MID-TERM QUIZ

Time allowed: 90 minutes (I hope many have finished by that time and will be ready to depart.) If you want to continue for an extra 25 minutes, you may remain. Please note that this is primarily a diagnostic exercise so that you and I find out how you are doing. It will count for 10-15% of the grade for the course.

1. Robinson Crusoe

Robinson Crusoe has a utility function \( U = \ln x_1 + 4 \ln x_2 - l \) where \( x_1 \) and \( x_2 \) are consumption goods and \( l \) is labor supply. Output on his South Pacific island lies in the production set \( Y = \{(x_1, x_2, l)|x_1^2 + x_2^2 \leq 2l\} \).

(a) Explain why the utility function is concave and the production set is convex.
   (You should state any theorems that you use.)

(b) Solve for the utility maximizing choice of input and outputs.

(c) Suppose you were asked to manage the firm. Robinson Crusoe instructs you to maximize profit. Assuming you act as a price-taker, which, if any, of the following price-wage vectors would lead you to make the same choice as your answer to part (b)?

(i) \((p_1, p_2, w) = (1, 3, 1)\) (ii) \((p_1, p_2, w) = (1, 2, 1)\) (iii) \((p_1, p_2, w) = (1, 1, 1)\).

(d) What profit would the firm make?

(e) Discuss carefully whether there is a Walrasian (price taking) equilibrium for the Robinson Crusoe economy.

2. Profit maximization and export restrictions

A monopoly sells its output both domestically and overseas. Domestic demand is given by \( p_1 = 41 - q_1 \) while foreign demand is given by \( p_2 = 51 - q_2 \). The cost of production \( C(q_1, q_2) = (1 + q_1)(1 + q_2) \).

(a) Show that the profit maximizing sales are \( q_1^* = 10 \) and \( q_2^* = 20 \).

The government intervenes and decrees that a minimum of \( Z \) units must be sold on the domestic market.
(b) Write down the Lagrangian for the firm. Explain carefully why the firm will now export only 17 units if Z=16.

(c) What is the shadow cost to the monopolist of further increasing the minimum domestic supply from Z=16?

(d) How would the answer to part (a) change if the cost function were instead

\[ C(q_1, q_2) = 2(1 + q_1)(1 + q_2) \]

3. Consumer and firm choice

A consumer has a utility function \( U(x_1, x_2) = (2 + x_1)x_2 \). She faces a price vector \( p = (p_1, p_2) \) and has an income of \( I \).

(a) Show that she will choose \( x_1^* \) and \( x_2^* \) satisfying

\[ 2 + x_1^* = \frac{2p_1 + I}{2p_1} \quad \text{and} \quad x_2^* = \frac{2p_1 + I}{2p_2} \]

(b) Hence solve for the indirect utility function.

(c) A firm has a production function \( q = (2 + z_1)z_2 \), where \( z_1 \) and \( z_2 \) are inputs. The price of input \( j \) is \( p_j, j=1,2 \). Solve for the firm's cost function.

4. Altruism

Do not attempt this until you have completed each of the above three questions.

Alex has a utility function \( U^A = \ln x_1 + \ln x_2 \), where \( x_i \) is Alex's consumption of commodity \( i \). His mother, Bev cares about her own consumption and about Alex's happiness. Her utility function is \( U^B = U^A + \ln y_1 + \ln y_2 \), where \( y_i \) is Bev's consumption of commodity \( i \). The price of each commodity is 1. Bev has an income of 200 and Alex has an income of 50.

(a) Solve for Alex's utility if he receives a gift of \( G \) from Bev.

(b) Hence, or otherwise, solve for Bev's optimal consumption of the two commodities and her gift to Alex.
(c) If Bev's income were to rise, what fraction of every extra dollar would she give to Alex?
EC 201A Microeconomic Theory
Final examination

Time allowed: Three (3) hours. The first 10 minutes are for reading only. Attempt four (4) questions only.

1. Equilibrium and Efficiency
Alex has a utility function \( U^A(c_1, c_2) = \ln c_1 + \ln c_2 \) while Bev has a utility function \( U^B(c_1, c_2) = c_1(50 + c_2) \). The aggregate endowment of the two commodities is \( \omega = (100, 150) \).

(a) Solve for the Pareto Efficient allocations and illustrate these in a neat figure.
(b) If the individual endowments are \( \omega^A = (20, 40) \) and \( \omega^B = (80, 110) \), what will be the Walrasian equilibrium price ratio \( \frac{p_1}{p_2} \) and what trade will take place at these equilibrium prices?
(c) Keeping aggregate endowments fixed, determine the range of possible equilibrium price ratios as the individual endowments are allowed to vary.
(d) Suppose that individual endowments are as in part (b), however preferences differ. Alex only likes commodity 1 and Bev only likes commodity 2. What will be the Walrasian equilibrium price ratio (if any.)

