The Economics of E-commerce and Technology

Dynamic Pricing
Peak Load Pricing

- Suppose a firm has zero marginal cost, with capacity $K$
  - Broadband capacity, cell phone towers, hotel rooms
  - Capacity costs $z$ per unit to build.
- There are two periods (or two equally likely states)
  - Period $L$ demand is low, $p_L(q)$
  - Period $H$ demand is high, $p_H(q)$
- Firm chooses $q_L, q_H$ and $K$ to maximize profits
  \[ \pi = q_L p_L(q_L) + q_H p_H(q_H) - zK \] subject to $q_L, q_H \leq K$
- Lagrangian: choose $q_L, q_H$ and $K$ to maximize
  \[ L = q_L p_L(q_L) + q_H p_H(q_H) - zK + \lambda_L [K - q_L] + \lambda_H [K - q_H] \]
Peak Load Pricing

- **Solution**
  - FOCs for $q_L, q_H$ and $K$: $MR_L(q_L^*) = \lambda_L$, $MR_H(q_H^*) = \lambda_H$, $z = \lambda_L + \lambda_H$
  - Optimal capacity: $K^* = q_H^*$

- **Idea:** Charge capacity when constraint binds.

- **Two cases:**
  1. Constraint slack in period $L$ (big difference in demands)
  2. Constraint binds in period $L$ (small difference in demands)

- **Price in $H$ higher for two reasons**
  (a) The demand is higher,
  (b) Charging more of the capacity

- **Examples:** cheap evening calls and Christmas flights
1. Constraint Slack in Period L ($q_L^* < q_H^*$)

- Optimal quantities: $\text{MR}_L(q_L^*) = 0$, $\text{MR}_H(q_H^*) = z$
2. Constraint Binds in Period L \( (q_L^{*} < q_H^{*}) \)

- Optimal quantities: \( q_L^{*} = q_H^{*} \), \( MR_L(q_L^{*}) = \lambda_L \), \( MR_H(q_H^{*}) = z - \lambda_L \)
Revenue Management

- A firm has K tickets to sell
  - Airline seats, hotel rooms, advertising slots
- Customers arrive over time
  - Customers have value v unknown to firm
- How should firm set prices over time? If lower price:
  (a) sell to marginal agents today
  (b) make less revenue from inframarginal agents
  (c) lose opportunity to sell tomorrow
Revenue Management: Example

- Example: one item to sell (K=1)
  - There are N customers with v~U[0, 1]

- Last customer
  - Choose \( p_N \) to maximize \( \Pi_N = (\text{prob sell}) \times \text{price} = (1 - p_N)p_N \).
  - Solution: \( p_N^* = 0.5 \), yielding \( \Pi_N^* = 0.25 \).

- Dynamic programming: suppose n\(^{th}\) customer arrives
  - Choose \( p_n \) to maximize \( \Pi_n = (1 - p_n)p_n + p_n \Pi_{n+1} \).
  - Solution: \( p_n^* = 0.5 \left[ 1 + \Pi_{n+1} \right] \), yielding \( \Pi_n^* = 0.25 \left[ 1 + \Pi_{n+1} \right]^2 \)

- Working backwards with 5 customers:

<table>
<thead>
<tr>
<th></th>
<th>5(^{th})</th>
<th>4(^{th})</th>
<th>3(^{rd})</th>
<th>2(^{nd})</th>
<th>1(^{st})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price, ( p_n^* )</td>
<td>0.5</td>
<td>0.63</td>
<td>0.70</td>
<td>0.74</td>
<td>0.78</td>
</tr>
<tr>
<td>Profit, ( \Pi_n^* )</td>
<td>0.25</td>
<td>0.39</td>
<td>0.48</td>
<td>0.55</td>
<td>0.60</td>
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Durable Goods and Price Commitment

- Apple is thinking how to price the iPhone
  - In the first year it sells to high value customers
  - Then lowers price to sell to low value customers
- Problem: Customers anticipate price will fall
  - Customer delay purchases until price falls
  - Monopolist competes with future selves
- Model applies to durable goods
  - Software, Xbox, Art
- Model applies to durable services
  - Movies, information goods.
Durable Goods: Example

- $N$ customers have $v=30$, $N$ customers have $v=10$.

- Suppose there are two periods, with discount rate $\delta$.
  - If commit to one price, charge $p=30$, profit $\Pi = 30N$.

