A Reformulation of
Orthodox Value Theory

by
Earl A. Thompson
University of California, Los Angeles

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A REFORMULATION OF ORTHODOX VALUE THEORY CONTAINING AN EXPLICIT INFORMATION STRUCTURE, INTERPERSONAL FORCE, IRRATIONALITY, AND BANKRUPTCY

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INTRODUCTION

This paper generalizes the standard competitive model by simultaneously (1) widening the class of admissible interpersonal information differences implicit in the standard model, (2) allowing for involuntary behavior and interpersonal force, (3) dropping the traditional rationality and non-saturation assumptions, and (4) allowing individuals to go bankrupt. The paper then shows the Pareto optimality of an equilibrium in the generalized competitive model.

This generalized model is a derivation from a model of N-person bargaining under private property constrained to satisfy only a certain, proper subset of the set of basic conditions of the standard competitive model.

But before any such derivation is possible, conceptual difficulties common to both standard N-person game theory (e.g., Owen) and the standard competitive model (e.g., Debreu) must be removed. This is done in Section 1 of the paper. One of these difficulties arises out of the incompleteness of the standard descriptions of "a given technology" necessary to the definition of individual and social optimality. The approach of Professor

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Radner, heretofore the only author to attempt to supply a complete, explicit structure of technological information for a standard competitive model, is found to contain serious methodological and technical errors.

Another of these conceptual difficulties in standard models is a lack of behavioral foundation for the use of Pareto optimality as a requirement for social optimality. Once the standard metaphysical foundation of Bergson and Samuelson is replaced with one based on a model of behavior in the real world, the behavioral rationality (and "local nonsaturation" [Koopmans]) of at least one individual becomes a requirement of any model in which social optimality implies Pareto optimality. There is also a conceptual difficulty in that the definition of a private property system of standard general equilibrium theory lacks the generality of the definition implicit in the common law and in orthodox economic discussions of the nature of externalities (e.g., Ellis and Fellner, Meade, and Coase). The more general definition of a private property system permits ownership to be divorced from control and explicitly admits the possibility of private interpersonal force as long as specified compensations are paid. Another difficulty is the absence of a concept of a transaction process and transaction costs, concepts which we find necessary to the derivation of solution sets in N-person bargaining theory, and, in particular, in deriving a standard competitive model. Finally, there is a difficulty in the specification of the costs of knowing the institutional system, costs which must be made explicit in order to derive transaction costs and in order to have a complete description of the information structure of an economic model.

Section 2 employs a model of N-person bargaining under private property. By specifying its different solution sets under different structures of transaction costs, this section establishes the transaction cost structure
and institutional information structure (i.e., the contract information structure) of a standard competitive model. The properties of zero costs of all transactions and costlessly perfect contract information typically assumed in informal discussions of the basis of the standard competitive model are found to be inconsistent with the standard competitive model as well as the recent models of Edgeworth processes generating the core and core-related solutions. Nevertheless, the less perfect contract/generating Edgeworth games and the core, and the still less perfect contract information structure of a standard competitive model, are seen to be very stringent. The claim of Arrow (1963, 1968) and others that the standard competitive model breaks down when individuals can control contingencies made the basis for contingent claim contracts, and the claim of Radner and others that the standard competitive model breaks down when there are costs of one's currently computing what would be his optimal decisions in the future in case of the occurrence of alternative future events, are seen to be incorrect.

Our generalization of the standard competitive model is carried out in Section 3, only after the basic concepts have been defined and the contract information structure specified. The generalized competitive model is a highly orthodox model in that it retains the familiar assumption of non-discriminatory and constant prices to all individuals as well as the stringent, but necessary, conditions on contract information costs implied by the standard model. Our reformulation of orthodox value theory is based solely upon generalizations of (1) the overly stringent conditions on technological information, (2) the overly stringent condition on rationality, and (3) the overly restrictive definition of private property implied by the standard competitive model. That is, the reformulated orthodox theory of value permits a much more general technological information structure than the
standard competitive models (although it retains the special conditions on contract information implied by the standard model), a more general definition of private property than appears in the standard competitive model, and a complete removal of the rationality assumption which has long been basic to the standard competitive model. Yet any equilibrium of the reformulated competitive model is still a Pareto optimum as long as Pareto optimality is a necessary condition for social optimality.

Section 4 of the paper is an extension and application of the general model to the phenomenon of bankruptcy. Bankruptcy, like a certain kind of irrationality, is seen to break down the Pareto Optimality of a general competitive equilibrium when and only when there are positive costs of "counterforce transactions." The policy implications of such market breakdowns are peculiar in that no tax-subsidy policies exist which will generally create a Pareto optimum.

Section 5 contains a summary of the propositions developed in this paper describing conditions sufficient for the Pareto optimality of generalized competitive equilibria with and without positive costs of applying counterforce.¹

¹We shall adopt a convention of capitalizing the first letters of terms, once they are defined, when the terms do not appear in conventional mathematical models but do appear in a mathematical description of our model. We hope that the use of this procedure instead of the usual symbolics will substantially promote understanding of the foundations and working of our model without sacrificing the clarity afforded by a mathematical exposition. However, the abstract mathematical description of the model will be sent on request.
1. BASIC CONCEPTS

   a. The meaning of a "given technology."

