HOMEWORKS Fall 99

Econ 201A Microeconomic Theory
Problem Set 1: Due Friday of Week 2

1. Concavity and Quasi-concavity

(a) Show that if \( f_i(x), \quad i = 1,\ldots,n \) is strictly concave on \( X \)
\[
U(x_1,\ldots,x_n) = \sum_{i=1}^{n} f_i(x) \quad \text{is strictly concave on } X.
\]

(b) Show that, for any increasing function \( V(\cdot) \), \( V(U(x)) \) is also strictly quasi-concave on \( X \).

(c) Show that the following functions are strictly concave for all \( x_1,\ldots,x_n > 0 \)
\[
(i) \quad U(x) = \sum_{i=1}^{n} \ln(y_i + x_i), \quad y_i \geq 0, \quad i = 1,\ldots,n \quad (ii) \quad U(x) = -\frac{1}{x_1} - \frac{1}{x_2}
\]

(iii) \( U(x) = \sum_{i=1}^{n} a_i x_i^{\frac{1}{\sigma}}, \quad \sigma \geq 0, \quad a_1,\ldots,a_n > 0 \).

Hint: A function \( f(x) \) is strictly concave on \( \mathbb{R}_+ \) (the non-negative real numbers) if \( f''(x) < 0 \), \( x \in \mathbb{R}_+ \).

(d) Hence show that the following functions are strictly quasi-concave, for all \( x_1,\ldots,x_n > 0 \)
\[
(i) \quad U(x) = x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}, \quad \alpha > 0 \quad (ii) \quad U(x) = \left( \sum_{i=1}^{n} a_i x_i^{\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0, \quad \sigma \neq 1, \quad \theta > 0, \quad a_1,\ldots,a_n > 0
\]

2. Consumer choice

(a) Prove the Equal Ratio Theorem \( \frac{a_1}{b_1} = \ldots = \frac{a_n}{b_n} \Rightarrow \frac{a_1}{\sum_{i=1}^{n} a_i} = \frac{b_1}{\sum_{i=1}^{n} b_i} \)

(b) A consumer with utility function \( U(x) \) and income \( I \) faces a price vector \( p > 0 \).

(i) If \( U(x) = x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}, \quad \alpha > 0 \) show that the optimal consumption vector satisfies
\[
\frac{\alpha_1}{p_1x_1} = \ldots = \frac{\alpha_n}{p_nx_n} = \frac{\sum_{i=1}^{n} \alpha_i}{I}
\]

(ii) If \( U(x) = \left( \sum_{i=1}^{n} a_i x_i^{\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0, \quad \sigma \neq 1, \quad \theta > 0, \quad a_1,\ldots,a_n > 0 \), show that the optimal consumption vector satisfies
\[
\frac{a_1}{p_1 x_1^{\frac{1}{\sigma}}} = \ldots = \frac{a_n}{p_n x_n^{\frac{1}{\sigma}}}.
\]

Hence show that
\[
\frac{(a_1)^{\sigma}}{p_1} = \ldots = \frac{(a_n)^{\sigma}}{p_n} = \frac{p_1 (a_1)^{\sigma}}{p_1 x_1} = \ldots = \frac{p_n (a_n)^{\sigma}}{p_n x_n} = \sum_{i=1}^{n} \frac{p_i (a_i)^{\sigma}}{I}.
\]

(c) For the utility function in (b) (ii) obtain an expression for the optimal ratio of consumption of commodities \(i\) and \(j\) in terms of parameters.
(d) What is the elasticity of this consumption ratio with respect to a change in \(p_k\), \(k = 1, \ldots, n\), that is \(E\left(\frac{x_i}{x_j}, p_k\right)\)?

3. Edgeworth Box

There are two consumers. Consumer 1 has utility function \(U^1(x, y) = \ln(4 + x) + y / 5\). His endowment is \((\bar{x}^1, \bar{y}^1) = (8, 20)\). Consumer 2 has utility function \(U^2(x, y) = \ln(6 + x) + y / 5\). His endowment is \((\bar{x}^2, \bar{y}^2) = (2, 20)\).

In parts (a)-(e) you may assume that, for each consumer, consumption of both commodities is strictly positive.

(a) Solve for each consumer's demand for commodity 1 and hence obtain the market demand. Show that the equilibrium price ratio is \(p_x / p_y = \frac{1}{2}\).
(b) What is the market demand for commodity 2?
(c) Use the Lagrange method to obtain the FOC for the maximization of \(U^1\) given that consumer 2 must have a utility of at least \(U^2\).
(d) Depict the Pareto efficient allocations which are in the interior of a neatly drawn Edgeworth Box diagram (with \(x\) on the horizontal axis.)
(e) In this same diagram depict the endowment point and the equilibrium budget constraints.
(f) What would happen to the equilibrium price ratio if the endowments were to be redistributed?
Ec 210A Microeconomic Theory - Problem set 2

1. Isoquants for CES production functions

For each of the following production functions, depict in a neat figure the isoquant through (1,1) (a) $q = \sqrt{z_1 z_2}, \ z \geq 0$ (b) $q = (z_1 ^{1/2} + z_2 ^{1/2})^2, \ z \geq 0$ (c) $q = \frac{1}{\frac{1}{z_1} + \frac{1}{z_2}}, \ z \geq 0$. In each case show clearly what happens as the input ratio goes to infinity.

