3. Saving in an uncertain world

A consumer lives for two periods. His riskless endowment in period $t$ is $W_t$, $t = 1, 2$. He can purchase an asset in period 1 which has a period 2 return per dollar of $z_s$ in state $s$. That is if he gives up one unit of period 1 consumption, his period 2 consumption rises by $z_s$ in state $s$. His 2 period utility function is $U(c_1, c_2) = u_1(c_1) + \sum_{s=1}^{S} \pi_s u_2(c_{2s})$, where $u_1(\cdot)$ and $u_2(\cdot)$ are both strictly concave.

(a) Write down his expected utility $\bar{U}(x)$ if he purchases $x$ units of the asset. Show that this indirect utility function is strictly concave.

(b) Provide necessary and sufficient conditions under which he will purchase some of the risky asset.

(c) Suppose instead that there is no uncertainty. The riskless asset has a return per dollar of $\sum_{s=1}^{S} \pi_s z_s$. Is the decision whether to purchase any units of this asset any different from your previous answer? Explain.

(d) Returning to the risky world, suppose that period 1 wealth rises. What is the effect on the slope of the indirect utility function. That is, does $\frac{\partial \bar{U}}{\partial x}$ rise or fall?

(e) At the old wealth level, the optimal choice $x^*$ satisfies $\frac{\partial \bar{U}}{\partial x}(x^*, w_1, w_2) = 0$. What is the sign of $\frac{\partial \bar{U}}{\partial x}(x^*, w_1 + a, w_2)$. Hence draw a conclusion about the effect of the increase of first period wealth on $x^*$.

(f) See if you can also show the effect on $x^*$ of an increase in second period wealth.

4. Equilibrium trading in state claims and asset markets

All consumers in an economy have the same VNM utility function $v(c) = \ln c$. There are 2 states. The states are equally likely. There are two types of asset in the economy. A unit of asset A has a return of $(1,1)$. There are 60 units of this asset in the economy. Asset B has a return of $(2,0)$. There are 60 units of this asset in the economy as well, so that the total endowment in the economy is $\omega = (180, 60)$. Each consumer has some initial endowment of assets $(q_{Ah}^h, q_{Bh}^h)$, $h = 1, ..., H$ where the sum of the asset holdings is 60.

(a) Solve for the state claims equilibrium price vectors.

(b) Solve for the market values of the two assets.

(c) Explain why trade in asset markets alone would achieve the same equilibrium outcome as trade in state claims markets.
(d) Show that each consumer will end up holding the same number of each asset.

Suppose that all assets have been traded so that each consumer holds a fraction of the “market portfolio” (i.e., a fancy way of saying a fraction of the total endowment of assets in the economy.) An unanticipated message is received which changes the probability of state 1 from \( \frac{1}{2} \) to \( \frac{3}{4} \).

(e) What will be the effect on state claims prices?
(f) What will be the effect on asset prices?
(g) What will be the effect on asset holdings?

Don’t hand this in but see what happens to the state claims prices and the ratio of asset prices if the utility function is CES with \( \sigma = 1/2 \), that is \( v'(c) = 1/c^2 \).