Partial Identification and Mergers

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1 Introduction

Inference on partially identified parameters has now become a standard tool kit for econometricians. Econometric models and properties of available data sets are such that it is often impossible to point-identify parameters of interest even if the sample size is infinity and the exact data generating process is available to econometricians without sampling noise. Numerous papers have been written on the issue, and Manski (2003) is probably a good starting point to have an idea about the fundamental issues there. Manski (2011) extends the analysis, and discusses policy choice when a planner faces ambiguity in policy outcomes.

In this note, we extend Manski’s insight to the problem of merger simulation by incorporating uncertainty about post-merger conduct into a simple simulation model. At a broad level, merger review by antitrust agencies requires the prospective evaluation of two types of likely merger effects. Unilateral effects refer to the incentives for the merged firms to unilaterally raise price in the absence of disciplining competition from the other firm. Coordinated effects refer to how the merger is expected to change the conduct, for example the opportunity and incentive to engage in tacit or explicit collusion, of the post-merger industry overall. In its typical implementation, merger simulation maps pre-merger estimates of demand and firm costs into forecasted, post-merger price changes via assumptions about post-merger costs and firm conduct. Since the assumption about conduct is almost always Nash in prices or quantities, this implementation of the simulation methodology is only useful, at best, for assessing unilateral effects.\(^1\) Coordinated effects must then be considered separately and, necessarily, in a non-integrated way (Kovacic et al. 2006, 2009).

Our focus of attention in this note is on the problem of uncertainty regarding overall post-merger conduct, i.e. integrated unilateral effects and coordinated effects, and the complications it entails for policy choice. In order to isolate this aspect of the problem, we work with a simple "conduct parameters" model in the spirit of Bresnahan (1982) and Lau (1982). This model allows integration of non-Nash conduct into the simulation but does not, under the functional restrictions imposed by Bresnahan (1982) and Lau (1982), face issues of partial identification of

\(^1\)Davis and Huse (2010) propose a merger simulation model that explicitly incorporates coordinated effects.
pre-merger demand, cost, or conduct parameters that would further complicate the analysis. If pre-merger data are large enough for sampling noise to be ignored, this framework would allow policy makers to focus only on the uncertainty about post-merger conduct and costs. It would be useful to consider decision making when there is uncertainty in other, pre-merger parameters as well as the post-merger conduct and cost. We chose a particular functional form only for the purpose of ensuring that all the pre-merger parameters are point identified. In more general models, those parameters would be partially identified (see Rosen 2006 for an example in the conduct parameters case), and the policy decision would be complicated even further.

While the convenience and clarity of the conduct parameters framework makes the approach appealing for our purposes, we are aware of the persuasive and widely known criticisms of the framework, as pointed out by Corts (1999). He shows the standard interpretation of the estimated conduct parameter as a time average of elasticity adjusted price-cost margins, i.e. average conduct, is only valid if the underlying model of competition satisfies certain high level conditions. He then goes on to argue that these conditions are unlikely to hold in most canonical models of tacit collusion. In our analysis, we assume the underlying model generating firm behavior satisfies Corts’ conditions, simply noting that in actual applications the researcher will have to confront this issue.

Corts’ (1999) criticism notwithstanding, the conduct parameters methodology has been appealing in practice because of its one-size-fits-all simplicity and ease of implementation, both strong desiderata in merger review where analysis time is short (Farrell and Shapiro 2011). The framework, however, has not been used in merger simulation precisely because the uncertainty in post-merger conduct we highlight is inherent to the approach. At their best, conduct parameters are the reduced form of a more detailed underlying model of competition and, as such, it is difficult to place a priori restrictions on how a major change in the economic environment, like a merger, would change those parameters. This makes the application of the framework to merger simulation a natural candidate for analyzing policy choice under ambiguity. Moreover, addressing this uncertainty may be necessary for effective merger simulation. Once one discovers that pre-merger conduct does not conform to the static Nash concept, it’s not clear that the standard merger simulation methodology provides a consistent estimate of unilateral effects in isolation since in general models there may be interactions between these and the coordinated effects.