      If everyone shared the same perceptions, i.e., made the same distinctions
between all conceivable events and placed the same subjective probabilities
on these events (including the emotional states of individuals), then it
would be a simple matter to describe a "given technology." It would be the
technology which people perceive, any individual's perceptions of tastes,
production possibilities, and occurrences. However, if everyone had the same
initial perceptions of the technology, there would be no Pareto superiority
of any allocation system over a dictatorship (assuming no diseconomies of
calculation or enforcement) since the latter would have no information problems
in such a world. Prices would lose their positive value as a device which
obviates or substitutes for the costly direct communication of perceived
preferences, production possibilities, and occurrences. Yet when some inter-
personal differences in initial perceptions exist, it is not immediately clear
how we should describe the "given technology" available to society, the
"social technology."

      The approach of Radner, the only author who has heretofore attempted to
develop a general equilibrium model with an explicit structure of technological
information, is to accept the set of initial technological perceptions of an
individual as the technology relevant to the individual's socially feasible
benefits. That is, Radner restricts each individual's socially feasible
consumption and production sets to functions of a set of perceptions containing
only the individual's own initial perceptions of the technology. The fault
with any such restriction on social information is that it cuts the Gordian
Knot. It fails to allow institutional systems their traditional role of
generating parameters or inducing behaviors which obviate or substitute for the costly direct communication of technological information.\(^1\) In an allocation which is socially optimal independent of the alternative systems of allocation, everyone’s complete set of perceptions of the technology must be costlessly available in making each social choice.

Following Savage, Radner allows interpersonal "differences in information" if and only if some people detect distinctions between truly different events while others do not. However, it is well-known (see, for example, Marschak) that even if people always make the same distinctions between events, they may place different subjective probabilities on the occurrence of these events and so have differences in information. The analyses of Koopmans and Debreu (1959, ch. 7) apparently permit differences in technological information of the latter variety. While the technological information structure is entirely implicit in these analyses in that it is embedded in the individual preference and production sets, we shall soon see that their results imply very severe restrictions on the extent of the interpersonal differences in technological information.

To help set the above ideas, and to help introduce another, consider the following examples of Radner and Koopmans-Debreu equilibria. Suppose an individual is unusual in that he can never perceive whether or not it is raining. In addition, suppose that if he is caught outside in the rain, he

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\(^1\)However, it is implicit in Radner's model, as it is in any standard general equilibrium model, that the competitive price system serves to obviate some of the costly direct communications of technological information that would generally appear in alternative systems of allocation. In particular, the competitive price system serves to obviate the costly communication of technological information useful to making the commodity production and distribution decisions in a dictatorship. This is only implicit in the standard model because it is tacitly assumed in defining both Pareto optimality and competitive equilibrium that no resources are used up in achieving alternative commodity productions and distributions. We shall more accurately characterize this implicit assumption in the analyses below.
will catch pneumonia because of his inability to perceive the rain and will subsequently suffer interminably. Say the individual rationally becomes a recluse whereas he certainly would not become such if only he could distinguish between rainy and dry weather. Radner would call the associated allocation a "competitive equilibrium" and a "Pareto optimum relative to an information structure." But it is intuitively clear that even under this technological information structure everyone can be made better off by taxing the recluse on rainy days for being out of doors. (Remember that the other people, who set the out-of-doors tax, can distinguish rainy from non-rainy days.) Now consider an example of a Koopmans-Debreu equilibrium. Suppose that the initially rich individuals in an initial competitive equilibrium suddenly dream that all chickens lay one million eggs per day and the well-informed, initially poor, chicken farmers are sure that chickens lay at most one egg per day (and everybody's information is the same otherwise.) Then the mistaken rich will proceed to bid up the current price of chickens (and down the future price of eggs) to fantastic levels and induce a great increase in the breeding of chickens. According to the analyses of Koopmans and Debreu, this represents an efficient response to the dream -- a move to a new competitive equilibrium and Pareto optimum. But we intuitively know it is not a Pareto optimum because the initially rich dreamers have inferior technological information. In this equilibrium, as in the first, everyone can be made better off if some individuals are induced to behave as if they had certain perceptions even if they don't have them.

For the first example, Radner's model formally precludes the possibility of social improvement because it artificially restricts the socially feasible consumption sets of the individual to those consistent with the individual's own technological information endowment. We shall allow initial information
differences such as appears in the first example, but shall not make Radner's methodological error. If a perception is had by one individual, we consider it potentially available to everyone as part of the "social technology."

If differing abilities to perceive certain distinctions were the only source of information differences, we would characterize the social technology as the union of perceived distinctions which the various individuals make between events.

