Econ 201A : Microeconomic Theory
Midterm Quiz and Modified Quiz

Answer three questions for 10 points each. While I doubt it will be the case, there may be a few students who are satisfied that they have completed all 3 questions with time to spare. In this case only, you may attempt a 4th question. If so, please label it "BONUS" and it will be graded out of 5.

1. **Equilibrium and Efficiency**

Alex has a utility function \( U^A = - \frac{1}{x_1} - \frac{1}{x_2} \) and an endowment \( \omega^A = (6,18) \). Bev has a utility function \( U^B = - \frac{1}{x_1} - \frac{1}{(x_2 + 10)} \) and an endowment \( \omega^B = (4,12) \).

(a) Characterize as completely as you can the Pareto Efficient allocations.
(b) What is the Walrasian equilibrium price ratio?
(c) What are the possible equilibrium price ratios in this economy as the individual endowments change (leaving the aggregate endowment \( \omega = (10,30) \) fixed?)

2. **Peak load pricing**

Demand in two periods is given as follows.

\[
p_1(q_1) = 60 - q_1, \quad p_2(q_2) = 156 - q_2.
\]

The cost of production is 30 per unit of output up to the capacity \( q_0 \). The cost of renting equipment with capacity \( q_0 \) is \( C(q_0) = q_0^2 \).

(a) Show that social surplus is maximized with a price vector \( (p_1, p_2) = (30,114) \).
(b) Suppose the cost of renting capacity doubles to \( 2q_0^2 \). What are the new socially optimal output levels and prices?

3. **Robinson Crusoe and Friday**

Robinson and Friday have the same utility function \( U^h = \ln x_1^h + \ln x_2^h, \quad h \in \{R,F\} \).

(a) if Robinson's endowment is \( \omega^R = (4,7) \) and Friday's is \( \omega^F = (1,3) \), show that the Walrasian equilibrium price ratio \( \frac{p_1}{p_2} = 2 \).

(b) Suppose that Robinson and Friday have equal shares in a new technology. Using \( z_1 \) units of commodity 1, their firm can produce \( y_2 = 10\sqrt{z_1} \) units of commodity 2. Endowments are as in part (a). Confirm that the new Walrasian equilibrium price ratio is 5.
4. Quasi-concavity and aggregation

A function $U(x)$ is strictly quasi-concave over $X$, if for any $x^0, x^1 \in X$, if $U(x^1) \geq U(x^0)$, then $U((1-\lambda)x^0 + \lambda x^1)$, where $0 < \lambda < 1$.

(a) Is this equivalent to the statement that, for any $x^0 \in X$, the set of consumption vectors $\{x \mid x \neq x^0\}$ is a strictly convex set?

(b) Define $U^*(x_j, y) = \max_x \{U(x_j, x_{-j}) \mid p_{-j} \cdot x_{-j} \leq y\}$, where $x_{-j}$ is the vector of all commodities except commodity $j$. If $U(\cdot)$ is strictly quasi-concave, show that the indirect utility function $U^*(x_j, y)$ is also a strictly quasi-concave function.

(c) Suppose $U^*(x_i, y) = x_i^a y^{\beta+\gamma} p_{x_i}^\beta p_{x_3}^{-\gamma}$. Show that this individual will always spend on commodities 2 and 3 in the same proportions, that is $\frac{p_{x_2}}{p_{x_3}} = k$.

11/11/99

1R. Equilibrium and Efficiency

Alex has a utility function $U^A = x_1 - \frac{1}{x_2}$ and an endowment $\omega^A = (6,28)$.

Bev has a utility function $U^B = 4x_1 - \frac{1}{(x_2 + 10)}$ and an endowment $\omega^B = (4,22)$.

(a) Characterize as completely as you can the Pareto Efficient allocations.

(b) What is the Walrasian equilibrium price ratio?

(c) What are the possible equilibrium price ratios in this economy as the individual endowments change (leaving the aggregate endowment $\omega = (10,50)$ fixed?)

2R. Peak load pricing

Demand in two periods is given as follows.

$p_1(q_1) = 60 - 2q_1$, $p_2(q_2) = 156 - 2q_2$. The cost of production is 30 per unit of output up to the capacity $q_0$. The cost of renting equipment with capacity $q_0$ is $C(q_0) = 2q_0^2$.

(a) What are the social surplus maximizing prices?

(b) Suppose the cost of renting capacity doubles to $4q_0^2$. What are the new socially optimal output levels and prices?

(c) What are the monopoly prices in each case?
3R. **Robinson Crusoe and Friday**

Robinson and Friday have the same utility function over two periods:

\[ U^h = u(x^h_1) + \frac{1}{2} u(x^h_2), \quad h \in \{ R, F \} \]

where \( x^h_t \) is individual \( h \)'s consumption vector in period \( t \) and

\[ u(x^h_t) = \ln x^h_{1t} + \ln x^h_{2t} \]

Robinson's first period endowment is \( \omega^R_1 = (4,12) \) and Friday's is \( \omega^F_1 = (1,18) \). There is no second period endowment. There are two firms. The first has production function \( y_{21} = 10\sqrt{z_{11}} \), the second has production function \( y_{22} = 2z_{21} \), where \( z_{ij} \) is the input of commodity \( j \) in period 1.

(a) Show that utility is maximized if \( z_{21} = 10 \),

(b) Show that \( z_{11} + 4\sqrt{z_{11}} = 5 \). This is a quadratic equation in \( r = \sqrt{z_{11}} \). Hence solve for \( r \) and \( z_{11} \).

(c) Hence, or otherwise, solve for the Walrasian equilibrium prices.

**COMMENT:** Questions 1R and 2R are similar to questions 1 and 2. Question 3R is somewhat harder than question 3, but similar to the exercise that we worked through in class.

If you scored less than half marks, I would like you to work out the answers on your own, taking as much time as you need. Then hand the answers in to me. Please put your email address on your answers.

JR