EC 104 Pricing and Strategy             John G. Riley
Spring 2001

HOMEWORK SET #2   Due at beginning of class, Monday Week 4

1.   Factory planning

Return to the problem of Homework 1.

(a) Solve part (a) using SOLVER. First download 2001_LP1.XLS. Then copy it as
YOURNAME.XLS Since the problem is 2x2 you can easily modify the data sheet to
solve. Submit a copy of your set-up page as well as a Sensitivity Report. Make sure you
click on “Options” in SOLVER and choose “Linear Model.” You will then get lots more
information when you get your sensitivity report.

(b) Compute revenue and the shadow cost of machine shop time for various values of machine
shop time (up to 500) and depict both as a function of machine shop time.

(c) Why does the revenue figure have three linear segments with kinks (one at b₁ = 50.)

(d) What is the connection between the revenue figure and shadow prices?

2.   Product Choice

Lowe Ying Lee Corp. can sell xₖ units of product j at a price of pₖ j =1,...,4. There are
three inputs used in the production of these products: input 1 - machine time, input 2 -
finishing time and input 3 - packing time. The matrix of input requirements [aᵢᵢ] is

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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
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<td>1</td>
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<td>2</td>
<td>4</td>
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<td>2</td>
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<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
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where aᵢᵢ is the number of units of input i needed to produce each unit of product j. There are
50 units of input 1 available, 140 units of input 2, and 90 units of input 3.

(a) If the output price vector (p₁, p₂, p₃, p₄) = (6,3,4,6) solve for the optimal choice of
products and hence show that maximized revenue is $240.
(You should set up the problem on a spreadsheet in such a way as to make it easy to change
parameter values. You will need to add a product to the 3x3 data sheet provided in
YOURNAME2.XLS. Submit a copy of your set-up page as well as your Sensitivity Report.)

(b) What are the shadow prices? Give an interpretation.
(c) Write down the dual Linear Programming Problem. Given the solution to (a) explain why it is not difficult to solve analytically for the shadow prices in this case? Solve and compare your answer with that in part (b).

(d) Looking back at the data, why is it “obvious” that \( x_4 \) must be zero?

(e) Use SOLVER to determine the lowest price \( p_4 \) at which it pays to produce a positive amount of product 4.

(f) At what price \( p_4 \) is it optimal to produce only product 4?

(g) Returning to the initial problem (with \( p_4 = 6 \)) re-solve the problem for different levels of finishing time (from 0 to 20). Plot finishing time on the horizontal axis and revenue on the vertical axis. How many kinks are there? What is the interpretation of the slope?

(h) In each interval use SOLVER to determine the shadow price of finishing time. Link this to your answer to part (g).

(i) If additional units of finishing time can be provided at $6 per unit, how many should be obtained?

3. Transportation Problem

The Hang Seng Co. has three pea canneries (in Canton, Auckland and Manila). The cans are all shipped to one of 4 distribution centers (in Los Angeles, Toronto, New York and Dallas). The following table indicates the capacity of each cannery, the allocation assigned to each distribution center and the cost per unit \( t_{ij} \) of shipping from source \( i \) to destination \( j \).

<table>
<thead>
<tr>
<th>Source i</th>
<th>destination j</th>
<th>capacity of source i</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>464 650 654 486</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>515 416 432 791</td>
<td>125</td>
</tr>
<tr>
<td>3</td>
<td>995 682 388 685</td>
<td>100</td>
</tr>
<tr>
<td>Allocation to destination j</td>
<td>80</td>
<td>65</td>
</tr>
</tbody>
</table>

(a) Write down the cost minimization problem. (There are seven constraints.)

(b) Explain why these constraints must, taken together, hold with equality.
(c) Expand the SOLVER program to allow for 3 rows and 4 columns. (You might wish to copy one of your current sheets to create a new sheet. To do so choose [Edit] then [Copy Sheet] then click on the box which says ‘Create a copy’. You can then choose where the copy of the sheet should go. Then solve numerically. (As a check, \( x_{11} = 20 \).)

(d) Present your results on a sensitivity sheet. Interpret the shadow prices.

(e) Determine numerically the effect on cost of increasing the allocation to destination 2 by 10 and reducing the allocation to destination 3 by 10.

4. Maximum Flow

The figure below shows the capacity of various channels of an electricity grid. Let \( x_i \) be the flow to the right along channel \( i, i=1,\ldots,8 \).

(a) Explain why there are 8 constraints of the form \(-c_i \leq x_i \leq c_i\). Write down the other (equality) constraints and the maximand if the objective is to solve for the maximum flow.

(b) Solve the problem using SOLVER. You should submit a printout of your set-up page as well as the Sensitivity Report.

(c) Interpret the shadow prices.

(d) How does the solution change if the capacity of channel \( N_1N_2 \) currently at 20 is increased by 10?