2. Capital accumulation

Demand for housing is given by

\[ p_t = 100 - \frac{1}{2} H_t \]

where \( H_t \) is the housing stock and \( p_t \) is the rental price in period \( t \). Housing depreciates at the rate \( \gamma \) so that if \( x_t \) units are produced in period \( t \), next period’s housing stock is

\[ H_{t+1} = H_t (1 - \gamma) + x_t \]

There is a single firm which can build new housing units in period \( t \) at a total cost

\[ C(x_t) = \frac{1}{2} (x_t)^2 \]

(a) If the monopoly discounts the future using interest rate \( r \), show that the first order conditions for maximizing profit over \( T \) periods require that

\[ (1 + r)x_{t-1} = 100 - H_t + (1 - \gamma)x_t. \]

(b) In a neat figure, depict the phase diagram.
(c) If the monopoly has an infinite time horizon, obtain an explicit solution, if you can, for the long run stationary state.
(d) Starting from an initial housing stock of zero, depict the optimal time path \( \{H_t, x_t\}_{t=1}^{\infty} \).
(e) Compare this with the time path for a long finite horizon.
(f) Starting from the stationary state, analyse the effect of a permanent decrease in the interest rate. Provide the intuition behind your result.

3. Theory of the Firm

A firm is a monopolist in output markets but a price taker in input markets. The input price vector is \( r \). Indicate whether each of the following statements is true (in which case prove it) or whether it is false (in which case provide a counter example.)

(a) If \( r_1 \), the price of input 1 rises, demand will either fall or remain constant.
(b) Input demand will be (weakly) less responsive to a change in the price of input 1 in the short run than in the long run.
(c) Maximized profit is a convex function of \( r \).
(d) When \( r_1 \) rises, demand for all inputs will either fall or remain constant.

4. Constant Returns to Scale Economy

Commodity 1 is produced to the Cobb-Douglas production function \( x_1 = \sqrt{L}K_1 \).
Commodity 2 is produced according to the Leontief production function \( x_2 = \min\{2L_2, K_2\} \). The aggregate supply of the two inputs is \( (\bar{L}, \bar{K}) = (500, 600) \).

(a) If the input price vector is \( (w, r) = (1, 1) \), solve for the equilibrium output prices
(b) In this case, is the production possibility frontier linear? If so why? If not, what is its shape? Explain.

Suppose that the country can trade the two outputs in world markets at the prices that you obtained in part (a).

(c) Discuss the qualitative effect on input prices and on the output of the two commodities if the aggregate stock of capital rises (i) from 600 to 900 (ii) from 900 to 1200.

5. Choice over time

An individual will live for \( T \) periods. He starts with no wealth and his income in each of the \( T \) periods is \( w \). He can borrow at an interest rate \( \beta \) and lend (save) at an interest rate \( r \). His utility function is

\[
U(c) = \sum_{t=1}^{T} \delta^{t-1}V(c_t)
\]

where \( V \) is strictly increasing and strictly concave with \( V'(0) = \infty \).

(a) For what values of the discount factor \( \delta \) will he save in the first period and for what values will he borrow? Derive your result in full.
(b) For all positive discount factors, discuss the time path of consumption and asset accumulation.
(c) In each case, how would first period consumption be affected by an increase in $T$, the length of his life?
(d) How would first period consumption be affected if, in the final period, the wage is zero rather than $w$?

6. Equilibrium under uncertainty

Individuals are expected utility maximizers. All have the same utility function over certain consumption but may have different beliefs, that is,

$$U^h = \sum_{s=1}^{S} \pi_s^h V(c_s), \ h = 1,...,H$$

Suppose all exhibit constant relative risk aversion so that $V'(c) = \frac{1}{c^\gamma}$.

(a) Show that if the aggregate endowment of state claims is $\omega = (\omega_1,...,\omega_S)$, and all share the same beliefs, equilibrium state claims prices satisfy

$$\frac{P_s}{P_1} = \frac{\pi_s}{\pi_1} \left(\frac{\omega_1}{\omega_s}\right)^{\frac{\gamma}{\gamma-1}}, \ s=2,...,S.$$ 

Henceforth assume that $R=2$. There are three states which all agree are equally likely $(\pi_1,\pi_2,\pi_3) = (\frac{1}{3},\frac{1}{3},\frac{1}{3})$. Individuals hold shares in each of three assets.

$$z_1 = (10,10,10)$$
$$z_2 = (5,15,10)$$
$$z_3 = (5,15,20)$$

There are no other endowments.

