1. Factory Planning

The Jong Wai Lei factory in Jung San can sell each unit of $x_1$ for $30 and each unit of $x_2$ for $12$. There are $b_1 = 150$ hours per day of available machine time (constraint 1) and $b_2 = 200$ hours per day of available finishing time (constraint 2). The matrix of unit requirements,

$$[ a_{ij} ] = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$$

where $a_{ij}$ is the time required of input $i$ to produce a unit of output $j$.

(a) Draw a neat figure to solve the problem and hence show that maximized revenue is $1950.

(b) Write down and interpret the dual problem. Draw a neat figure to solve the dual (minimization) problem. Confirm that minimized imputed cost is equal to maximized revenue.

(c) Appealing to part (b), show that there is a range of values of machine time ($b_1$) over which the shadow prices are unaffected. What is this range? Show also that outside this range one of the shadow prices drops to zero.

(d) Try to explain this by showing the effect of increasing $b_1$ in a copy of the figure you used in part (a).

(e) Suppose packing time is also limited ($a_{31} = 2, a_{32} = 3$). If there are 400 hours of packing time, is the optimum affected? If so, how? If not, why not?

(f) What is the imputed value of packing time?

(g) If you, as manager, could obtain additional machine time at a cost of $7 per hour, would this be a good idea? Explain. How many units would you purchase? What if you could obtain additional finishing time at the same cost?

(h) Henceforth ignore any constraint on packing time. A third product, $x_3$ has input requirements $a_{13} = 2, a_{23} = 3$. Each unit sells for $28$. Write out the new maximization problem and also the new dual problem.

(i) Solve the dual problem graphically.

(j) Which duel constraint is an inequality constraint at the minimum? What level of output is optimal if (imputed) cost strictly exceeds revenue per unit?

(k) Make use of your answer to part (i) to draw a conclusion as to the optimal choice of $x_1, x_2$ and $x_3$.

Filename: 2001SHW1.doc