Second Degree Price Discrimination

- This note (a) explains why SDPD beats normal monopoly pricing.

(b) examine the optimal SDPD scheme with 2 types of consumers.

- Model: Suppose there are equal nos. of 2 types of consumers.
  - high demand have demand \( p = a_h - q \)
  - low demand have demand \( p = a_L - q \)

- assume \( a_h > a_L > \frac{a_h}{2} \). Assume \( MC = 0 \).
  (for simplicity)

- Standard Monopoly price:

\[
\text{Max} \quad p(a_h - p) + p(a_L - p) \\
\text{For}\,(p) \quad a_h - 2p + a_L - 2p = 0 \\
p^* = \frac{a_h + a_L}{4}
\]
- Nonlinear price $\tilde{p}(q)$ improves on $p^m$.
- Firm makes extra profit $A$.
- High type's demand rises $\hat{q}_n \rightarrow \hat{q}_n^*$.

- Nonlinear price $\tilde{p}(q)$ improves on $\bar{p}(q)$.
- Firm makes extra profit $B$.

- Nonlinear price $\tilde{p}(q)$ improves on $\bar{p}(q)$.
- Firm makes extra profit $C$.

In fact, given firm sells $\hat{q}_L$, this is the best the firm can do.
- The last pricing scheme, \( p(q) \), looks quite complicated.

- Is there another way the firm can implement this?

- Suppose firm sells 2 bundles:
  1. Buy \( q^L \) units at price given by area D.
  2. Buy \( q^H \) units at price given by area D + F.

- Low agents will buy bundle (1), while high types buy (2).

- We can use this picture to show this pricing scheme is the best the firm can do, conditional on selling \( q^L \) to low types.

- Suppose firm sells \( q^L \) to low demand agents.

- Most firm can extract is D.

- If high types copy low types, they can always guarantee themselves come up plus E.

- Hence most firm can charge for \( q^H \) units is D + F.
What is optimal choice of \( q^*_L \)?

- What quantity should firm sell to low types?
- First suppose \( q^*_L = q^*_L^* \), the socially optimum quantity.
  - Again firm sells two bundles:
    1. \( q^*_L \) units at \( p = D \)
    2. \( q^*_L \) units at \( p = D + F \)
  - Total profits \( 2D + F \).

- Now suppose firm \( q^*_L \) by a little.
  - Suppose firm sells \( q^*_L + \delta \) to low agents.
  - Change in profit:
    - Lost \( AD \) on low types since sell less of good
    - Made \( AE \) on high types since consumer replies lower
    - Observe \( AE > AD \). Hence reduction in \( q^*_L \) increases profit.
We know the firm wants to undersupply the low agent. That is, they supply the agent with less than the efficient amount, \( q^* \). Intuitively, the lost profit on low agents is less than the extra profit made from high types. [recall the duopoly quote]

**How far should the firm reduce \( q^* \)?**

The answer is easy: they should equate marginal benefit and marginal costs, i.e., \( AD = AE \).

- \( AD \) is proportional to the difference between the demand curves, \( a_H - a_L \).
- \( AE \) is proportional to the height of the low demand curve.

Here, \( AE = AD \) when the low demand curve has height \( a_H - a_L \). That is, \( \bar{q}_L = 2a_L - a_H \).

Intuitively, if \( 2L > \bar{q}_L \), then \( AE > AD \) and the firm should sell \( \bar{q}_L \).

If \( 2L < \bar{q}_L \), then \( AE < AD \) and the firm should sell \( 2L \).