2. Cost functions

In each of the three cases in question 1, solve for the cost function $C(q, r_1, r_2)$ for all output levels and all positive input prices.

3. Marginal Cost

(i) For each of the following production functions solve for the cost function and hence marginal cost (a) $q = z_1 + \ln(1 + z_2)$ (b) $q = 4z_1^2 + z_2^2$.

(ii) In each case depict the marginal cost curve in a neat figure and hence discuss the effect of input price changes on the profit maximizing output.

4. Joint Costs and independent demand

Demand in three sub-periods (of the day) is given as follows.

$$p_1 = 100 - q_1, \ p_2 = 200 - 2q_2, \ p_3 = 300 - 3q_3.$$  
The marginal operating cost is 20 per unit per sub-period. The marginal cost of capacity is 6 per unit per day.

(i) Solve for the profit maximizing output levels. Clearly state any theorems that you appeal to in your derivation.

(ii) Solve for the socially optimal prices and output levels.

(iii) How would your answer to part (i) change if the marginal cost of capacity were to be 40 per day?

5. Joint costs and interdependent demand

Suppose demand in 2 sub-periods is given as follows.
\[ p_1 = 50 - 4q_1 + 2q_2, \quad p_2 = 180 + q_1 - 4q_2. \]

The marginal operating cost is 30 per unit per sub-period. The marginal cost of capacity is 20.

(i) Is the profit maximizing output vector \((q_1, q_2) = (10, 20)\) ?

(ii) What are the socially optimal outputs and prices?

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**Econ 201A Homework 3**

1. **Equilibrium in a CES economy**

There are \(H\) individuals in an economy all with the same CES preferences

\[
U^h(x^h) = \frac{1}{1-\sigma} \sum_{j=1}^{n} a_j x_j^{1-\sigma}, \quad \sigma > 0, \sigma \neq 1.
\]

The endowment of individual \(h\) is \(\omega^h = (\omega_1^h, \ldots, \omega_n^h)\).

(a) Solve for equilibrium prices.

(b) What is the elasticity of equilibrium relative prices with respect to the aggregate endowment of one of the commodities?

2. **Generous Mum?**

(a) Solve for the indirect utility function if \(U(x_1, x_2) = (x_1x_2)^{\frac{1}{3}}\).

Suppose Alex has an income \(I_1\) and utility function as given in part (a). Her mother Bev, on the other hand cares about her daughter as well as herself. Bev's income is \(I_2\). Bev's utility is

\[
u(x^1, x^2) = U(x^2) + \alpha U(x^1).
\]

(b) For what values of \(\alpha\) and income levels will it be optimal for Bev to give part of her income to Alex?

(b) Show that there is an upper bound to the ratio of after gift incomes of the two agents.

3. **Exercise B.5 (check)**

Further hint: It may not be optimal to consume strictly positive amounts of all commodities

4. **Exercise C.2**

5. **Exercise C.3**
Econ 201A: Microeconomic Theory Problem Set 4

1. An individual has a wage \( w_t, \ t = 1, \ldots, T \). He begins with no assets. Let \( W \) be the present value of his wage stream. He can borrow and lend at the interest rate \( r \). His lifetime utility is

\[
U(c_1, \ldots, c_T) = \sum_{t=1}^{T} \delta^{t-1} \ln c_t
\]

(a) Write down the individual's life-time budget constraint.
(b) Hence, or otherwise show that

\[ c_{t+1}^* = (1 + r) \delta c_t^* \]

(c) Substitute back into the life-time budget constraint and hence show that

\[ c_t^* = \frac{(1 - \delta)}{1 - \delta^T} W. \]

HINT: Let \( S(T) = 1 + \delta + \delta^2 + \ldots + \delta^{T-1} \). Then \( 1 + \delta S(T) = 1 + \delta + \delta^2 + \ldots + \delta^{T-1} + \delta^T \).

Subtracting the first equation from the second we obtain \( S(T) = \frac{1 - \delta^T}{1 - \delta} \).

(d) By how much would first period consumption rise if the first period wage were to rise by $1?
(e) By how much would first period consumption rise if the wage in every period were to rise by $1?
(f) Do these results bear out Milton Friedman's claim that if a long lived consumer were to have an unanticipated one period increase in his wage income, his current period spending would be very little affected?

2. Chapter 7, Exercise C-4

3. Chapter 7, Exercise E-3

Note: The modified Midterm can be found on my web-site.

http://www.econ.ucla.edu/riley/201