EC 104 Pricing and Strategy             John G. Riley
Fall 2002

HOMEWORK SET #1       Due at beginning of class on Tuesday of week 3.

1.   Factory Planning

The Fong Lowe factory can produce 5 types of chair. The price per unit for each chair type is
given below.

<table>
<thead>
<tr>
<th>Price of x₁</th>
<th>Price of x₂</th>
<th>Price of x₃</th>
<th>Price of x₄</th>
<th>Price of x₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>80</td>
<td>35</td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

There are 100 hours per day of available machine time (constraint 1) and 140 hours per day of
available sanding time (constraint 2). The matrix of unit requirements is ,

<table>
<thead>
<tr>
<th>Machine time (hours)</th>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
<th>x₄</th>
<th>x₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Sanding time (hours)</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Write down a table of “costs per dollar of revenue.” For example consider product 1.
The machine time cost of generating 1 unit of output and hence $60 is 2. Thus the
machine time cost per dollar of revenue is 2/60. Also the sanding time cost per dollar
of revenue is 1/60. Given your answer, which types of chair will not be produced?

In the next sections assume that the prices on chairs 3-5 are so low that only the first 2 chairs
are under consideration.
(b) Draw a neat diagram and hence solve for the optimal number of each type of chair.
(c) Suppose we assign shadow prices to each constraint. What are the conditions that
must be satisfied if these prices are to support the optimum?
(d) Solve for the shadow prices.
(e) By how much can the price of a type 1 chair rise or fall without there being any effect
on the optimal production plan.
(f) Discuss what happens when the price of chair 1 moves outside this range. Appeal to
a neat figure to clarify your answer.
(g) If you, as manager, could obtain additional machine time at a cost of $18 per hour,
would this be a good idea? Explain. What if you could obtain additional finishing
time at the same cost?
(h) By how much would the price of a type 3 chair have to rise before it was profitable to
switch production?
### 2. Factory Planning with more than 2 products

Consider the same problem with the following new data.

<table>
<thead>
<tr>
<th>Price of $x_1$</th>
<th>Price of $x_2$</th>
<th>Price of $x_3$</th>
<th>Price of $x_4$</th>
<th>Price of $x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>80</td>
<td>70</td>
<td>100</td>
<td>170</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine time (hours)</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Sanding time (hours)</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Finishing time (hours)</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

There are 120 hours of finishing time available.

(a) Use solver to compute the revenue maximizing outputs of the 5 types of chair.
(b) What are the shadow prices for each constraint?
(c) How much could you reduce finishing time without affecting revenue?
(d) What would you do if you could rent (or rent out) machine time at $20?  
(e) What would your answer be if could rent (or rent out) machine time at $50?  
(f) What would your answer be if could rent (or rent out) machine time at $80?  
(g) Depict in a neat figure (with machine hours on the horizontal axis) the marginal revenue of additional machine time.

Returning to the original data of this question, what happens to the optimal outputs and revenue if the price of all 5 types of chair doubles?