2 Model

We will illustrate the post-merger ambiguity problem, by assuming that there are three firms operating in a market with a homogeneous product, e.g., three gas stations in a city, and that two of them petition for a merger claiming that the merger will result in cost efficiencies. From a policy maker’s perspective, this has to be weighed against decreased competition. In order to focus on the ambiguity of post-merger conduct, we will adopt a conduct parameter based approach, and adopt specifications in Jans and Rosenbaum (1996).

We assume that the demand is characterized by a linear demand function

\[ Q = \alpha_0 - \alpha_1 P + \alpha_2 X \]

We also assume that the marginal cost is characterized by a linear marginal cost function; for
each firm $i = 1, 2, 3$

$$MC_i = \beta_0 + \beta_1 \frac{Q_i}{CAP} + \beta_2 Z,$$

Here, the value $CAP$ denotes the market level capacity, which does not change due to merger. Note that we are imposing the assumption that all firms have identical marginal costs.

The inverse demand function is

$$P = \frac{\alpha_0}{\alpha_1} - \frac{1}{\alpha_1} (Q_1 + Q_2 + Q_3) + \frac{\alpha_2}{\alpha_1} X$$

The perceived marginal revenue for firm $i$ with conduct parameterized by $\lambda$ is:

$$MR_i = P - \frac{\lambda}{\alpha_1} Q_i$$

The parameter $\lambda \in [0, 1]$ indexes the competitiveness of the oligopolistic market. If $\lambda = 0$, we have perfect competition. If $\lambda = 1$, we have a perfect cartel. Note that we are imposing the assumption that all firms have the same conduct.

Using $MC_i = MR_i$, we obtain the reduced form supply relation for each firm $i$:

$$P = \beta_0 + \left( \frac{\beta_1}{CAP} + \frac{\lambda}{\alpha_1} \right) Q_i + \beta_2 Z$$

or defining $D = \frac{\lambda}{\alpha_1} + \frac{\beta_1}{CAP}$

$$Q_i = -\frac{\beta_0}{D} + \frac{1}{D} P - \frac{1}{D} \beta_2 Z$$

for each $i$. The market supply relation is

$$Q = -\frac{3\beta_0}{D} + \frac{3}{D} P - \frac{3}{D} \beta_2 Z$$

As noted in the beginning of this section, Jans and Rosenbaum’s (1996) estimate is based on a generalized version of this model, which means that all the parameters are identified. For the purpose of merger simulation, we will assume that sampling noise is negligible, and that we know $(\alpha, \beta, \lambda)$. The $\lambda$ that we identified is the pre-merger value. This follows from the fact that $Z$ is excluded from the demand equation and $X$ from the supply equation. The conduct parameter is identified by variation in $CAP$.²

Equating the market supply and demand, we get

$$\sum_{i=1}^{3} Q_i = \alpha_0 - \alpha_1 P + \alpha_2 X$$

or

$$-\frac{3\beta_0}{D} + \frac{3}{D} P - \frac{3}{D} \beta_2 Z = \alpha_0 - \alpha_1 P + \alpha_2 X$$

Solving for the price, we get

$$P = \frac{\alpha_0 D + 3\beta_0}{\alpha_1 D + 3} + \frac{3}{\alpha_1 D + 3} \beta_2 Z + \frac{D}{\alpha_1 D + 3} \alpha_2 X$$

²Note that the condition in Lau (1982) is not satisfied, because there is only one $X$. Still the conduct parameter is identified.
3 Merger Simulation

Merger simulation is basically an exercise forecasting the price after a hypothetical merger. If firms 1 and 2 merge and the output of the merged firm is $Q_{12}$, the cost function of the merged firm is

$$C_{12}(Q_{12}) = \min_{Q_1} C(Q_1) + C(Q_{12} - Q_1) = 2C \left( \frac{Q_{12}}{2} \right)$$

