1. Nash Equilibrium

For product 1 the demand price function is \( p_1 = 12 + \alpha - \frac{4}{3} q_1 - \frac{1}{3} q_2 \). For product 2 the demand price function is \( p_2 = 12 - \alpha - \frac{1}{3} q_1 - \frac{4}{3} q_2 \). Firm 1 produces product 1 and firm 2 produces product 2. The cost per unit of output is 3.

(a) What are the best response functions for each firm? Hence obtain two equations that the Nash equilibrium outputs must satisfy.
(b) Add these equations and hence show that the sum of the two outputs must be 18.
(c) Also obtain an expression for the difference between the two outputs as a function of the parameter \( \alpha \).
(d) For what values of \( \alpha \) will both firms produce a positive output?
(e) Suppose that the two firms merge. What will the merged firm do if \( \alpha = 0 \).
(f) Is there an incentive to merge if \( \alpha \geq 7 \)?

Note: You can solve this problem analytically or using a spread-sheet or using a combination of each.

2. First mover advantage

Industry demand is \( p = c + \alpha - q \). Each firm has the same unit cost \( c \). There are \( n \) firms.

Firm \( j \) has the \( j \)th move. Initially assume that there are two firms.

(a) Show that \( q_2 = \frac{1}{2} (\alpha - q_1) \) and hence that \( \Pi_i(q_1, q_2^{BR}(q_1)) = \frac{1}{2} (\alpha - q_1) q_1 \).
(b) Show that \( (q_1, q_2) = \left( \frac{1}{2} \alpha, \frac{1}{2} \alpha \right) \).
(c) If there are three firms show that the profit of firm 3 is \( \Pi_3 = (\alpha_3 - q_2 - q_3) q_3 \), where \( \alpha_3 = \alpha - q_1 \).
(d) Appealing to parts (a) and (b) or otherwise, show that \( (q_2, q_3) = \left( \frac{1}{2} \alpha_3, \frac{1}{2} \alpha_3 \right) \).
(e) Analyze the optimal strategy of firm 1 and hence show that each firm produces half as much as the one moving just before it.
(f) By looking at the pattern of outputs for 2 and 3 firms, make a conjecture about the output of each firm when there are 4 firms.
(g) Examine the sequence of equilibrium prices and hence determine whether it approaches unit cost as the number of firms continues to grow.
3. Optimal Indirect Price Discrimination

There are three types of buyer with demand price functions \( p_t = a_t - b_t q_t \). The unit cost of production is 20. Initially the number of each types (in thousands) is as given below.

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept parameter</td>
<td>( a_t )</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>number of buyers</td>
<td>( f_t )</td>
<td>20</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>slope parameter</td>
<td>( b_t )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>cost parameter</td>
<td>( c_t )</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>quantity</td>
<td>( q_t )</td>
<td>30</td>
<td>30</td>
<td>180</td>
</tr>
</tbody>
</table>

(a) Confirm that the plans indicated above are the profit maximizing plans.
(b) Present a table showing the effect on the three plans \((q_t, R_t)\), \( t = 1, 2, 3 \) of steadily reducing the number of type 1 and increasing the number of type 2 so that the total is unchanged.
(c) What is the range of numbers of type 1 for which the monopolist does not sell to type 1?
(d) What is the range of numbers of type 1 for which there are three plans? (Don’t go beyond 1 decimal place.)
(e) What happens outside these ranges?
(f) Provide the intuition behind your results.

4. Optimal 2-part pricing

(a) Using the data of part (a), solve for the profit maximizing 2 part pricing strategy.
(b) Show that the low types have exactly the same outcome as in 3(a) but high types are strictly better off.
(c) Provide the intuition for these conclusions.