2}(s'')\eta_{t+2}(s'') + h^*_t \left( \sum_{s'} q_{t+1}(s') \left[ p_{t+1}^h(s') + \rho_{t+1}(s') \right] - p_t^h \right) + \]

\[ \sum_{s'} q_{t+1}(s')h_{t+2}^*(s') \left( \sum_{s''} q_{t+2}(s'') \left[ p_{t+2}^h(s'') + \rho_{t+2}(s'') \right] - p_{t+1}^h \right). \]

Repeating these substitutions, we obtain the following inequality at time \( t \):

\[ \Pi_t \left[ \{c + \rho h^\tau\} \right] \leq W_t - \eta_t + \Pi_t \left[ \{\eta\} \right], \quad (2) \]

where we have used: (1) the transversality condition

\[ \lim_{j \to \infty} \sum_{s^{t+j}} p_{t+j}(s^{t+j})W_{t+j}(s^{t+j}) = 0, \quad (3) \]

3
and (2) a no-arbitrage condition:

\[ p_{t+j-1}^h(s^{t+j-1}) = \sum_{s\in s^{t+j-1}} q_{t+j}(s^{t+j}) \left[ p_{t+j}^h(z^{t+j}) + \rho_{t+j}(z^{t+j}) \right], \quad \forall j \geq 0, \forall s^{t+j} \]  \hspace{1cm} (4)

If the latter condition were not satisfied, a household could achieve unbounded consumption by investing sufficiently high amounts in housing shares \( h^o \) and financing this by borrowing. This is a feasible strategy because ownership shares in the housing tree are collateralizable.

Because \( W_0 = \eta_0 + \ell \), and relabelling \( h_t^o = h_t \), we recover from equation (2) the Arrow-Debreu budget constraint at time 0:

\[ \Pi_{s^0} \{ c + \rho h \} \leq \ell + \Pi_{s^0} \{ \eta \}, \]

where we have used the assumption that interest rates are sufficiently high (see Assumption 1). This implies that the AD budget constraint is satisfied, if the sequential budget constraints are satisfied.

**Collateral Constraints** Second, we show the equivalence between the collateral constraints of the sequential markets setup and the solvency constraint in the static economy. The sequential collateral constraints are:

\[ \left[ p_{t}^h(z^{t}) + \rho_{t}(z^{t}) \right] h_{t-1}^o(s^{t-1}) + a_{t-1}(s^{t-1}, s_t) \geq 0, \]

and the collateral constraints in a history \( s^t \):

\[ \Pi_{s^t} \{ c + \rho h \} \geq \Pi_{s^t} \{ \eta \}. \] \hspace{1cm} (5)

The equivalence follows if and only if

\[ a_{t-1}(s^{t-1}, s_t) + h_{t-1}^o(s^{t-1}) \left[ p_{t}^h(z^{t}) + \rho_{t}(z^{t}) \right] = \Pi_{s^t} \{ c + \rho h - \eta \}. \]

But this follows immediately from the budget constraint (2) holding with equality and the definition of \( W \):

\[ W_t(s^t) - \eta_t(s) = a_{t-1}(s^{t-1}, s_t) + h_{t-1}^o(s^{t-1}) \left[ p_{t}^h(z^{t}) + \rho_{t}(z^{t}) \right]. \]

Under conditions (3) and (4) an allocation that is feasible and immune to the threat of default in sequential markets is feasible and immune to the threat of default in time-zero markets.

The equivalence implies that the allocation of home-ownership \( h^o \) is indeterminate in the sequential economy.
2.2 The Time-Zero Problem

**Household Problem** A household of type \((\ell, s_0)\) purchases a complete contingent consumption plan \(\{c(\ell, s_0), h(\ell, s_0)\}\) at time-zero market state prices \(\{p, pp\}\). The household solves:

\[
\sup_{\{c, h\}} U(c(\ell, s_0), h(\ell, s_0))
\]

subject to the time-zero budget constraint

\[
\Pi_{s_0} [\{c(\ell, s_0) + \rho h(\ell, s_0)\}] \leq \ell + \Pi_{s_0} [\{\eta\}],
\]

and an infinite sequence of collateral constraints for each \(t\) and \(s^t\)

\[
\Pi_{s^t} [\{c(\ell, s_0) + \rho h(\ell, s_0)\}] \geq \Pi_{s^t} [\{\eta\}], \forall s^t.
\]

