1. Product Choice

Lowe Ying Lee Corp. can sell \( x_j \) units of product \( j \) at a price of \( p_j \), \( j = 1, \ldots, 4 \). There are three inputs used in the production of these products: input 1 - machine time, input 2 - finishing time and input 3 - packing time. The matrix of input requirements is

\[
\begin{array}{cccc}
1 & 1 & 2 & 4 \\
4 & 2 & 1 & 2 \\
2 & 1 & 1 & 3 \\
\end{array}
\]

where \( a_{ij} \) is the number of units of input \( i \) needed to produce each unit of product \( j \). There are 50 units of input 1 available, 140 units of input 2, and 90 units of input 3.

(a) If the output price vector \((p_1, p_2, p_3, p_4) = (6, 6, 10, 15)\) solve for the optimal choice of products and hence solve for maximized revenue. (You should set up the problem on a spreadsheet in such a way as to make it easy to change parameter values. Submit a copy of your set-up page as well as your Sensitivity Report.)

(b) What are the shadow prices? Give an interpretation.

(c) Write down the dual Linear Programming Problem. Given the solution to (a) explain why it is not difficult to solve analytically for the shadow prices in this case.? Solve and compare your answer with that in part (b).

(d) Looking back at the data, why is it “obvious” that \( x_4 \) must be zero?

(e) Use SOLVER to determine the lowest price \( p_4 \) at which it pays to produce a positive amount of product 4.

(f) To what price must \( p_4 \) rise before it is optimal to produce only product 4?

(g) Returning to the initial problem (with \( p_4 = 6 \)) plot finishing time on the horizontal axis and the shadow price of finishing time on the vertical axis. How many jumps are there?

(h) Also draw a neat figure underneath with finishing time on the horizontal axis and revenue on the vertical axis. How many kinks or jumps does this plot have?

(i) In simple words explain the jumps and kinks on parts (g) and (h).
(i) If additional units of finishing time can be provided at $6 per unit, how many should be obtained?

2. Transportation Problem

The Hang Seng Co. has three pea canneries (in Canton, Auckland and Manila). The cans are all shipped to one of 4 distribution centers (in Los Angeles, Toronto, New York and Dallas). The following table indicates the capacity of each cannery, the allocation assigned to each distribution center and the cost per unit $t_{ij}$ of shipping from source $i$ to destination $j$.

<table>
<thead>
<tr>
<th>Source i</th>
<th>destination j</th>
<th>capacity of source i</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>463 650 654 486 75</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>515 422 441 791 130</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>995 682 388 685 100</td>
<td></td>
</tr>
</tbody>
</table>

Allocation to destination j

| 80 65 70 85 |

(a) Write down the cost minimization problem. (There are seven constraints.)

(b) Enter the data in Solver and then enter an initial feasible set of shipments.

(c) What would be the effect on revenue of reducing supply by 5 at (i) source 1 (Canton) (ii) Source 2 (iii) source 3.

(d) Do shadow prices help to answer part (d)? Is it surprising that one of the shadow prices is zero? Explain.

(d) Present your results on a sensitivity sheet. Interpret the shadow prices.

(e) What is the imputed profit or loss on shipping from canton to Toronto?

(f) If the cost of shipping from Canton to Toronto falls by (i) 250 (ii) 300 what is the effect on the solution?
3. **Maximum Flow**

The figure below shows the capacity of various channels of a water supply grid. Let \( x_{ij} \) be the flow to from node \( i \) to node \( j \).

(a) Explain why there are 8 constraints of the form \( x_{ij} \leq c_{ij} \) and 8 more of the form \( x_{ij} \geq -c_{ij} \).

(b) You may assume that there is an overflow valve at each node so that the total flowing away from a node is less than or equal to the inflow. Write down the constraint at each node (inflow – outflow \( \geq 0 \)) and the maximand if the objective is to solve for the maximum flow from \( N_1 \) to \( N_6 \).

(c) Solve the problem using SOLVER. You should submit a printout of your set-up page as well as the Sensitivity Report.

(d) Interpret the shadow prices. Explain each of the zero prices and why the others are all the same.

(e) How does the solution change if the capacity of channel \( N_1N_2 \) currently at 20 is increased by 10?