Therefore the marginal cost of the merged firm is

$$MC_{12} = MC_1 \left( \frac{Q_{12}}{2} \right) = \beta_0 + \frac{\beta_1}{2} \frac{Q_{12}}{CAP} + \beta_2 Z$$

if nothing else changes. The unmerged firm has the same marginal cost function as before. If the two merged firms have an extra source of efficiency, we will model it as

$$MC_{12} = \beta'_0 + \frac{\beta_1}{2} \frac{Q}{CAP} + \beta_2 Z$$

with $\beta'_0 < \beta_0$.

In the best case, the estimated pre-merger conduct parameter is a reduced form of a more complicated underlying model of firm conduct. Since models of tacit or explicit collusion are often what we have in mind for this underlying model and these models usually predict more collusive outcomes are more likely with fewer firms, we will make the natural assumption that the post-merger conduct parameter is equal to $\lambda' \in [\lambda, 1]$. For the merged firm, we have the supply relation

$$P = \beta'_0 + D_m Q_{12} + \beta_2 Z$$

with

$$D_m = \frac{\beta_1}{2CAP} + \frac{\lambda'}{\alpha_1}$$

so that for the merged firm

$$Q_{12} = -\frac{\beta'_0}{D_m} + \frac{1}{D_m} P - \frac{1}{D_m} \beta_2 Z$$

and for the unmerged firm, we have

$$Q_3 = -\frac{\beta_0}{D'} + \frac{1}{D'} P - \frac{1}{D'} \beta_2 Z$$

with

$$D' = \frac{\lambda'}{\alpha_1} + \frac{\beta_1}{CAP}$$

Because the demand equation is unchanged the effect of the merger on the price depends on the shift in the supply $Q_{12} + Q_3$ (see Fig 1). If $\lambda' = 0$, i.e., if before and after the merger the market is perfectly competitive, then the supply is the same before and after the merger and there is no effect on price. If $\lambda' > 0$, then

$$\frac{1}{D_m} < \frac{2}{D'}$$
Hence even if $\lambda' = \lambda$, supply will be lower due to the merger and the market price higher. However, for every $\lambda'$ there is a value of $\beta'_0 < \beta_0$ such that $P$ post merger is less than pre merger. If $P_p, Q_p$ are the pre-merger equilibrium price and quantity, the post-merger price is lower than the pre-merger price if

$$\beta'_0 < -D_m Q_p + \left(1 + \frac{D_m}{D'}\right) P_p - \beta_0 \frac{D_m}{D'} - \left(1 + \frac{D_m}{D'}\right) \beta_2 Z$$

The equilibrium price is equal to

$$P = \frac{\alpha_0 D_m D'}{\alpha_1 D_m D' + D' + D_m} + \frac{\beta'_0 D' + \beta_0 D_m}{\alpha_1 D_m D' + D' + D_m} \alpha_2 X + \frac{D' + D_m}{\alpha_1 D_m D' + D' + D_m} \beta_2 Z$$

(1)

A merger will be approved if the projected reduction in marginal cost (due to increased efficiency) exceeds the upward pressure in price (due to decreased competition). The set of $(\lambda', \beta'_0)$ combinations consistent with merger approval is

$$\left\{ (\beta'_0, \lambda') \mid \beta'_0 < -\left(\frac{\beta_1}{2CAP} + \frac{\lambda'}{\alpha_1}\right) Q_p + \left(1 + \frac{\beta_1}{\alpha_1 + CAP} \right) P_p - \beta_0 \frac{\beta_1}{\alpha_1 + CAP} - \left(1 + \frac{\beta_1}{\alpha_1 + CAP}\right) \beta_2 Z \right\}$$

3.1 Parameter Calibration

We use the parameter estimates in Jans & Rosenbaum (1996) as a guidance, and assume that the parameter values are known to be the following (tables are in Jans & Rosenbaum):