**Dual Problem** Given Arrow-Debreu prices \(\{p, \rho\}\) the household with label \((\ell, s_0)\) minimizes the cost \(C(\cdot)\) of delivering initial utility \(w_0\) to itself:

\[
C(w_0, s_0) = \min_{\{c, h\}} (c_0(w_0s_0) + h_0(w_0, s_0)\rho_0(s_0)) + \sum_{s^t} p(s^t|s_0) \left( c_t(w_0, s^t|s_0) + h_t(w_0, s^t|s_0)\rho_t(s^t|s_0) \right)
\]

subject to the promise-keeping constraint \(U_0(\{c\}, \{h\}; w_0, s_0) \geq w_0\), and the collateral constraints \(\Pi_{s^t} [\{c(w_0, s_0) + \rho h(w_0, s_0)\}] \geq \Pi_{s^t} [\{\eta\}], \forall s^t\). The initial promised value \(w_0\) is determined such that the household spends its entire initial wealth: \(C(w_0, s_0) = \ell + \Pi [\{\eta\}].\) There is a monotone relationship between \(\ell\) and \(w_0\).

The above problem is a convex programming problem. We set up the saddle point problem and then make it recursive by defining cumulative multipliers (Marcet and Marimon (1999)). Let \(\gamma\) be the Lagrange multiplier on the promise keeping constraint and \(\gamma_t(w_0, s^t)\) be the Lagrange multiplier on the collateral constraint in history \(s^t\). Define a cumulative multiplier at each node: \(\zeta_t(w_0, s^t) = 1 - \sum_{s^{t'} \leq s^t} \gamma_{t'}(w_0, s^{t'}).\) Finally, we rescale the market state price \(\hat{p}_t(s^t) = p_t(s^t)/\delta^t \pi_t(s^t|s_0).\)

By using Abel’s partial summation formula (see Ljungqvist and Sargent (2000), Chapter 15) and the law of iterated expectations to the Lagrangian, we obtain an objective function that is a function of the cumulative multiplier process \(\zeta_t:\)

\[
D(c, h, \zeta; w_0, s_0) = \sum_{t \geq 0} \sum_{s^t} \left\{ \delta^t \pi(s^t|s_0) \left[ \zeta_t(w_0, s^t|s_0)\hat{p}_t(s^t) \left( c_t(w_0, s^t) + h_t(w_0, s^t)\right) + \gamma_t(w_0, s^t)\Pi_{s^t} [\{\eta\}] \right] \right\}
\]

such that

\[
\zeta_t(w_0, s^t) = \zeta_{t-1}(w_0, s^{t-1}) - \gamma_t(w_0, s^t), \quad \zeta_0(w_0, s_0) = 1
\]
Then the **recursive dual** saddle point problem is given by:

\[
\inf_{\{c,h\}} \sup_{\{\zeta\}} D(c, h, \zeta; w_0, s_0)
\]

such that

\[
\sum_{t \geq 0} \sum_{s^t} \delta^t \pi(s^t|s_0) u(c_t(w_0, s^t), h_t(w_0, s^t)) \geq w_0
\]

To keep the mechanics of the model in line with standard practice, we re-scale the multipliers. Let

\[
\xi_t(\ell, s^t) = \frac{\chi}{\zeta_t(w_0, s^t)}
\]

The cumulative multiplier \(\xi(\ell, s^t)\) is a non-decreasing stochastic sequence (sub-martingale). If the constraint for household \((\ell, s_0)\) binds, it goes up, else it stays put.

**First Order Necessary Conditions**  The f.o.c. for \(c(\ell, s^t)\) is:

\[
\hat{p}(s^t) = \xi_t(\ell, s^t) u_c(c_t(\ell, s^t), h_t(\ell, s^t)).
\]

Upon division of the first order conditions for any two households \(\ell'\) and \(\ell''\), the following restriction on the joint evolution of marginal utilities over time and across states must hold:

\[
\frac{u_c(c_t(\ell', s^t), h_t(\ell', s^t))}{u_c(c_t(\ell'', s^t), h_t(\ell'', s^t))} = \frac{\xi_t(\ell', s^t)}{\xi_t(\ell'', s^t)}.
\]  

(6)