(b) What are the equilibrium prices in the state claims market?

Henceforth assume that there are no state claims markets.

(c) Suppose individuals trade their shareholdings of the three assets. Would they be any worse off in equilibrium? Explain. What would be the equilibrium asset prices?

(d) If the second and third assets were to be merged into a single asset with return $(10,30,40)$ before individuals have traded in asset markets, would there be any effect on the asset market equilibrium?

(e) What if the merger were to take place after a first round of trading in asset markets?

(f) Suppose that the three individuals have widely differing beliefs about the probability of the three states and, if there were state claims markets, equilibrium state claims prices would therefore be $(7,3,4)$. In the absence of state claims markets, what can you now say about the equilibrium prices of the assets?
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Time allowed: 10 minutes for thinking (no writing) and then 100 minutes for writing. Don't worry if you run out of time. But please try to spend a significant amount of time on each question.

1. Utility maximization

There are two commodities. Bev has a utility function \( U(x) = x_1^\alpha x_2^\beta \), \( \alpha, \beta > 0 \). She has an endowment of \( \omega = (\omega_1, \omega_2) \).
(a) Solve for her optimal consumption choice and her maximized (or "indirect") utility as a function of her endowment and the price vector \( p \).
(b) Solve also for her expenditure function, that is, the income needed to achieve a utility \( \bar{U} \).
(c) More generally, let \( V(p, \omega) \) be a consumer's indirect utility function. Appealing to the Envelope Theorem, explain carefully how it is possible to obtain the consumer's demand functions from this indirect utility function.
(d) Confirm your result for Bev.
(e) Suppose Bev's endowment increases from \( \omega \) to \( k\omega \), \( k > 1 \). Under what conditions is her indirect utility a convex function of \( k \)? Does this seem implausible? Comment briefly.

2. Concave and quasi-concave functions

(a) Define a concave function and a quasi-concave function.
(b) Give an example of a quasi-concave function which is not concave when \( x \) is one dimensional.
(c) Give another example when \( x \) is 2-dimensional.

Let \( f(x) \) and \( g(x) \) be concave functions defined for all n-vectors \( x \geq 0 \).

(d) Is the sum \( f(x) + g(x) \) concave?
(e) Is \( \text{Min}\{f(x), g(x)\} \) concave?
(f) Is the difference \( f(x) - g(x) \) concave?

(For (d)-(f), if your answer is "Yes", provide a proof, if your answer is "No", provide a counter-example.)

3. Monopoly choice with alternative technologies

A monopolist has a cost of production

\[
C(Q, z) = F(z) + c(z)Q = (26 - 10z + z^2) + 2(1 + z)Q
\]

The demand price function for the product is \( p(Q) = 10 - \frac{1}{2}Q \). Note that \( z \) is a parameter.

(a) If the parameter \( z = 1 \), solve for the profit maximizing output and price. How much profit does the firm make?
(b) What would be the marginal effect on profit of an increase in the parameter \( z \)?
(c) Suppose \( z \) is not a parameter but a choice variable of the firm. Solve for the profit maximizing choice of \( z \) and \( Q \).
(d) Compare the profit levels in parts (a) and (c).
FINAL EXAMINATION

Time allowed: First 10 minutes for reading (no writing allowed). 3 hours for writing. Answer four (4) questions only. There is NO extra credit for attempting additional questions.

1. Demand

(a) Sketch a proof of the Envelope Theorem
(b) State and prove Roy's Identity
A firm is a monopolist in output markets and a price taker in input markets.
(c) Show that the profit function \( \Pi(r) \) is convex.
(d) Derive the first law of input demand.
(e) Show that there is a simple relationship between the cross elasticities of demand \( E(z_i, r_j) \) and \( E(z_j, r_i) \).

2. An island economy

Robinson and Friday live on an island in which coconuts can be produced from labor. The production function is \( y = 108\sqrt{L} \). Robinson owns the entire island. He likes coconuts but is unable to work so hires Friday, who has utility function \( U_F = x_F - L^2 \), where \( x_F \) is Friday's consumption of coconuts.
(a) Characterize as completely as you can the Pareto Efficient allocations in this economy.
(b) What is the Walrasian equilibrium allocation and "real wage" w/p?
(c) Suppose instead that Robinson can work. His utility function is \( U_R = x_R - 20L_R \). Again solve for the Walrasian equilibrium.

