**Strategic Production Game**

Consider two firms, which have to make production decisions without knowing what the other is doing. For simplicity we shall suppose that the product is essentially identical. If outputs levels are $q_1$ and $q_2$ so that the total supply is $q = q_1 + q_2$, the market-clearing price is

$$p = a - bq = a - b(q_1 + q_2)$$

Firm i has a production cost of $C_i = c_i q_i$. Then if firm 2 produces $q_2$, firm 1’s profit is

$$\Pi_i(q_1, q_2) = pq_i - c_i q_i = (a - c_i - bq_2)q_i - bq_i^2$$

Equal-profit curves for firm 1 are depicted in Figure E-1. To understand why they must have the indicated shape, first note that as $q_2$ increases, firm 1’s profit must decline. Expressing this mathematically,

$$\frac{\partial \Pi_i}{\partial q_2} < 0$$

Next differentiating firm 1’s profit by $q_1$,

$$\frac{\partial \Pi_i}{\partial q_1} = (a - c_i - bq_2) - 2bq_i = 2b[(a - c_i - bq_2) / 2b - q_i]$$

Hence $\frac{\partial \Pi_i}{\partial q_1} > 0$ if $q_i < (a - c_i - bq_2) / 2b$

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1 The earliest discussion of this game goes all the way back to Augustin Cournot (1848).
Figure E-1: Iso-profit curves for firm 1

\[ \frac{\partial \Pi}{\partial q_i} > 0 \] if \( q_i < (a-c_i - bq_2)/2b \)

Best response:

Choose output such that marginal profit \( \frac{\partial \Pi}{\partial q_i} \) is zero.

\[ BR_1 : \quad q_i^B = (a-c_i - bq_2)/2b \]

Similarly for player 2:

\[ BR_2 : \quad q_2^B = (a-c_2 - bq_1)/2b \]
For equilibrium,

\[ BR_1 : \quad q_1^{BR} = \frac{(a - c_1 - bq_2)}{2b} = q_1 \]

Similarly for player 2:

\[ BR_2 : \quad q_2^{BR} = \frac{(a - c_2 - bq_1)}{2b} = q_2 \]

Rearranging,

\[ 2bq_1 + bq_2 = a - c_1 \quad \times 2 \quad 4bq_1 + 2bq_2 = 2(a - c_1) \]

and

\[ q_1 + 2bq_2 = a - c_2 \quad \times 1 \quad q_1 + 2bq_2 = a - c_2 \]

Subtracting:

\[ 3bq_1 = 2(a - c_1) + (a - c_2) \]

Arguing symmetrically:

\[ 3bq_2 = (a - c_1) + 2(a - c_2) \]

Dividing by 3 and summing:

\[ b(q_1 + q_2) = \frac{1}{3}(a - c_1) + \frac{1}{3}(a - c_2) \]

Then the equilibrium price

\[ p = a - b(q_1 + q_2) = \frac{1}{3}a + \frac{1}{3}c_1 + \frac{1}{3}c_2 \]

Class Exercise: With 3 firms, and \( c_1 < c_2 < c_3 \) under what conditions will there be (i) 1 firm (ii) 2 firms producing a positive output?
N identical firms

\[ p = a - bQ \]
\[ Q = q_i + z_i, \quad z_i = \text{output of all other firms} \]

profit of firm 1, \( \Pi_i = pq_i - cq_i = (p - c)q_i = (a - c - b(q_i + z_i))q_i \)

\[ = (a - c - bz_i)q_i - bq_i^2 \]

marginal profit, \( \frac{\partial \Pi}{\partial q_i} = a - c - bz_i - 2bq_i. \)

Then firm 1’s best response \( BR_i : q_i^{BR} = \frac{1}{2b}(a - c - bz_i). \)

Given the symmetry we seek a solution in which each firm chooses the same output.

Then \( q_i = \frac{1}{N}Q \) and \( z_i = \frac{N-1}{N}Q \)

But \( 2bq_i = a - c - bz_i \)

Substituting, \( \frac{2}{N}Q = a - c - \frac{N-1}{N}Q. \) Hence \( \frac{N+1}{N}bQ = a - c \)

Then

\[ Q = \frac{N}{N+1} \frac{(a-c)}{b} \]

and

\[ p = a - bQ = (\frac{1}{N+1})a + (\frac{N}{N+1})c \]

Note that as the number of firms rises, the price falls. In the limit as the number of firms gets large, the equilibrium price approaches the unit cost \( c \).
Sequential rather than simultaneous play

The oligopoly game is a natural one to use to introduce games in which moves are sequential, rather than simultaneous. (The classic childhood sequential game is 0’s and X’s or tic-tac-toe)

If firms are very similar in size, the simultaneous model is more likely to be appropriate. However, when one firm is larger, or the incumbent, it may be in a position to make the first move, leaving other firms to follow.

What should the first mover do?

RULE: Look ahead and work backwards

Once firm 1 has made its move, firm 2 simply gets to maximize profit conditional upon firm 1’s move. That is, it will choose its best response:

\[ q_2^* = \frac{(a - c_2 - bq_1)}{2b} \]  (Best Response by firm 2)

Figure F-1: Leader-Follower equilibrium

has the same best response rule as before, firm 1 is not simply responding to firm 2’s choice. Suppose that firm 1 initially chooses the same output as in the simultaneous move game. Then the outcome is the
point E in the figure. But firm 1 knows that firm 2 will choose a response along the heavy response curve. Thus firm 1 optimizes by picking the most profitable point on this curve. As depicted it is the point A.

Example:

\[ a = 100, \ b = 1, \ c_1 = c_2 = 10. \]

**Simultaneous moves:**

\[
BR_1 : \quad q_2^\star = (a - c_1 - bq_2) / 2b = (90 - q_2) / 2
\]

\[
BR_2 : \quad q_2^\star = (a - c_2 - bq_1) / 2b = (90 - q_1) / 2
\]

Solving, \( q_1 = q_2 = 30 \)

**Sequential moves:**

\[
BR_2 : \quad q_2^\star = (a - c_2 - bq_1) / 2b = (90 - q_1) / 2
\]

\[ p = a - b(q_1 + q_2) = 100 - (q_1 + \frac{1}{2}(90 - q_1)) = 55 - \frac{1}{2}q_1 \]

\[ \Pi_1 = pq_1 - cq_1 = (p - c)q_1 = (45 - \frac{1}{2}q_1)q_1. \]

Differentiating,

\[
\frac{\partial \Pi}{\partial q_1} = 45 - q_1. \quad \text{Thus} \quad q_1^\star = 45 \quad \text{and} \quad q_2^\star = (90 - q_1) / 2 = 22.5
\]
Entry Game

Incumbent monopolist and a potential entrant.

Incumbents payoffs indicated first.

Nash equilibrium 1:
Incumbent announces that he will fight if there is entry. Entrant’s BR is to choose OUT. Given that Entrant stays OUT, the strategy FIGHT is a BR for the Incumbent.

Nash equilibrium 2:
Entrant believes Incumbent will share. Then BR is COME IN. Given that Entrant does come in, Incumbent’s BR is SHARE.

Which equilibrium is more plausible?

If you look down the tree and ask what each player would actually do at a decision node, one equilibrium is eliminated