Growth rates of marginal utility of non-durable consumption, weighted by the multipliers, are equalized across agents:

\[
\frac{\xi_{t+1}(\ell', s^{t+1})}{\xi_t(\ell', s^t)} \frac{u_c(c_{t+1}(\ell', s^{t+1}), h_{t+1}(\ell', s^{t+1}))}{u_c(c_t(\ell', s^t), h_t(\ell', s^t))} = \frac{\hat{p}_{t+1}(s^{t+1})}{\hat{p}_t(s^t)} = \frac{\xi_{t+1}(\ell'', s^{t+1})}{\xi_t(\ell'', s^t)} \frac{u_c(c_{t+1}(\ell'', s^{t+1}), h_{t+1}(\ell'', s^{t+1}))}{u_c(c_t(\ell'', s^t), h_t(\ell'', s^t))}.
\]

There is a mapping from the multipliers at \(s^t\) to the equilibrium allocations of both commodities. We refer to this mapping as the risk-sharing rule.

\[
c_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{1/2}}{\xi_t^a(z^t)} c_a^t(z^t) \text{ and } h_t(\ell, s^t) = \frac{\xi_t(\ell, s^t)^{1/2}}{\xi_t^a(z^t)} h_a^t(z^t).
\]  

(7)

It is easy to verify that this rule satisfies the optimality condition and market clearing follows immediately from the definition of \(\xi_t^a\).

At \(t = 0\), the ratio of marginal utilities is pinned down by the ratio of multipliers on the promise-keeping constraints. For \(t > 0\), it tracks the stochastic weights \(\xi_t\). From the first order condition
w.r.t. $\xi_t(\ell, s^t)$ of the saddle point problem, we obtain a reservation weight policy:

$$
\xi_t = \xi_{t-1} \text{ if } \xi_{t-1} > \xi(y_t, z^t), \\
\xi_t = \xi(y_t, z^t) \text{ otherwise.}
$$

(8) \\

(9)

and the collateral constraints hold with equality at the bounds:

$$
\Pi_s \left\{ c_t(\ell, s^t; \xi(y_t, z^t)) + \rho h_t(\ell, s^t; \xi(y_t, z^t)) \right\} = \Pi_s \{ \{\eta\} \}.
$$

Lustig and Van Nieuwerburgh (2005) prove that the cutoff consumption share $\xi$ can only depend on the current idiosyncratic income realization $y_t$, and not on the entire history $y^t$.

3 Model with Recursive Preferences

In the main text, we assume additive utility. In this section, we show how the model’s stochastic discount factor changes when preferences are of the Kreps and Porteus (1978) type. We show that this type of preferences is important to generate a less volatile risk-free rate and low risk premia on long horizon assets.

3.1 Model

Preferences The household’s utility at time $t$, $V_t$, is given by a composite of the utility it derives from current consumption and its future expected utility:

$$
V_t = \left[ (1 - \delta) \left( c_t^{\varepsilon} + \psi h_t^{\varepsilon} \right)^{\frac{(1-\phi)\varepsilon}{1-\varepsilon}} + \delta (R_t V_{t+1})^{1-\phi} \right]^{\frac{1}{1-\phi}},
$$

where future expected utility is defined by $R_t V_{t+1} = \left( E[V_{t+1}^{1-\gamma}] \right)^{\frac{1}{1-\gamma}}$. The coefficient $\phi$ is the inverse of the intertemporal elasticity of substitution, $\gamma$ measures the risk aversion and $\varepsilon$ measures the intratemporal elasticity of substitution between non-durable and housing services consumption. Additive utility is a special case with $\gamma = \phi$.

Risk-Sharing Rule The risk-sharing rule with recursive preferences takes the same form as in the main text, but the stochastic consumption weights are different:

$$
c_t(\ell, s^t) = \frac{\hat{\xi}_t(\ell, s^t)^{\frac{1}{2}}}{\xi_t^a(z^t)} c_t^a(z^t) \text{ and } h_t(\ell, s^t) = \frac{\hat{\xi}_t(\ell, s^t)^{\frac{1}{2}}}{\xi_t^a(z^t)} h_t^a(z^t).
$$

(10)
The new stochastic consumption weight $\hat{\xi}_t(\ell, s^t)$ equals the old stochastic consumption weight $\xi_t(\ell, s^t)$ multiplied by the utility gradient between period 0 and period $t$, $M_{0,t}(\ell, s^t)$:

$$M_{0,t}(\ell, s^t) = \prod_{s^\tau \leq s^t} M_{\tau}, \quad M_{\tau} = \left( \frac{V_{\tau}}{R_{\tau-1}V_{\tau}} \right)^{\phi-\gamma}.$$ 

The new consumption weights still have a recursive structure:

$$\hat{\xi}_{t+1} = \left( \frac{\hat{\xi}_{t+1}}{\hat{M}_{t+1}} \right) \hat{\xi}_t.$$ 

While the individual consumption weights $\{\xi\}$ were non-decreasing processes, this is no longer true for the new stochastic weight shocks $\{\hat{\xi}\}$, because $M_{t+1}$ may be less than one. Furthermore, even if the solvency constraints never bind, the new consumption weights change over time. As before, the aggregate weight shock is the cross-sectional average of the individual stochastic consumption weights: $\hat{\xi}_t^a(z^t) = E[\hat{\xi}_t^1(\ell, s^t)]$.

