STRATEGIC DECISION-MAKING

or

“How to play the game.”

“Business is a game - - the greatest game in the world if you know how to play it!”

Thomas J. Watson (founder of I.B.M.)
A. Elements of a Game

Players

We will label the $n$ players/agents/participants $i=1,...,n$

Strategy Set $S_i$ -- the set of all feasible choices/alternatives/options available to player $i$.

Example 1: Rock, Scissors, Paper (Bau, Jin, Dup)

$$S_i = \{\text{Paper, Scissors, Rock}\}$$

Example 2: Open bidding for an art object

A strategy is a choice of when to drop out of the bidding.

$$S_i = \{\text{all possible bids called by the auctioneer}\}$$

Player $i$, $i=1,...,n$ plays some strategy $s_i$ from his strategy set $S_i$

Pure strategy: A single choice from $S_i$

Mixed strategy: Player $i$ plays probabilistically, assigning positive probabilities to some of the alternatives in $S_i$

Note that while each player must play a single strategy $s_i$ in $S_i$, he can choose to play a mixed strategy using some form of randomizing device such as a coin toss or throw of a die.

Example: Rock, Scissors Paper

As everyone who ever played it quickly realizes, it pays to randomize.

HOW?
Payoffs

If the strategies actually chosen by the \( n \) players are \( s_1, s_2, \ldots, s_n \), then the outcome of the game leads to a payoff for each player. We write this as

\[ u_i(s_1, \ldots, s_n). \]

Rules of play - Timing of Moves

Sequential or simultaneous

Equivalently, each player moves sequentially but choices must be made without knowing the choices of those who have already moved.

Example: Rock, Scissors, Paper

The payoffs for a 2 person game can be conveniently depicted in a matrix as shown in Table B-1. Player 1’s payoffs are denoted in *italic* and **player 2**’s in **bold**

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Paper</th>
<th>Scissors</th>
<th>Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper</td>
<td>0, 0</td>
<td>-10,10</td>
<td>10,-10</td>
<td></td>
</tr>
<tr>
<td>Scissors</td>
<td>10,-10</td>
<td>0, 0</td>
<td>-10, 10</td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td>-10, 10</td>
<td>10,-10</td>
<td>0, 0</td>
<td></td>
</tr>
</tbody>
</table>

TABLE B-1: Rock, Scissors, Paper Game in “Normal form”

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Paper</th>
<th>Scissors</th>
<th>Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>Paper</td>
<td>0, 0</td>
<td>-30,30</td>
<td>10,-10</td>
</tr>
<tr>
<td>( q )</td>
<td>Scissors</td>
<td>30,-30</td>
<td>0, 0</td>
<td>-10, 10</td>
</tr>
<tr>
<td>( 1-p-q )</td>
<td>Rock</td>
<td>-10, 10</td>
<td>10,-10</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

TABLE B-1: Modified Rock, Scissors, Paper Game
Description of the Game in “tree” form

From the initial “node” of the tree, player 1 chooses an action. The game then continues at one of three nodes for player 2. She then takes her action. The payoffs are indicated at the “terminal” nodes.

Figure B-1: Tree representation of the game if played sequentially

If play is sequential, the tree representation of the game is often very helpful. What will player 2 do?
Player 1 is thus indifferent between Rock and Scissors.

If player 2 does not know what player 1 has done, we can still represent the game in tree form.

Figure B-3: Player 2 does not know player 1’s move

Player 2 must move not knowing which of the nodes connected by the dotted line she has reached.
\[U_2(\text{Paper}) = 0(p) - 30(q) + 10(1 - p - q) = 10 - 10p - 40q\]
\[U_2(\text{Scissors}) = 30(p) + 0(q) - 10(1 - p - q) = -10 + 40p + 10q\]
\[U_2(\text{Rock}) = -10(p) + 10(q) + 0(1 - p - q) = -10p + 10q\]

**RULE**: If a player is using a mixed strategy, the strategies which he plays with positive probability must yield the same payoff.

In our example, for player 2 to be willing to play both Rock and Paper we require
\[U_2(\text{Paper}) = 10 - 10p - 40q = -10p + 10q = U_2(\text{Rock})\]
Hence \(10 - 40q = 10q\), and so \(q = 0.2\)

Similarly, for player 2 to be willing to play both Scissors and Rock,
\[U_2(\text{Scissors}) = -10 + 40p + 10q = -10p + 10q = U_2(\text{Rock})\]
Thus \(p = 0.2\). It follows that player plays Rock with probability 0.6.

End of Lec13