1. **Joint Costs**

A firm faces the following demand price functions for its two products.

\[ p_1 = 500 - 10q_1, \quad p_2 = 400 - 6q_2. \]

The production costs of the two products are

\[ C_1 = 20q_1, \quad C_2 = 40q_2. \]

Both products are produced from an input which has a cost \( C_0(q_0) = 60q_0 \). Each unit of the input yields enough to produce one unit of each product. Thus \( q_1 \leq q_0 \) and \( q_2 \leq q_0 \).

(a) Solve for the profit maximizing outputs and prices.
(b) Solve for the social surplus maximizing outputs and prices.
(c) Solve for the profit maximizing outputs if the "capacity cost" rises to \( C_0 = 136q_0 \).

2. **Peak-load pricing**

A monopoly supplier of electricity faces the following day and night demands.

\[ p_1 = 580 - 9q_1, \quad p_2 = 400 - 4q_2 \]

The operating cost of producing electricity is \( C_1 = 100q_1 + q_1^2 \) and \( C_2 = 40q_2 + 2q_2^2 \). The cost of turbine capacity is \( C_0 = 60q_0 \).

(a) Solve for the profit maximizing outputs and prices.
(b) Can you see why the outputs are the same as those in question 1?
(c) Solve for the profit maximizing outputs if the capacity cost rises to \( C_0 = 136q_0 \).

3. **Peak-load pricing with interdependent demands**

The government takes over the local electrical utility and you are asked to choose the levels of electricity production and prices. The day is divided into 2 sub-periods "night" and "day." Your goal is to maximize social surplus.

Demand price functions are as follows.

\[ p_1 = 42 - b_1 \cdot q, \quad p_2 = 88 - b_2 \cdot q \quad \text{where} \]
and \( x \cdot y \) is the "sumproduct" of the arrays \( x \) and \( y \).

The operating cost in each period is 10 per unit. In addition, the capital cost is 20 per unit per day.

(a) Consider the benefit function

\[
B(q_1, q_2) = (42 - \frac{1}{2} b_1 \cdot q)q_1 + (88 - \frac{1}{2} b_2 \cdot q)q_2
\]

Write out this expression and differentiate it by \( q_1 \) and \( q_2 \) to show that the marginal benefits are indeed the demand price functions.

(b) Solve for the social surplus maximizing prices and quantities.

(c) Suppose that the local government becomes short of funds and asks you to maximize the profit of the utility. What would be the new outputs and prices?

(d) Modify your analysis to take account of the following constraint. The local government indicates that the period 2 price, \( p_2 \), cannot exceed \( d + p_1 \), where \( d \) is a parameter. Solve for the new profit maximum if the parameter \( d = 0 \).

(e) Give an interpretation of the shadow prices printed on the sensitivity sheet.

Please note that you may hand in answers to the first two problems which show how you got your answers using either graphical/analytical methods or a spread-sheet. The third problem is intended as a spread-sheet problem. However I encourage you to try to solve the first two problems both ways.

The homework is due on the Monday of week 6 at the beginning of class.