1. Debt Contracts and Information Acquisition

An Entrepreneur seeks financing $I$ from a competitive market of Investors. E has a project that pays off random return $q$ that is observed by both players ex-post, but not before the contract $\langle r(q) \rangle$ is signed.

Unlike previous models, the project will take place whether or not the investor makes the investment. Rather, the funding is used to pay E, who gains utility $\alpha I$. We assume $\alpha \geq 1$, so that investment is efficient.

If the investor makes the investment, she is repayed according to $r(q) \in [0, q]$. Hence her payoff is $U_I = r(q) - I$, while E’s utility is $U_E = q - r(q) + \alpha I$. In the first-best contract, any feasible $r(q)$ such that $E[r(q)] = I$ maximises E’s utility.

(a) Suppose that, just before they sign the contract, the investor can pay to observe the success of the project, $q$. Suppose we wish to choose $r(q)$ to minimize the incentive for the investor to acquire information (i.e. minimize the increase in their payoffs obtained by acquiring). Argue that the optimal contract has $r(q) \geq \min\{q, I\}$ and therefore that a debt contract is optimal (although not necessarily uniquely optimal).

(b) Now suppose that, just before they sign the contract, the Entrepreneur can pay to observe the success of the project, $q$. Suppose we wish to choose $r(q)$ to minimize the incentive for the entrepreneur to acquire information. Again, argue that a debt contract is optimal.

(c) Suppose the cost of information acquisition are $c_E$ and $c_I$ for the entrepreneur and investor. What is the highest level of investment $I$ that is sustainable if we do not wish either party to acquire information?

[Note: If you get stuck, you might find it easier to work with $\alpha = 1$.]
2. Revenue Management

A firm has one unit of a good to sell over $T$ periods. One agent enters each period and has a value drawn IID from $F(\cdot)$ on $[0,1]$. Agents values are privately known. The discount rate is $\delta \in (0, 1)$.

First, assume there is no recall, so agent $t$ leaves at the end of period $t$ if they do not buy. A mechanism $\langle P_t, Y_t \rangle$ gives the probability of allocating the object in period $t$ as a function of the reports until time $t$, and the corresponding payment. Agent $t$’s utility is then given by

$$u_t = v_t \delta^t P_t - Y_t$$

and the firm’s profits equal $\Pi = \sum_t Y_t$. Assume $\text{MR}(v) = v - (1 - F(v))/f(v)$ is increasing.

(a) Argue that the firm’s profit is given by

$$\Pi = E_0 \left[ \sum_t \delta^t P_t \text{MR}(v_t) \right]$$

where $E_0$ is the expectation at time 0, over all sequences of values. [Note: you don’t have to be formal]

(b) What is the optimal allocation in period $T$? [Hint: it is easy to think in terms of cutoffs $v^*_T$, where the firm is indifferent between allocating the object and not]

(c) Using backwards induction, characterize the optimal cutoff in period $t < T$? What happens to the cutoffs over time?

Next, suppose there is recall. Hence agent $t$ can buy in any period $\tau \geq t$. A mechanism $\langle P_{t,\tau}, Y_t \rangle$ gives the probability of allocating the object to agent $t$ in period $\tau$ as a function of the reports until time $\tau$, and the corresponding payment. Agent $t$’s utility is then given by

$$u_t = \sum_{\tau \geq t} v_t \delta^\tau P_{t,\tau} - Y_t$$

and the firm’s profits equal $\Pi = \sum_t Y_t$. 
(d) Argue that the firm’s profit is given by

\[ \Pi = E_0 \left[ \sum_t \sum_{\tau \geq t} \delta^{\tau} P_{t,\tau} MR(v_t) \right] \]

(e) When there is recall, what is the optimal allocation in period \( T \)?

(f) Using backwards induction, what is the optimal allocation in period \( t < T \)? What happens to the cutoffs over time? [Hint: try \( t = T - 1 \) and \( t = T - 2 \), and the general case only if you have time]

3. Moral Hazard with Persistent Effort

An agent chooses effort \( e \in \{e_L, e_H\} \) at time 0 at cost \( c(e) \in \{0, c\} \). At time \( t \in \{1, 2\} \), output \( y_t \in \{y_1, \ldots, y_N\} \) is realized according to the IID distribution \( \Pr(y_t = y_n|e) = f(y_n|e) \).

A contract is a pair of wages \( \langle w_1(y_1), w_2(y_1, y_2) \rangle \). The agent’s utility is then

\[ u(w_1(y_1)) + u(w_2(y_1, y_2)) - c(e) \]

where \( u(\cdot) \) is increasing and concave, while the firm’s profits are

\[ y_1 + y_2 - w_1(y_1) - w_2(y_1, y_2) \]

where we ignore discounting. The agent has outside option \( 2u_0 \). Also, assume the principal wishes to implement effort \( e_H \).

(a) What is the first best contract, assuming effort is observable? [Note: A formal derivation is not necessary].

(b) Suppose the firm cannot observe the agent’s effort. Set up the firm’s problem.

(c) Characterise the optimal first-period and second-period wages.

(d) Suppose output is binomial, \( y_t \in \{y_L, y_H\} \). Let \( f(y_H|e_L) = \pi_L \) and \( f(y_H|e_H) = \pi_H \). How do wages vary over time? In particular, can you provide a full ranking of wages across the different states and time periods?