2001 sample answers

1. Non Linear Pricing

(b) \( p_L(5) = 25 - 2(5) = 15, \quad p_H(10) = 50 - 4(10) = 10 \)

(c) Need to show the Low demander, with flatter indifference curve preferring (5,30) to (10,50) and (0,0). Also the High demander preferring the larger bundle.

(d) \( U_i(q,R) = B_i(q) - \frac{R}{payment} \)

\( U_L(5,30) = B_L(5) - 30 = 70 \)
\( U_H(5,30) = B_H(5) - 30 = 170 \)
\( U_H(20,50) = B_H(10) - 50 = 250 \)

Suppose that we increase the payment on both bundles by 70, that is, to 100 and 120. Then

\( U_L(5,100) = B_L(5) - 100 = 0 \)
\( U_H(5,100) = B_H(5) - 100 = 100 \)
\( U_H(10,120) = B_H(10) - 120 = 180 \)

We cannot squeeze the Low types any further. However the High types are willing to pay 80 more for the big bundle. Thus we can raise the payment for the big bundle to 200.

(e) There is a more profitable alternative. The package for the high demander is optimally set so that \( MB_H(q_H) = p_H(q_H) = c = 6 \). Since \( p_H(10) = 10 \) the profit maximizing bundle is larger.

(f) In this case squeeze the little guys out completely and offer only one package. The size of this package satisfies the conditions of part (e). Using the data of part (b)

\( p_H(q_H) = 50 - 4q_H = 6 \).

Thus \( q_H = 11 \).

2. Bidding

(a) If my opponent uses the strategy \( b_2 = a_2v_2 \), I win if my bid \( b \) is larger. That is

\( b_2 = a_2v_2 < b \), that is \( v_2 < b/a_2 \).

Given the even (uniform) distribution, \( \Pr\{v_2 < x\} = x/100 \). Thus

\( \Pr\{v_2 < b/100\} = b/a_2100 \).
(b) \( U = \Pr\{\text{win with bid of } b\}(v_1 - b) = b(v_1 - b) \frac{1}{100a_2} \).

(c) \( \frac{\partial U}{\partial b} = (v_1 - 2b) \frac{1}{100a_2} = 0 \) at \( b = \frac{1}{2} v_2 \).

(d) Best reply to any linear strategy of buyer 2 is for buyer 1 to bid half her valuation. It follows immediately that if both use the strategy “bid half your valuation” each strategy is a best response.

(e) With 3 bidders, I win with a bid of \( b \) if both bid less, that is my win probability is \( \Pr\{b_2 < b\} \Pr\{b_3 < b\} = (b/a_2100)(b/a_3100) = b^2 /10,000a_2a_3 \).

(f) \( U = \Pr\{\text{win with bid of } b\}(v_1 - b) = b^2(v_1 - b) \frac{1}{10,000a_2a_3} \).

(g) \( \frac{\partial U}{\partial b} = \frac{1}{10,000a_2a_3} \frac{\partial}{\partial b} (b^2v_1 - b^3) = \frac{1}{10,000a_2a_3} (2bv_1 - 3b^2) \)
\( = 0 \) at \( b = \frac{1}{3} v_1 \).

(h) Arguing as above, each bidder’s best response to any linear strategy is to bid two-thirds of his valuation.

3. Joint Costs and Fixed Costs

\[ MR_1 = 110 - 2q_1, \quad \text{then } MNR_1 = 100 - 2q_1 \]
\[ MR_2 = 120 - 4q_2, \quad \text{then } MNR_2 = 100 - 4q_2 \]
\[ MR_3 = 200 - 2q_3, \quad \text{then } MNR_3 = 160 - 2q_3 \]

Note that, for any \( q \) \( NMR_3(q) \) is at least 60 greater. Thus the other products do not use all the logs. Hence, for the profit maximum,

\[ MNR_1 = 100 - 2q_1 = 0. \quad \text{Solving, } q_1 = 50. \]
\[ MNR_2 = 100 - 4q_2 = 0. \quad \text{Solving, } q_2 = 25. \]
\[ MNR_3 = 160 - 2q_3 = 40. \quad \text{Solving, } q_3 = 60. \]

Profits are computed as follows:

Net revenue

\( (p_1 - c_1)q_1 = 50 \times 50 = 2500, \)
\( (p_2 - c_2)q_2 = 50 \times 25 = 1250, \)
\( (p_3 - c_3)q_3 = (200 - 60 - 40) \times 60 = 6000 \)

Capacity Cost = \( 60 \times 40 = 2400 \)

(c) All products are profitable.
(d) Eliminate product 2 since \( F_2 > 1250 \)
It is tempting to say “eliminate product 3” since, if we subtract off the capacity costs in the peak period, \( NR_3 - C_0 = 3600 \) and the fixed cost \( F_3 = 4000 \). However, period 3 net revenue is 6,000, more than enough to cover the fixed cost \( F_3 \). So the profit maximizing plan is unaffected.

4. Transportation Problem

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<th>Final Value</th>
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(a) Using the program, the sources are A,B,C.

Destination constraints:

\[ x_{A1} + x_{B1} + x_{C1} \geq 150 \]
\[ x_{A2} + x_{B2} + x_{C2} \geq 150 \]
source constraints

\[ x_{A1} + x_{A2} \leq 75 \]
\[ x_{B1} + x_{A2} \leq 125 \]
\[ x_{C1} + x_{C2} \leq 110 \]

All shipments must be non-negative.

Minimize \( T = \sum t_{ij} x_{ij} \)

(b) \( \lambda_i \) is the shadow price of a purchase at source \( i \) (the reduction in shipping costs if there is one additional unit at this source.) \( \mu_j \) is the shadow price of a unit at a destination (the reduction in cost if there is one less unit demanded at this destination.)

