STRATEGIC DECISION-MAKING

or

“How to play the game.”

“Business is a game - - the greatest game in the world if you know how to play it!”

Thomas J. Watson (founder of I.B.M.)
A. Elements of a Game

Players

We will label the $n$ players/agents/participants $i=1,...,n$

**Strategy Set** $S_i$ -- the set of all feasible choices/alternatives/options available to player $i$.

Example 1: Rock, Scissors, Paper (Bau, Jin, Dup)

$S_i = \{\text{Paper, Scissors, Rock}\}$

Example 2: Open bidding for an art object

A strategy is a choice of when to drop out of the bidding.

$S_i = \{\text{all possible bids called by the auctioneer}\}$

Player $i$, $i=1,...,n$ plays some strategy $s_i$ from his strategy set $S_i$

**Pure strategy:** A single choice from $S_i$

**Mixed strategy:** Player $i$ plays probabilistically, assigning positive probabilities to some of the alternatives in $S_i$

Note that while each player must play a single strategy $s_i$ in $S_i$, he can choose to play a mixed strategy using some form of randomizing device such as a coin toss or throw of a die.

Example: Rock, Scissors Paper

As everyone who ever played it quickly realizes, it pays to randomize.

HOW?
Payoffs

If the strategies actually chosen by the $n$ players are $s_1, s_2, \ldots, s_n$, then the outcome of the game leads to a payoff for each player. We write this as

$$u_i(s_1, \ldots, s_n).$$

Rules of play - Timing of Moves

Sequential or simultaneous

Equivalently, each player moves sequentially but choices must be made without knowing the choices of those who have already moved.

Example: Rock, Scissors, Paper

The payoffs for a 2 person game can be conveniently depicted in a matrix as shown in Table B-1. Player 1’s payoffs are denoted in italic and player 2’s in bold.

<table>
<thead>
<tr>
<th>Player 2</th>
<th>Paper</th>
<th>Scissors</th>
<th>Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper</td>
<td>0,0</td>
<td>-10,10</td>
<td>10,-10</td>
</tr>
<tr>
<td>Scissors</td>
<td>10,-10</td>
<td>0,0</td>
<td>-10,10</td>
</tr>
<tr>
<td>Rock</td>
<td>-10,10</td>
<td>10,-10</td>
<td>0,0</td>
</tr>
</tbody>
</table>

TABLE B-1: Rock, Scissors, Paper Game in “Normal form”

<table>
<thead>
<tr>
<th>Player 2</th>
<th>Paper</th>
<th>Scissors</th>
<th>Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Paper</td>
<td>0,0</td>
<td>-30,30</td>
</tr>
<tr>
<td>q</td>
<td>Scissors</td>
<td>30,-30</td>
<td>0,0</td>
</tr>
<tr>
<td>1-p-q</td>
<td>Rock</td>
<td>-10,10</td>
<td>10,-10</td>
</tr>
</tbody>
</table>

TABLE B-1: Modified Rock, Scissors, Paper Game
Description of the Game in “tree” form

From the initial “node” of the tree, player 1 chooses an action. The game then continues at one of three nodes for player 2. She then takes her action. The payoffs are indicated at the “terminal” nodes.

Figure B-1: Tree representation of the game if played sequentially

If play is sequential, the tree representation of the game is often very helpful. What will player 2 do?
Player 1 is thus indifferent between Rock and Scissors.

If player 2 does not know what player 1 has done, we can still represent the game in tree form.

Player 2 must move not knowing which of the nodes connected by the dotted line she has reached.
\[ U_2(\text{Paper}) = 0(p) - 30(q) + 10(1 - p - q) = 10 - 10p - 40q \]
\[ U_2(\text{Scissors}) = 30(p) + 0(q) - 10(1 - p - q) = -10 + 40p + 10q \]
\[ U_2(\text{Rock}) = -10(p) + 10(q) + 0(1 - p - q) = -10p + 10q \]

**RULE:** If a player is using a mixed strategy, the strategies which he plays with positive probability must yield the same payoff.

In our example, for player 2 to be willing to play both Rock and Paper we require

\[ U_2(\text{Paper}) = 10 - 10p - 40q = -10p + 10q = U_2(\text{Rock}) \]

Hence \( 10 - 40q = 10q \), and so \( q = 0.2 \)

Similarly, for player 2 to be willing to play both Scissors and Rock,

\[ U_2(\text{Scissors}) = -10 + 40p + 10q = -10p + 10q = U_2(\text{Rock}) \]

Thus \( p = 0.2 \). It follows that player plays Rock with probability 0.6.

End of Lec13