- Using Table 2a, we take $\alpha_0 = 5425 \approx \frac{5377+5471+5428}{3}$
- Using Table 2a, we take $\alpha_1 = 34 \approx \frac{33.23+35.02+34.29}{3}$
- Using Table 2b, we take $\beta'_0 = 55 \approx \frac{59.05+52.44+52.41}{3}$
- Using Table 2b, we take $\beta_1 = 15 \approx \frac{10.75+17.46+17.65}{3}$
- Using Appendix A and taking Detroit as our market of interest, we take $\lambda = 0.12$
- The rest of the numbers are made up. The numbers are for Detroit. First, from Appendix A in Detroit $Q = 3701$ and we pick $CAP = 4000$. Because in Detroit, $P = 46.7$ substitution in the market demand function gives

$$3701 = 5425 - 34(46.7) + \alpha_2 X$$

so that

$$\alpha_2 X = -136.2$$

5
Because

\[
P = \frac{\alpha_0}{\alpha_1 + \frac{3\beta_0}{cAP + \frac{\lambda}{\alpha_1}}} + \frac{\frac{3\beta_0}{cAP + \frac{\lambda}{\alpha_1}}}{\alpha_1 + \frac{3}{cAP + \frac{\lambda}{\alpha_1}}} \beta_2 Z + \frac{1}{\alpha_1 + \frac{3}{cAP + \frac{\lambda}{\alpha_1}}} \alpha_2 X
\]

we find upon substitution of the parameter values

\[
46.7 = \frac{5425}{34 + \frac{3}{1000 + \frac{0.17}{44}}} + \frac{\frac{3(55)}{3000 + \frac{0.17}{44}}}{34 + \frac{3}{3000 + \frac{0.17}{44}}} \beta_2 Z + \frac{1}{34 + \frac{3}{1000 + \frac{0.17}{44}}} (-136.2)
\]

so that

\[
\beta_2 Z = -17.28
\]

### 3.2 Merger Simulation: Numerical Illustration

If we substitute these parameter values in (1) we can express the post-merger price \( P (\lambda', \beta_0') \) as a function of the post-merger conduct \( \lambda' \) and the post-merger \( \beta_0' \). Therefore we can consider the merger approval as a function of these parameters.

1. Suppose that \( \beta_0' = 55 = \beta_0 \). Then the price is equal to 48.961 > 46.7 so that even if the conduct remains at \( \lambda = 0.12 \). There is no uncertainty in decision making: the merger is not approved (see Fig 1).

2. Suppose that \( \beta_0' = 50 < \beta_0 \). We then have

\[
P (0.12, 50) = 46.365 < 46.7
\]

but

\[
P (0.13, 50) = 46.882 > 46.7
\]

So the merger is approved if the competition in the market remains the same, but the decision changes for the minor deterioration of the conduct to \( \lambda = 0.13 \)

3. Suppose that \( \beta_0' = 40 < \beta_0 \). Then

\[
P (\lambda', 40) = 46.7
\]

for \( \lambda' = 0.2241 \) and the merger is approved if the competition decreases up to this level, but is not approved if the the post-merger conduct is known to be such that \( \lambda' > 0.2241 \).

We now note that the analysis above is predicted on the assumption that the post-merger conduct \( \lambda' \), as well as the post merger \( \beta_0' \), is known to the policy maker. Assume that the two firms petitioning for merger have produced an honest estimate of \( \beta_0' \). Whether the merger will be approved or not is now a function of the post-merger conduct \( \lambda' \), which is not known to the policy maker.