**Stochastic Discount Factor** The stochastic discount factor still equals the marginal utility growth of the unconstrained households. It is of the same form as in the additive utility case

$$m_{t+1} = \delta \left( \frac{c_{t+1}^a}{c_t^a} \right)^{-\phi} \left( \frac{\alpha_{t+1}^a}{\alpha_t^a} \right)^{\frac{\phi-1}{\phi-1}} \left( \frac{\xi_{t+1}}{\xi_t} \right)^{\phi-\gamma}. \quad (11)$$

The first two terms reflect aggregate consumption growth risk and composition risk. The last term however embodies both long-run consumption growth risk and the risk of binding solvency constraints. The long-run consumption growth risk is the last term of the representative agent’s SDF:

$$m_{t+1}^a = \delta \left( \frac{c_{t+1}^a}{c_t^a} \right)^{-\phi} \left( \frac{\alpha_{t+1}^a}{\alpha_t^a} \right)^{\frac{\phi-1}{\phi-1}} \left( \frac{V_{t+1}^e}{R_{t+1}V_{t+1}^e} \right)^{\phi-\gamma},$$

where $V_t^e$ denotes the continuation utility of a representative agent who consumes the aggregate non-durable and housing services endowment. Epstein and Zin (1991) show that the representative agent SDF can be rewritten as a function of the gross return on an asset paying the aggregate non-durable and the aggregate housing endowment stream, $R^{c,h}$.

$$m_{t+1}^a = \delta \left( \frac{c_{t+1}^a}{c_t^a} \right)^{-\phi} \left( \frac{\alpha_{t+1}^a}{\alpha_t^a} \right)^{\frac{\phi-1}{\phi-1}} \left( \frac{R_{t+1}^{c,h}}{R_{t+1}^{c,h}} \right)^{\phi-\gamma}.$$

The SDF of the economy is still the product of the representative agent economy’s SDF and a liquidity shock.

$$m_{t+1} = m_{t+1}^a \phi.$$

Contrary to the aggregate weight shock $g = \frac{\xi_{t+1}}{\xi_t}$ in the case of additive utility, the new liquidity
shock $\tilde{g}$ is no longer theoretically restricted to be greater than or equal to one.

**Calibration** For the economy with recursive preferences, we use $\phi = 5$, where $\phi$ is the inverse of the intertemporal elasticity of substitution.

### 3.2 Unconditional Asset Pricing Moments

Table 3 shows the unconditional asset pricing statistics for the collateral model under recursive preferences. The equity premium on a levered consumption claim for the economy with 5 percent collateral and $\gamma = 8$ is 6 percent, excess stock returns have a volatility of 18 percent and the Sharpe ratio of the stock return is 0.33. The risk-free rate is 6.8 percent on average. These moments are of the same magnitude as the ones we found for additive preferences. However, the volatility of the risk-free rate is only half as large: 6.1 percent versus 12.5 percent under additive preferences.

[Table 3 about here.]

### 3.3 Conditional Asset Pricing Moments

All relationships between the housing collateral ratio and the conditional asset pricing moments carry over to the model with recursive preferences.

### 3.4 Cross-Sectional Asset Pricing Moments

Figure 3 plots the risk premia and Sharpe ratios on the same strips for an inverse intertemporal elasticity of substitution $\phi = 5$. We recall that when $\phi = \gamma$, we are back in the case of additive preferences. If $\gamma$ is sufficiently above $\phi$, the decline in equity premia and Sharpe ratios for long horizon strips is large. The equity premium on a claim to aggregate consumption two periods from now is 2%, the risk premium on the period 20 consumption strip is 5%, while the equity premium on a claim to aggregate consumption 30 years from now is 0%. Claims on payoffs far in the future have a lower expected excess return than claims on payoffs in the near future. Likewise, the Sharpe ratio, plotted in the bottom panel, falls from 0.25 to 0. Claims on payoffs far in the future have a lower conditional Sharpe ratios than claims on payoffs in the near future.