- Suppose sell to high agents in period 1.
  - Charge $p_2=10$ and sell to low agents in period 2.

- High agents anticipate price will fall and may wait.
  - Charge at most $p_1=30-20\delta$, for high agents to buy in period 1.
  - Total profits $\Pi = (30-20\delta)N + \delta(10)N = (30-10\delta)N$.

- Firm suffers because it cannot commit.
  - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.
Durable Goods: Solutions

- Solution 1: Destroy the mould (e.g. artist)
  - Without mould cannot create quantity in second period
- Solution 2: Reputation (e.g. Xbox games)
  - Develop reputation for not dropping prices
- Solution 3: Renting (e.g. Xerox)
  - Good no longer “durable”, so sell static monopoly quantity each period
- Solution 4: Best-price provision (e.g. iPhone)
  - If firm lowers price then customers get rebate
  - Firm never any incentive to lower price below monopoly price since lose money in rebates
Behavior Based Pricing and Commitment

- Suppose a firm sells to customers multiple times
- Purchasing behavior in early period tells firm about values
  - Firm tempted to condition price on past behavior
- Problem: Customers anticipate “ratchet effect”
  - Customers delay purchases to get lower prices later
  - Monopolist competes with her future selves
- Applications
  - Online sites with cookies, magazine subscriptions, cable TV
Behavior Based Pricing: Example

- N customers have v=30, N customers have v=10.

- Suppose there are two periods, with discount rate $\delta$
  - If cannot see past behavior, charge $p=30$, profit $\Pi_0 = 30(1+\delta)N$.

- Suppose sell to high agents in period 1
  - Charge $p_1=10$ if did not buy in period 1
  - Charge $p_2=30$ if bought in period 1 (ratchet effect)

- If customers myopic charge $p_1=30$
  - Total profits $\Pi_M = 30N + \delta(30+10)N = (30+40\delta)N > \Pi_0$

- If customers forward looking, anticipate price fall if don’t buy
  - Charge at most $p_1=30 - 20\delta$, for high agents to buy in period 1
  - Total profits $\Pi_F = (30-20\delta)N + \delta(30+10)N = (30+20\delta)N < \Pi_0$

- Firm suffers because it cannot commit
  - Firm cannot resist lowering price in period 2, exerting a negative externality on its former self.
Why are there Introductory Discounts?

- **Behavioral-based pricing view**
  - Firms can’t resist giving discount to people who don’t purchase
  - These discounts hurt the firm if
    (a) Consumers are forward looking
    (b) Consumers get annoyed

- **Introductory discounts may be good idea**
  - Network effects (see network slides)
  - Overcome switching costs (see lockin slides)
  - Encourage customer experimentation (next slide)
Customer Experimentation

- **Product is “experience good”**
  - Don’t know taste until tried it

- **Customers have value $v=30$ or $v=10$ with equal prob.**
  - Optimal pricing: niche market strategy
  - Period $1$, charge price $p_1=20$, and everyone buys
  - Period $t \geq 2$, charge price $p_t=30$, and high value agents buy

- **Customers have value $v=30$ or $v=20$ with equal prob.**
  - Optimal pricing: mass market strategy
  - Period $1$, charge price $p_1=25$, and everyone buys
  - Period $t \geq 2$, charge price $p_t=20$, and everyone buy
Firm Experimentation

- How does a firm price when it does not know demand?
  - Firm wishes to sell a unique good.
  - Customers enter each period (not forward looking)
  - Each buyer has the same value, \( v \), unknown to firm

- Optimal policy: start price high and lower slowly.
  - Solve through backwards induction.
  - Rate of decrease depends on firm’s patience.

- What if have good each period to sell?
  - Price may go up or down.
  - But should move prices around to experiment.

- Experimentation very easy online
  - Run A/B tests. Seems scientific, but can be misleading.
Sticky Prices

- For single firm, choose price so $d\pi(p^*)/dp = 0$.
- Suppose there is inflation and cost $k$ of changing prices
  - E.g. Physical cost of printing menu
- No reason to change price when almost optimal.
  - Costs $k$, but only second order gain.
- Optimal policy is called $(S,s)$ rule.
  - When real price hits $p_L < p^*$ then increase to $p_H > p^*$.
- Similar story when demand subject to shocks.
  - If demand has transitory increase, bring
  - If demand has permanent increase, also increase $S$. 