This scenario can be mapped into Manski’s (2011) framework. A policy maker can choose one of the two actions, i.e., either approve (A) or deny (D) the proposed merger. If her choice
(C) is to approve the merger, then the price would change to \( P(\lambda', \beta'_0) \), but if it is not approved, then the price does not change. Because the post merger \( \beta'_0 \) is assumed to be known, the only state of nature (S) that is ambiguous to the policy maker is the post-merger conduct parameter \( \lambda' \). For simplicity, suppose that the policy maker has the following simple utility function

\[
u(C, S) = \begin{cases} 
1 & \text{if } C = A \text{ and } P(\lambda', \beta'_0) < 46.7 \\
0 & \text{if } C = A \text{ and } P(\lambda', \beta'_0) = 46.7 \\
-1 & \text{if } C = A \text{ and } P(\lambda', \beta'_0) > 46.7 \\
0 & \text{if } C = D
\end{cases}
\]

In other words, the policy maker has the positive utility if the post-merger price is below the pre-merger price, 0 if it is unchanged, and -1 if the post-merger price is above the pre-merger price. Note that \( u(C, S) = 0 \) if the merger is denied, because the price does not change.

Suppose now that the policy maker adopts the maxmin criterion, i.e., she choose

\[
\max_C \min_S u(C, S).
\]

In the second and third scenarios above, there is an uncertainty in \( \lambda' \), but the maxmin criterion points to a simple solution; given that there are values of \( \lambda' \) that makes \( P(\lambda', \beta'_0) > 46.7 \), the merger should be denied.

Now suppose that the policy maker adopts the minimax-regret criterion, i.e., he choose to

\[
\min_C \max_S \left( \max_{C'} u(C', S) - u(C, S) \right)
\]

Consider \( \max_S (\max_{C'} u(C', S) - u(C, S)) \) for \( C = D \). Because \( u(D, S) = 0 \), this amounts to

\[
\max_S \left( \max_{C'} u(C', S) - u(D, S) \right) = \max_S \max_{C'} u(C', S)
\]

Suppose that \( \beta'_0 = 40 < \beta_0 \). Because \( P(\lambda', 40) < 46.7 \) for \( \lambda' < 0.2241 \), we see that

\[
\max_S \left( \max_{C'} u(C', S) - u(D, S) \right) = 1.
\]

Now consider \( \max_S (\max_{C'} u(C', S) - u(C, S)) \) for \( C = A \). Because \( P(\lambda', 40) > 46.7 \) for \( \lambda' > 0.2241 \), we also have

\[
\max_S \left( \max_{C'} u(C', S) - u(A, S) \right) = 1.
\]

In other words, both \( C = A \) and \( C = D \) results in the same value of \( \max_S (\max_{C'} u(C', S) - u(C, S)) \), and the policy maker should be indifferent between \( C = A \) and \( C = D \).

Finally, suppose that the policy maker adopts the expected utility criterion based on subjective distribution \( \pi \) on \( \lambda' \). As before, suppose that \( \beta'_0 = 40 < \beta_0 \). Given that \( P(\lambda', 40) = 46.7 \) for \( \lambda' = 0.2241 \), we can easily see that

\[
\int u(A, \lambda') \pi(d\lambda') \geq \int u(D, \lambda') \pi(d\lambda')
\]
if and only if \( \pi (\lambda' < 0.2241) \geq \pi (\lambda' > 0.2241) \). If the prior distribution \( \pi \) on \( \lambda' \) is uniform on \((0.12, 1)\), where 0.12 is the pre-merger conduct parameter, then we can see that

\[
\pi (\lambda' < 0.2241) = \frac{0.2241 - 0.12}{1 - 0.12} = 0.11830
\[
< 0.8817 = \frac{1 - 0.2241}{1 - 0.12} = \pi (\lambda' > 0.2241)
\]

and the merger would not be approved according to the expected utility criterion.

### 4 Summary

We mapped the merger simulation as a special case of dealing with uncertainty in the policy outcome. We summarized the uncertainty as arising from the post-merger conduct, and proposed a decision scientific analysis as in Manski (2011). We believe that extensions of our analysis in several dimensions would be useful. First, we may want to consider a more realistic specification of the utility function of the policy maker. We chose to work with a simple utility function only for simplicity. Second, it would be useful to combine parameter uncertainty in addition to the ambiguity in post-merger conduct.

### References


