Econ 201A  Microeconomic Theory  
Due Friday October 3.

1. Concavity and Quasi-concavity

Tip: Begin by working out your answer with $n=2$. Then go back and generalize to the n variable case.

(a) Show that if $f_i(x_i), i=1,...,n$ is concave on $X$

$$U(x_1,...,x_n) = \sum_{i=1}^{n} f_i(x)$$

is concave on $X$.

Show also that if $U$ is concave on $X$, then for any increasing function $V$, $V(U(x))$ is quasi-concave on $X$.

(b) Show that the following functions are concave over some domain $D_U$. In each case identify the domain.

(i) $U(x) = \sum_{i=1}^{n} \ln(y_i + x_i)$  

(ii) $U(x) = -\frac{1}{x_1} - \frac{1}{x_2}$  

(iii) $U(x) = \sum_{i=1}^{n} a_i x_i^{1-\frac{1}{\sigma}}, \quad \sigma \geq 0, \quad a_1,...,a_n > 0$.

(c) Show that the following functions are quasi-concave, for all $x \in \mathbb{R}^n_+$

(i) $U(x) = x_1^{\alpha_1} x_2^{\alpha_2} ... x_n^{\alpha_n}, \quad \alpha_j > 0, \quad j=1,...,n$

(ii) $U(x) = \left(\sum_{i=1}^{n} a_i x_i^\sigma\right)^{\frac{\sigma-1}{\sigma}}, \quad \sigma > 0, \quad \sigma \neq 1, \quad \theta > 0, \quad a_1,...,a_n > 0$

2. Consumer choice

(a) If $U(x) = x_1 + \ln x_2 + 2\sqrt{x_3}$ solve for the consumer’s demand functions assuming income $I$ is large. What are the own price elasticities of demand for $x_2$ and $x_3$.

(b) Show that demand for one of the commodities will be zero if income is small. If this is the case solve again for the demand functions.

(c) What is the condition under which only two products are purchased?

3. Equilibrium

There are two consumers. Alex has utility function $U^A = \ln(4 + x_1^A) + \ln x_2^A$. His endowment is $\omega^A = (20,12)$. Bev has utility function $U^B = \ln x_1^B + \ln(6 + x_2^B)$. Her endowment is $\omega^B = (36,12)$.

(a) Solve for each consumer's demand for commodity 1 and hence obtain the market demand. Show that the equilibrium price ratio is $p_1 / p_2 = \frac{1}{2}$.

(b) Use the Lagrange method to obtain the FOC for the maximization of $U^A$ given that Bev must have a utility of at least $\bar{U}^B$. 
(c) Depict the Pareto efficient allocations which are in the interior of a neatly drawn Edgeworth Box diagram (with $x_i$ on the horizontal axis.)

(d) In this same diagram depict the endowment point and the equilibrium budget constraints.

(e) What would happen to the equilibrium price ratio if the aggregate endowment were to remain constant but Bev’s endowment were to increase by 1 unit?

(f) What is the largest equilibrium price ratio in this economy, as individual endowments vary but the aggregate endowment remains fixed?

4. Expenditure and Cost Functions

An individual with utility function $U(x) = \left( \sum_{i=1}^{3} x_i^{-1} \right)^{-1}$ faces prices $p = (p_1, p_2, p_3)$.

(a) Solve for the individual’s demand functions $x(p, I)$ and hence for maximized utility as a function of income and prices.

(b) Hence solve for the smallest income $M(p, \bar{U})$ required to achieve a utility level $\bar{U}$.

(d) A firm has a production function $q = F(z) = \frac{1}{\sum_{j=1}^{n} \frac{1}{z_j}}$. The input price vector is $r$.

What is the cost function of the firm?