1. Cost Functions

A firm has cost function \( c(q) = 100 - 10q + 5q^2 \).

a) Find the fixed cost.\(^1\)

b) Find the variable cost.\(^2\)

c) Find the average cost.

d) Find the marginal cost.

e) Draw the relationship between MC and AC. Prove that they always intersect at the minimum AC.

2. Cost Functions

A firm has production function

\[ f(z_1, z_2) = z_1^{1/2}(z_2 - 1)^{1/2} \]

The prices of the inputs are \( r_1 \) and \( r_2 \).

(a) Find \( MP_1 \), \( MP_2 \), and \( MRTS \).

(b) If \( z_2 \) is fixed at 5, what is the short-run cost function? Find the short-run marginal cost and average cost.

(c) What are the long-run input demand functions? What is the long-run cost function? Find the long-run marginal cost and average cost.

(d) Does the production function exhibit increasing, constant or decreasing returns to scale.

\(^1\) Definition: The fixed cost is the cost that is independent of output.

\(^2\) Definition: The variable cost is the cost that varies with the level of output.
3. Cost Minimisation: Cobb Douglas

Suppose that a firm production function is given by the Cobb-Douglas function: \( f(z_1, z_2) = z_1^\alpha z_2^\beta \). The cost of the inputs is \( z_1 \) and \( z_2 \).

a) Find marginal and average productivity of the two factors.

b) Does this production function have increasing, constant or decreasing returns to scale?

c) Show that cost minimisation requires \( \beta r_1 z_1 = \alpha r_2 z_2 \).

d) Suppose \( \alpha = \beta = 1/4 \). Find the cost function.

4. Returns to Scale

A firm has production function \( f(z) = z^\alpha \). The price of the input is \( r \).

(a) For which values of \( \alpha \) does the technology have increasing, decreasing and constant returns?

(b) Show that the cost function is convex when technology has constant or decreasing returns. Provide an intuition.

5. Average Cost and Marginal Cost

(a) Show that \( AC(q) \) is increasing when \( MC(q) \geq AC(q) \), and \( AC(q) \) is decreasing when \( MC(q) \leq AC(q) \). [Hint: Differentiate \( AC(q) \).]

(b) Provide an intuition for the result in part (a).

(c) Suppose \( AC(q) \) is U-shaped. Argue that \( AC(q) = MC(q) \) when \( AC(q) \) is at it’s lowest point.

6. Profit Maximisation

A firm has production function \( f(z) = 2z^{1/2} \). The output price is \( p \); the input price is \( r \).
What is the firm’s optimal output? What is the optimal input? What is the firm’s profit?

7. Revenue Maximisation

A firm has cost function \( c(q) = 3750 + \frac{1}{2}q^2 \). The price of output is \( p = 100 \).

(a) What is the profit maximising quantity?

(b) What are the maximal profits?

(c) Which quantity maximises revenue,\(^3\) subject to the constraint that profits are positive?

Note: For this question, you may find the quadratic formula useful. If \( ax^2 + bx + c = 0 \) then

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\(^3\)Definition: revenue equals the money the firm gets from selling it’s goods, ignoring the cost.