Economics 11: Homework 2

October 19, 2008

Due date: Tuesday 28th October.

Instructions: You are required to write up your solution separately and independently, although you are encouraged to discuss and work in groups. Please write your name, student ID number, and the name of your TA on the front page of the assignment that you hand in. Also, please put boxes around your final answer to each part.

1. Budget Sets with Price Discounts

There are two goods: $x_1$ and $x_2$. The seller of $x_2$ charges $p_2 = 2$. The seller of $x_1$ offers price discounts. If the agent buys $x_1 < 10$ she pays $p_1 = 2$ for every unit. If the agent buys $x_1 \geq 10$ she pays $p_1 = 1$ for every unit (including the first 10).

(a) Suppose $m = 30$. Draw the agent’s budget set.

(b) Assume the agent has monotone preferences. Do preferences exist such that the agent chooses (i) $x_1 = 3$, (ii) $x_1 = 7$ and (iii) $x_1 = 12$? Explain your answers.

2. Intertemporal Choice with Differential Interest Rates

An agent allocates consumption across two periods. Let the consumption in period $t$ be $x_t$, and the income in period $t$ be $m_t$. The agent’s utility is

$$u(x_1, x_2) = \ln(x_1) + \frac{3}{4} \ln(x_2)$$

The agent is poor in period 1 but wealthy in period 2. In particular, she has income $m_1 = 3$ in period 1 and $m_2 = 4$ in period 2.

(a) Suppose the agent can borrow and save at interest rate $r = 1/3$, so $\$1$ in period 1 is worth $\$(1+1/3)$ in period 2. Sketch the agent’s budget constraint. Solve for her optimal consumption.
(b) Suppose the agent can still save at \( r = 1/3 \), but can only borrow at \( r = 1/2 \). Sketch the agent’s budget constraint. Solve for her optimal consumption.

(c) Suppose the agent can still save at \( r = 1/3 \), but can only borrow at \( r = 1 \). Sketch the agent’s budget constraint. Solve for her optimal consumption. [Hint: Beware of kinks.]

3. Consumer Problem

An agent has utility \( u(x_1, x_2) = (x_1^{-1} + x_2^{-1})^{-1} \) for goods \( x_1 \) and \( x_2 \). The prices of the goods are \( p_1 \) and \( p_2 \). The agent has income \( m \).

First, we show that these preferences are convex. We do this three ways.

(a) (i) Solve for equation of a typical indifference curve, specifying \( x_2 \) in terms of \( x_1 \). (ii) Using the fact that \( MRS = -\frac{dx_2}{dx_1} \) find the MRS. (iii) Show that MRS is decreasing in \( x_1 \).

(b) (i) Using the fact that \( MRS = \frac{MU_1}{MU_2} \) find the MRS. (ii) Using the equation for the indifference curve (see part (a)), substitute for \( x_2 \) and write MRS in terms of \( x_1 \). (iii) Show that MRS is decreasing in \( x_1 \).

(c) (i) Sketch a typical indifference curve. (ii) State the definition of convexity. (iii) Graphically verify the indifference curve you have drawn satisfies convexity.

(d) Write down the agent’s budget constraint.

(e) Solve for the agent’s optimal choice of \((x_1, x_2)\).

(f) Show the agent’s indirect utility function is given by

\[
v = \frac{m}{(p_1^{1/2} + p_2^{1/2})^2}
\]

(g) Solve for the agent’s Hicksian demand.
(h) Solve for the expenditure function.

(i) Verify the Slutsky equation for good $x_1$.

4. Labour Supply

Suppose an agent has the same utility function as in question 3

$$u(x_1, x_2) = (x_1^{-1} + x_2^{-1})^{-1}$$

Suppose that $x_1$ is hours of leisure and $x_2$ is quantity of food. The agent is endowed with $T$ hours to divide between work and leisure. Her wage rate is $w$, while the price of $x_2$ is $p_2 = 1$. The agent has no outside income, $m = 0$.

(a) Derive the budget constraint of the agent.

(b) Solve for the agent’s optimal choice of $(x_1, x_2)$. [Hint: the answer is very similar to part (e) of question 3.]

(c) How does the agent’s leisure consumption change as a function of her wage? Explain this intuitively in terms of income and substitution effects.

For your own interest...

The following questions are optional.

(d) Derive the Hicksian demands.

(e) Verify the Slutsky equation with endowments. That is,

$$\frac{\partial x_1^*}{\partial w} = \frac{\partial h_1}{\partial w} - (x_1^* - T) \frac{\partial x_1^*}{\partial m}$$

Note: The final term $\partial x_1^*/\partial m$ involves ‘m’. For this term, you should use the demand calculated in part (e) of Question 3.