The Information Economy

Dynamic Prices
Peak Load Pricing

- Suppose a firm has zero marginal cost, with capacity $K$
  - Broadband capacity, cell phone towers, number of tickets
  - Capacity costs $z$ per unit to build.
- There are two periods (or two equally likely states)
  - Period L demand is low, $p_L(q)$
  - Period H demand is high, $p_H(q)$
- Firm chooses $q_L$, $q_H$ and $K$ to maximize profits
  \[
  \pi = q_L p_L(q_L) + q_H p_H(q_H) - zK \quad \text{subject to } q_L, q_H \leq K
  \]
- Lagrangian: choose $q_L$, $q_H$ and $K$ to maximize
  \[
  L = q_L p_L(q_L) + q_H p_H(q_H) - zK + \lambda_L [K - q_L] + \lambda_H [K - q_H]
  \]
Peak Load Pricing

Solution

- FOCs for $q_L, q_H$ and $K$: $MR_L(q_L^*) = \lambda_L$, $MR_H(q_H^*) = \lambda_H$, $z = \lambda_L + \lambda_H$
- Optimal capacity: $K^* = q_H^*$

Idea: Charge capacity when constraint binds.

Two cases:

1. Constraint slack in period L (big difference in demands)
2. Constraint binds in period L (small difference in demands)

Price in H higher for two reasons

(a) The demand is higher,
(b) Charging more of the capacity

Examples: cheap evening calls and Christmas flights
1. Constraint Slack in Period L ($q_L^*<q_H^*$)

- Optimal quantities: $MR_L(q_L^*)=0$, $MR_H(q_H^*)=z$
2. Constraint Binds in Period L \((q_L^*<q_H^*)\)

- Optimal quantities: \(q_L^* = q_H^*\), \(\text{MR}_L(q_L^*) = \lambda_L\), \(\text{MR}_H(q_H^*) = z - \lambda_L\)
Revenue Management

- A firm has $K$ tickets to sell
  - Airline seats, hotel rooms, advertising slots
- Customers arrive over time
  - Customers have value $v$ unknown to firm
- How should firm set prices over time?
  - If lower price then:
    1. sell to marginal agents today
    2. make less revenue from inframarginal agents
    3. lose opportunity to sell tomorrow
Revenue Management: Example

- Example: one item to sell (K=1)
  - There are N customers with v~U[0,1]

- Last customer
  - Choose $p_N$ to maximize $\Pi_N = (\text{prob sell}) \times \text{price} = (1-p_N)p_N$.
  - Solution: $p_N^* = 0.5$, yielding $\Pi_N^* = 0.25$.

- Dynamic programming: suppose $n^{th}$ customer arrives
  - Choose $p_n$ to maximize $\Pi_n = (1-p_n)p_n + p_n \Pi_{n+1}$.
  - Solution: $p_n^* = 0.5[1 + \Pi_{n+1}]$, yielding $\Pi_n^* = 0.25[1 + \Pi_{n+1}]^2$

- Working backwards with 5 customers:

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<thead>
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<th>4\textsuperscript{th}</th>
<th>3\textsuperscript{rd}</th>
<th>2\textsuperscript{nd}</th>
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<tbody>
<tr>
<td>Price, $p_n^*$</td>
<td>0.5</td>
<td>0.63</td>
<td>0.70</td>
<td>0.74</td>
<td>0.78</td>
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<tr>
<td>Profit, $\Pi_n^*$</td>
<td>0.25</td>
<td>0.39</td>
<td>0.48</td>
<td>0.55</td>
<td>0.60</td>
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Durable Goods and Price Commitment

- Apple is thinking how to price the iPhone
  - In the first year it sells to high value customers
  - Then lowers price to sell to low value customers

- Problem: Customers anticipate price will fall
  - Customer delay purchases until price falls
  - Monopolist competes with future selves

- Model applies to durable goods
  - Software, Xbox, Art

- Model applies to durable services
  - Movies, information goods.
Durable Goods: Example