[Figure 3 about here.]

### References


Figure 1: Housing Collateral Ratio and Long-Horizon Sharpe Ratio in Model.

The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion is 8. This the Sharpe ratio on a 10 year and 5 year cumulative excess return on a non-levered consumption claim (dotted line), and the collateral ratio $m_y$ is the ratio of housing wealth to total wealth (full line) for a one hundred period model simulation.
Figure 2: Housing Collateral Ratio and Long-Horizon Sharpe Ratio in Data.

This is the Sharpe ratio on 5-year and 10-year cumulative stock market returns in the data for 1928-1997. The housing collateral measure \( \tilde{m}_i \) measures the scarcity of collateral and is scaled to be between 0 and 1.
The average collateral share is 5 percent, the discount factor is .95 and the coefficient of risk aversion $\gamma = 8$. Preferences are of the Epstein-Zin type with inverse intertemporal elasticity of substitution of $\phi = 5$. The figure plots the expected excess return on a claim to aggregate consumption $k$ periods from now, $k = 2, 3, \ldots, 45$, and the Sharpe ratio.
Table 1: Expenditure Share and Rental Price Regression Results.

Panel A reports regression results for $\log(r_{t+1}) = \theta \log(r_t) + \lambda \Delta \log(c_{t+1}) + \epsilon_{t+1}$, where $r$ is the expenditure share of nondurable consumption. Panel B reports results for the regression $\log(\rho_{t+1}) = \rho - r \log(\rho_t) + b_r \Delta \log(c_{t+1}) + \epsilon_{t+1}$, where $\rho$ is the rental price. Below the OLS point estimates are HAC Newey-West standard errors. The left panel reports the results for the entire sample, while the right panel reports the results for the post-war sample. The variables with superscript 1 are available for 1926-2002. The variables with superscript 2 are only available for 1929-2002. The data appendix contains detailed definitions and data sources for these variables.

| Expl. Var. | $\rho_r$ | $b_r$ | $ho_r$ | $b_r$ |
|------------|---------|-------|---------|-------|
|            | 1926/9-2002 | 1945-2002 |       |       |

**Panel A: Expenditure Share**

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<td>(.033)</td>
<td>(.180)</td>
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<td>.952</td>
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**Panel B: Rental Price**

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<td>$\log(\rho^1)$</td>
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OLS regression of excess returns of the 10 book-to-market deciles on a constant, a scaled version of the collateral measure \( \bar{m}_y \), the aggregate consumption growth rate \( \Delta \log(c_{t+1}) \), the interaction term \( \bar{m}_y \Delta \log(c_{t+1}) \), the aggregate expenditure share growth rate \( \Delta \log(e_{t+1}) \), and the interaction term \( \bar{m}_y \Delta \log(e_{t+1}) \). These are the five risk factors in the collateral model. The first line of each panel is for the lowest book-to-market decile (growth), the last line for the highest book-to-market decile (value). The number is brackets are OLS t-statistics. In the first panel the housing collateral ratio is based on the value of outstanding mortgages and in the second panel, the housing collateral ratio is based on the value of residential real estate wealth. The data are annual for the period 1930-2003.

**Table 2: Beta Estimates for Book-to-Market Decile Returns in Data - 5 Factors.**

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**Panel 1: Mortgage-Based Collateral**

**Panel 2: Residential Wealth-Based Collateral**

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Table 3: Unconditional Asset Pricing Moments - Recursive Utility.

Same as Table 6 in main text, but preferences are of the recursive utility type with inverse intertemporal elasticity of substitution $\phi = 5$ and $\epsilon = .05$. The risk aversion parameter $\gamma$ is reported in the first column.

\[
\begin{array}{cccccccc}
\gamma & E(R^{l,e}) & E(R^{c,e}) & E(r^f) & \sigma(R^{l,e}) & \sigma(R^{c,e}) & \sigma(r^f) & \frac{E(R^{l,e})}{\sigma(R^{l,e})} & \frac{E(R^{c,e})}{\sigma(R^{c,e})} \\
7 & 0.067 & 0.058 & -0.008 & 0.278 & 0.222 & 0.137 & 0.242 & 0.260 \\
8 & 0.049 & 0.033 & 0.048 & 0.195 & 0.144 & 0.069 & 0.252 & 0.229 \\
\end{array}
\]