If the shipper pays the source cost and receives the destination price, he breaks even or loses money on all routes and does not ship on any loss making route.

(c) \( \mu_j \leq \lambda_i + r_{ij} \) with equality on routes that are used.

(d) See above \( T=1400 \)

(e) This is in the range of allowable increase thus the quantities shipped do not change.

(f) Shipments from A to 1 are 75 so cost goes up by 150. This is easily checked on Excel.

(g) This is in the range of allowable increase thus the shadow prices do not change. For every unit, \( T \) goes down by \( 4 \) so \( \lambda_i = 4 \) thus \( T \) drops by \( 20 \times 4 = 80 \).

5. Choice of Technique

(a) Let \( x_j \) be the output using technique \( j \). Total output is

\[ x_1 + x_2. \quad \text{Slope = -1} \]

Capital use \( 4x_1 + 2x_2 \leq K \) slope = -2

Labor use \( 2x_1 + 4x_2 \leq L \) slope = -1/2

(b) Given the slopes, both constraints will be binding

\[ 4x_1 + 2x_2 = 70 \]
\[ 2x_1 + 4x_2 = 50 \]

subtracting: \( 3x_1 = 45 \) \( x_1 = 15 \), \( x_2 = 5 \)

(c) Dual Problem

Shadow prices are shadow costs of resources. Value measured in units of output.
\[ MR_1 \leq MC_1 : \quad 1 \leq 4\lambda_1 + 2\lambda_2, \quad \text{Slope} = -2 \]
\[ MR_2 \leq MC_2 : \quad 1 \leq 2\lambda_1 + 4\lambda_2, \quad \text{Slope} = -1/2 \]

Minimize \( 70\lambda_1 + 50\lambda_2 \quad \text{Slope} = -\bar{K}/\bar{L} = -70/50. \)

Given slopes, both constraints are binding. Thus

\[
\begin{align*}
4\lambda_1 + 2\lambda_2 &= 1 \\
2\lambda_1 + 4\lambda_2 &= 1
\end{align*}
\]
Solving \( \lambda_1 = \lambda_2 = 1/6. \)

Slope \( -\bar{K}/\bar{L} \) can vary from \(-1/2\) to \(-2\).

(d) Shadow prices are unaffected. Thus the solution of the dual is \( 70\lambda_1 + 80\lambda_2 = 150(\frac{1}{6}) = 25. \) Since this must be the same as the solution of the original problem it follows that \( x_1^* + x_2^* = 25. \)

6. Prisoner’s Dilemma

(a) NE pair of strategies are each best responses to the other.
(b) For each player A is a dominated strategy.
(c) Round 2 is the same as a 1 round game. But once we know what will happen in round 2, the same logic applies to round 1.
(d) Again A and B are dominated strategies so the unique NE is C.
(e) Yes, arguing as in (c).
(f) Not credible. For if Player 1 uses B in round 2 he knows Player 2 will choose C. But if Player 2 chooses C, player1’s best response is not B after all.

A more interesting case is the following modification which we discussed in class.

\[
\begin{array}{c|c|c|c}
\text{Player 2} & \text{A} & \text{B} & \text{C} \\
\hline
\text{A} & 20, 20 & 5, 25 & 0, 0 \\
\text{B} & 25, 5 & 10, 10 & 0, 0 \\
\text{C} & 0, 0 & 0, 0 & 1,1 \\
\end{array}
\]

Now there are 2 Nash equilibria in pure strategies: (B,B) and (C,C).

You should check and show that there is also a mixed strategy equilibrium in which the mix is over B and C. (Since A is dominated, it will never be played.)

(g) Suppose Player 1 adopts his proposed strategy. If Player 2 chooses A in round 1 and B in round 2 he gets 20+10=30. If he chooses B in round 1 his best response in round 2 is C so he ends up with 25+1 =26. Thus Player 2’s best response is (A,B). Arguing symmetrically, if Player 2 announces the same rule as player 1, each end up playing (A,B) and get 30 each.
(h) The problem with equilibria like this is that both parties have an incentive to renegotiate when round 2 begins if a player “cheats” in round 1.

7. Monopoly with Asymmetric Information

Consumer benefit $B_L(q) = \int_0^q p_L(x)dx = 100q - q^2$, $B_H(q) = \int_0^q p_H(x)dx = 200q - \frac{1}{2}q^2$

If a consumer pays $R$ it follows that his net payoff or consumer surplus is

$U_L(q, R) = 100q - q^2 - R$ and $U_H(q, R) = 200q - \frac{1}{2}q^2 - R$.

(a) Choose a payment to extract all consumer surplus.

For type L,

$U_L(q, R) = 100q - q^2 - R = 0$. Then Revenue $R = 100q - q^2$ Cost is $10q$

For type H,

$U_H(q, R) = 200q - \frac{1}{2}q^2 - R = 0$. Then Revenue $R = 200q - \frac{1}{2}q^2$ Cost is $10q$.

Choose $q$ so that $MR = MC$.

$100 - 2q_L = 10$, $q_L = 45$

$200 - q_H = 10$, $q_H = 190$.

(b) The solution is depicted in the figure below. First fix the quantities and then squeeze each buyer types as hard as possible. The low type is just indifferent between $(q_L, R_L)$ and $(0, 0)$. The high type is just indifferent between the high and low quantities. This determines the payments $R_L$ and $R_H$. Step 2 is to note that it pays to increase $q_H$ until $MR = MC$, exactly as in part (a). Step 3 is to recognize that in choosing $q_L$ there is a tradeoff. A higher $q_L$ extracts more from the low demanders (since they are willing to pay more) but is more attractive to high demanders so they are more willing to switch to the small bundle. Thus the more H types there are, the lower is the profit maximizing $q_L$. 