3. Risk bearing

There are 1 million people like Bev in the economy. Each Bev type is risk neutral. There are also a million people like Alex. Each Alex type is risk averse with preference scaling function \( v(c) = \ln c \). For each pair of Alex and Bev types, the total endowment of claims to state 1 is 100 and to state 2 is 200. The two states are equally likely.

(a) Depict the PE allocation is a neat Edgeworth-box diagram.
(b) Hence solve for Walrasian equilibrium state claims prices if Alex's endowment is sufficiently small.
(c) Explain why the relative price of state 1 claims must be higher if the endowment of Alex types is sufficiently large.
(d) Holding fixed the endowment of each pair so that \( (\omega_1^A, \omega_2^A) + (\omega_1^B, \omega_2^B) = (100,200) \), what is the range of possible equilibrium state claims prices in this economy? (continued over page)
(c) Suppose that each Bev types owns a firm with return \((100,100)\) and each Alex type owns a firm with returns \((0,100)\). If all trading takes place in asset markets rather than state claims markets, what will be the final ownership of each Alex and Bev type?

4. Timely consumption

A consumer has a strictly quasi-concave utility function \(U(x_1,\ldots,x_n)\). She faces a price vector \(p\) and has income \(I\). Consumption of each commodity takes time. Each unit she consumes of commodity \(j\) takes up \(t_j\) units of time. The total available time is \(T\).

(a) Obtain and interpret the first order conditions for a maximum (ignore corner solutions).

(b) Depict the solution in a neat figure for the \(n=2\) case.

(c) Suppose that we also introduce leisure (commodity \(x_0\)). Modify the model to incorporate the work leisure choice, as well as non-wage income. You should assume that all consumption takes place away from the work-place.

(d) Again obtain and interpret the first order conditions for a maximum.

(e) Discuss the income and substitution effects of an increase in the wage on labor supply and consumption of other commodities.

5. The educated consumer

An individual lives for \(S+T\) periods. He can borrow at a rate \(\rho\) and lend at a rate \(r < \rho\). His wage during the first \(S\) periods is zero. For the remaining (post school years) his wage each period is \(w\). His utility function is \(U(c) = \sum_{t=1}^{S+T} \delta^{t-1} v(c_t)\), where \(v(\cdot)\) is an increasing strictly concave function, \(v'(0) = \infty\) and \(1+r < \delta^{-1} < 1+\rho\).

Initially suppose that his parents support him at school and he ends period \(S\) with no capital.

(a) Explain why for the \(T\) years in the work force, his capital must satisfy the growth equation
\[
K_{t+1} = K_t + w + rK_t - c_t, \quad \text{if} \quad K_t > 0
\]
where \(K_t\) is the principal invested in the previous period for use in period \(t\).
What if \(K_t\) is negative?

(b) Explain also why, in the final period, it must be the case that
\[
c_{S+T} = (1+r)K_{S+T} + w \quad \text{if} \quad K_T > 0 \quad \text{and} \quad c_{S+T} = (1+\rho)K_{S+T} + w \quad \text{if} \quad K_T < 0
\]

(c) Depict the phase diagram.

(d) Hence or otherwise characterize the optimal time path of consumption and capital from time \(S\) to time \(S+T\).
Henceforth suppose instead that the individual has to support himself during those $S$ years.

(e) Depict the phase diagram for the $S$ school years.
(f) Characterize, as completely as you can, the optimal time path of consumption and capital over the $S+T$ years of his life.

6. Constant Returns to Scale economy

Consider a $2 \times 2$ CRS economy in which the aggregate supply of labor and capital $(\bar{L}, \bar{K}) = (100,100)$. The production functions for the two commodities are as follows.

$$x_1 = L_1^{1/3} K_1^{2/3}, \quad x_2 = (\frac{2}{3} L_2^{2/3} + \frac{1}{3} K_2^{2/3})^3.$$

Suppose initially that the economy only produces commodity 2 and that the world price of this commodity is 1.

(a) Depict the $1$ isoquant and explain why the $1$ cost line must go through the point $(1,1)$.
(b) Show that the equilibrium input prices must be $(w,r) = (\frac{2}{3}, \frac{1}{3})$.
(c) Show that it will not be profitable to produce commodity 1 unless $p_1 > \frac{2\sqrt{2}}{3}$.

Suppose, henceforth, that the price of commodity 1 rises to 1 and the price of commodity 2 falls. As a result, the economy specializes in the production of commodity 1.

(d) Solve for the new equilibrium input prices.
(e) For what range of prices $p_2$ would it be unprofitable to produce both commodities?