- N customers have v=30, N customers have v=10.
- Suppose there are two periods, with discount rate $\delta$
  - If commit to one price, charge $p=30$, profit $\Pi = 30N$.
- Suppose sell to high agents in period 1
  - Charge $p_2 = 10$ and sell to low agents in period 2.
- High agents anticipate price will fall and may wait
  - Charge at most $p_1 = 30-20\delta$, for high agents to buy in period 1
  - Total profits $\Pi = (30-20\delta)N + \delta(10)N = (30-10\delta)N$
- Firm suffers because it cannot commit
  - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.
Durable Goods: Solutions

- **Solution 1: Destroy the mould (e.g. artist)**
  - Without mould cannot create quantity in second period

- **Solution 2: Reputation (e.g. record companies)**
  - Develop reputation for not dropping prices

- **Solution 3: Renting (e.g. Xerox)**
  - Good no longer “durable”, so sell static monopoly quantity each period

- **Solution 4: Best-price provision (e.g. iPhone)**
  - If firm lowers price then customers get rebate
  - Firm never any incentive to lower price below monopoly price since lose money in rebates
Behavior Based Pricing and Commitment

- Suppose a firm sells to customers multiple times
- Purchasing behavior in early period tells firm about values
  - Firm tempted to condition price on past behavior
- Problem: Customers anticipate “ratchet effect”
  - Customers delay purchases to get lower prices later
  - Monopolist competes with her future selves
- Applications
  - Online sites with cookies, magazine subscriptions, cable TV
Behavior Based Pricing: Example

- N customers have v=30, N customers have v=10.
- Suppose there are two periods, with discount rate $\delta$.
  - If cannot see past behavior, charge $p=30$, profit $\Pi_0 = 30(1+\delta)N$.
- Suppose sell to high agents in period 1.
  - Charge $p_2=10$ if did not buy in period 1.
  - Charge $p_2=30$ if bought in period 1 (ratchet effect).
- If customers myopic charge $p_1=30$.
  - Total profits $\Pi_M = 30N + \delta(30+10)N = (30+40\delta)N > \Pi_0$.
- If customers forward looking, anticipate price fall if don’t buy.
  - Charge at most $p_1=30-20\delta$, for high agents to buy in period 1.
  - Total profits $\Pi_F = (30-20\delta)N + \delta(30+10)N = (30+20\delta)N < \Pi_0$.
- Firm suffers because it cannot commit.
  - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.
Why are there Introductory Discounts?

- Behavioral-based pricing view
  - Firms can’t resist giving discount to people who don’t purchase
  - These discounts hurt the firm if
    - (a) Consumers are forward looking
    - (b) Consumers get annoyed
- Introductory discounts may be good idea
  - Network effects (see network slides)
  - Overcome switching costs (see lockin slides)
  - Encourage customer experimentation (next slide)
Customer Experimentation

- Product is “experience good”
  - Don’t know taste until tried it
- Customers have value $v=30$ or $v=10$ with equal prob.
  - Optimal pricing: niche market strategy
    - Period 1, charge price $p_1=20$, and everyone buys
    - Period $t \geq 2$, charge price $p_t=30$, and high value agents buy
- Customers have value $v=30$ or $v=20$ with equal prob.
  - Optimal pricing: mass market strategy
    - Period 1, charge price $p_1=25$, and everyone buys
    - Period $t \geq 2$, charge price $p_t=20$, and everyone buy
How does a firm price when it does not know demand?

- Firm wishes to sell a unique good.
- Customers enter each period (not forward looking)
- Each buyer has the same value, $v$, unknown to firm

**Optimal policy: start price high and lower slowly.**

- Solve through backwards induction.
- Rate of decrease depends on firm’s patience.

What if have good each period to sell?

- Price may go up or down.
- But should move prices around to experiment.

Experimentation very easy online
Inventories

- Need inventories because
  - Fixed costs to order
  - Take time for delivery to arrive
- Firm should adopt \((S,s)\) rule.
  - If inventories fall below \(s\) then bring back to \(S\).
- Demand shocks
  - If demand has transitory increase, bring inventory back to \(S\)
  - If demand has permanent increase, also increase \(S\).
- If it take time to place new order, then use revenue management to find optimal prices.