Lecture 3: Review

Optimal use of scarce Capacity

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Revenue per unit</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>Total time available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Units of machine shop time</td>
<td>6</td>
<td>2</td>
<td>$b_1 = 160$</td>
</tr>
<tr>
<td>2</td>
<td>Units of sanding time</td>
<td>3</td>
<td>3</td>
<td>108</td>
</tr>
<tr>
<td>3</td>
<td>Units of polishing time</td>
<td>2</td>
<td>4</td>
<td>160</td>
</tr>
</tbody>
</table>

Solution $(x_1^*, x_2^*) = (22, 14)$ $(\lambda_1^*, \lambda_2^*) = (0.25, 0.5)$

Shadow price of input 1 for different levels of the input.
Machine time constraint

\(6x_1 + 2x_2 = 60\)

\(b_1 < 72\)
Machine time constraint

$6x_1 + 2x_2 = 90$

$160 > b_1 > 72$
Machine time constraint

$6x_1 + 2x_2 = 160$

$b_1 = 160$

((22, 14))

$x_1$

$x_2$
Machine time constraint

\[ 6x_1 + 2x_2 = 192 \]

\[ 216 > b_1 > 160 \]
Machine time constraint

\[ 6x_1 + 2x_2 = 240 \]

\( b_1 > 216 \)

Point \((22, 14)\)
Choice of technique

A firm uses capital equipment and skilled labor to produce a single product. There are three possible techniques of production. In each case these are the unit input requirements needed to produce 1 (million) units of output.

### Unit input requirements

<table>
<thead>
<tr>
<th>Technique</th>
<th>Capital equipment</th>
<th>Skilled labor</th>
</tr>
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<tbody>
<tr>
<td>Technique 1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Technique 2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Technique 3</td>
<td>2</td>
<td>5</td>
</tr>
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In the short-run the supply of each input is fixed at \((\bar{K}, \bar{L}) = (90, 60)\).

\[
\begin{align*}
\text{Max} \quad & x_1 + x_2 + x_3 \\
4x_1 + 3x_2 + 2x_3 & \leq \bar{K} \\
2x_1 + 3x_2 + 5x_3 & \leq \bar{L} \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]

Dual problem

\[
\begin{align*}
\text{Min} \quad & \lambda_1 \bar{K} + \lambda_2 \bar{L} \\
4\lambda_1 + 2\lambda_2 & \geq 1, \quad m_1 = -2 \\
3\lambda_1 + 3\lambda_2 & \geq 1, \quad m_2 = -1 \\
2\lambda_1 + 5\lambda_2 & \geq 1, \quad m_3 = -\frac{2}{5} \\
\lambda_1, \lambda_2 & \geq 0
\end{align*}
\]
Dual problem

$$\text{Min } \lambda_1 \bar{K} + \lambda_2 \bar{L}$$

$$4\lambda_1 + 2\lambda_2 \geq 1, \ m_1 = -2$$
$$3\lambda_1 + 3\lambda_2 \geq 1, \ m_2 = -1$$
$$2\lambda_1 + 5\lambda_2 \geq 1, \ m_3 = -\frac{2}{5}$$
$$\lambda_1, \lambda_2 \geq 0$$

Suppose $2 > \frac{\bar{K}}{\bar{L}} > 1$. The third constraint is not binding and so $x_3^* = 0$.

Since $(\lambda_1^*, \lambda_2^*) = (\frac{1}{6}, \frac{1}{6}) > 0$, the two input constraints are binding.

(1) $4x_1 + 3x_2 = \bar{K} = 90$
(2) $2x_1 + 3x_2 = \bar{L} = 60$

Solving $2x_1 = 30$ and so $x_1 = 15$.
Then, from (2) $3x_2 = 60 - 2x_1 = 30$ and so $x_2 = 10$
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**Isoquant**

The isoquant diagram shows the combinations of capital (K) and labor (L) that can produce a certain level of output. The isoquant curves represent different levels of capital-to-labor ratios: $\frac{K}{L} = 2$, $\frac{K}{L} = 1$, and $\frac{K}{L} = 0.4$. The shaded region represents the set of all feasible combinations of K and L that satisfy the production function $x \geq 1$. Points $T_1$, $T_2$, and $T_3$ are points on the isoquant curves for the respective capital-to-labor ratios.