HOMEWORK SET #2  Due at beginning of class, Thursday, October 23

1.  Transportation Problem

The Hang Seng Co. has three pea canneries (in Canton, Auckland and Manila). The cans are all shipped to one of 4 distribution centers (in Los Angeles, Toronto, New York and Dallas). The following table indicates the capacity of each cannery, the allocation assigned to each distribution center and the cost per unit $t_{ij}$ of shipping from source $i$ to destination $j$.

<table>
<thead>
<tr>
<th>Source $i$</th>
<th>destination $j$</th>
<th>capacity of source $i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>482</td>
<td>670</td>
</tr>
<tr>
<td>2</td>
<td>525</td>
<td>440</td>
</tr>
<tr>
<td>3</td>
<td>925</td>
<td>632</td>
</tr>
<tr>
<td>Allocation to destination $j$</td>
<td>230</td>
<td>420</td>
</tr>
</tbody>
</table>

(a) Write down the cost minimization problem. (There are seven constraints.)
(b) Explain how to write down some feasible shipments.
(c) Solve for the cost minimizing shipments.
(d) Present your results on a sensitivity sheet. Interpret the shadow prices.
(e) Determine numerically the effect on cost of increasing the allocation to destination 2 by 10 and reducing the allocation to destination 3 by 10.
(f) If you could move a unit of production capacity from one source to another, what change would you make? Do the shadow prices provide any help here?
(g) Explain why one of the shadow prices must be zero.

2.  Choice of Technique

Output can be produced with different combinations of machines and skilled workers. Each column in the following array indicates the amount on input $i$ needed to produce output using technique $j$.

<table>
<thead>
<tr>
<th>techniques</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine A</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Machine B</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Labor</td>
<td>6</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>
Short-run analysis
(a) Suppose that in the short-run the supply of each of the inputs is fixed. There are 80 hours of Machine A time, 52 hours of machine B time and 128 hours of skilled labor. How much output should the firm produce and which techniques should be utilized?
(b) Henceforth assume that the output price is $100. What is the range of wage rates for skilled labor such that it would be profitable to hire more workers?

Medium run analysis
(c) If the cost per machine hour is $10 and the firm can hire as much as it wishes at this rental rate, what is the profit maximizing strategy of the firm?
Hint: You might eliminate the machine B constraint. Then you have to incorporate the cost of machine B rentals in some way.

Long run analysis
(d) The cost per hour of machine A is $20, for Machine B it is $10 and skilled labor costs $40. Suppose that the firm is a monopolist and can sell q units of output at a price

\[ p(q) = 1000 - 2q \]

In the long run, with no constraints on the supply of any input, what is the profit maximizing strategy of the firm?
(e) Would the techniques used change if the demand curve were to change? Explain.

3. Peak load pricing

The day is divided into 4 periods. Demand in each period is given by the following demand price functions.

\[
\begin{align*}
p_1 &= 262 - 2q_1 \\
p_2 &= 150 - q_2 \\
p_3 &= 172 - 2q_3 \\
p_4 &= 110 - q_4
\end{align*}
\]

The operating cost per unit in each period is 40. The unit cost of capacity is 50.

(a) Solve analytically for the profit maximizing outputs and prices.
HINT: Why is it helpful to look at capacity where, for \( q_t \), \( MR_t(q) = 40, \ t = 1,...,4, \)
(b) Solve for the surplus maximizing outputs and prices.
(c) In the second case, is this marginal cost pricing? Explain.
(d) Confirm your results by using a spreadsheet. ce?
4. Peak load pricing

The day is divided into 3 periods. Demand in each period is given by the following demand price functions.

\[
p_1 = 200 - 4q_1 - q_2 - 2q_3 \\
p_2 = 300 - q_1 - 3q_2 - q_3 \\
p_3 = 300 - 2q_1 - q_2 - 6q_3
\]

The operating cost per unit in each period is 20. The unit cost of capacity is 30.

(a) Solve (in any way you wish) for the profit maximizing outputs and prices.
(b) A regulator intervenes and requires that the difference in prices should not exceed $50. What is the new profit maximizing strategy?