However, as the second example illustrates, individuals may place different subjective probabilities on the same socially perceivable event. Debreu and Koopmans do not account for the possibility of the Paretoian improvement of their equilibria in the presence of these information differences. One might attempt to rationalize their results on the Pareto optimality of competitive equilibrium by a strong Radner-type assumption applied to differences in subjective probabilities. In particular, if no institutions could ever prevent one from gambling resources on his information as he does in the standard model (or permit an individual to benefit from the "superior" subjective probability distributions of others), then it would be impossible to improve an equilibrium such as appears in the example. But the antecedent would be simply incorrect, for a dictatorship or a democracy could (and does) obviously restrict such gambles. The correct rationalization of the results of Koopmans and Debreu-- a rationalization which applies as well to the results of the Walrasian "certainty models" of Arrow (1951) and Arrow-Debreu (1954) and to the model of the standard Marshallian textbook-- is that the models contain the tacit assumption that every individual has the same technological information regarding production possibilities and the occurrence of future events but has "superior" information regarding his own tastes. Our objection to these models is then simply that the technological information structure is
much too narrow. We shall, however, use this stringent technology in Section II in inferring the non-technological information structure of the standard competitive model.

The specification of a social technology once we generalize the narrow structure of technological information implicit in the Walras-Marshall-Arrow-Debreu-Koopmans model is not as direct as the above mentioned correction of the methodological error in Radner's definition of social information. For a union of individual subjective probabilities does not represent sufficient data to determine socially optimal behavior. And there is no obvious function from the various individual subjective probabilities to the social probabilities used in determining socially optimal behavior.

The difficulty of depicting social information in terms of individual information in the form of subjective probability distributions should be expected as it reflects the following, elementary result in the theory of inference: If a relationship between the observations in different samples exists so as to affect the posterior probability distribution inferred from the joint sample, then efficient estimators are not simply combinations of the posterior probability distributions resulting from the different samples.

It thus provides more valuable information, and is more basic, to define social information in terms of sources of individual posterior probability judgments themselves. We call any current source of these judgments (and of the distinctions between events) a Technological Fact. A Technological Fact may be either an observation, model, or theorem which is available in making current decisions. We assume that each set of Technological Facts uniquely determines a set of distinctions between events and the probability distributions over the events. Each individual is given an initial set of
Technological Facts, his Technological Information. The set of these sets of Technological Facts of the individuals is called the Technological Information Structure of the economy. The union of the sets of Technological Facts of all individuals at any given date is the Social Technology at that date—the "given technology" employed in the determination of socially optimal behavior at that date.

On the other hand, the set of Technological Facts of an individual is the "given technology" available to the individual for the determination of his privately optimal behavior at the corresponding date once we specify the constraints imposed upon him by an institutional system. The Technological Information Structure and the Social Technology may vary over time in any way; for example, in contrast to Radner's model, an individual may forget old distinctions or learn of new possibilities. But we assume (as does Radner) that no individual can costlessly, directly communicate a Technological Fact to another individual. This last assumption is very important for it allows institutions to permit the achievement of social optimality in a general technology only if these institutions serve to obviate or substitute for the direct communication of Technological Information.

b. The characterization of "social optimality."

Social optimality will be assumed to imply Pareto optimal current behavior where the "given technology" for any period is the period's Social Technology, the union of all of the individuals' Technological Facts in the period. We call such a Pareto optimum a Pure Pareto Optimum because it is determined without reference to institutional constraints. Thus a Pure Pareto Optimum exists when there is no alternative set of (current) individual behaviors which, under the current Social Technology, everyone prefers to the existing set of behaviors.¹

¹We use the following expressions synonymously: He prefers A to B; he receives more benefits under A than under B; his utility is higher under A than B; he is better-off with A than with B.
Future Technological Facts and future actions are relevant to the determination of a Pure Pareto Optimum only in so far as there are current Technological Facts concerning these future events. In particular, "current behavior," or simply "behavior," here includes any strategy—any currently determined future action contingent upon a future Technological Fact or action which is currently viewed as a possibility. Thus, we employ a standard Arrow (1953)-type of analysis with respect to the description of intertemporal equilibria.\footnote{However, these analyses do not imply, as Radner has assumed, that markets do not exist in the future (even when memory is perfect and the set of perceived possibilities is stationary). We shall see in Section 3 that markets in the future generally emerge in a standard competitive model, obviate the costly direct communication of Technological Facts, and expand the set of feasible current behaviors to the extent that all Radner-Savage information differences (where individuals perceive different distinctions with respect to a given set of events) are completely obliterated in a correctly described intertemporal equilibrium of a standard competitive model.}

Since the union of everybody's Technological Facts characterizes the Social Technology, neither the Radner equilibrium nor the Koopmans-Debreu equilibrium in the examples above are Pure Pareto Optima. But we should not be ready to conclude from these examples that there is a "market failure" and a reason for "government intervention." The failure may reside only with the models. There may be no such market failures in a competitive equilibrium derived from a model of N-person bargaining under private property.

What precisely is social optimality? Why is Pareto optimality in any form a requirement of social optimality? It is important to have answers to these questions because a model of the real world supplying answers to these questions must also imply restrictions on the environment of the entire discussion. The currently accepted answer, that of Bergson (1939) and Samuelson (1947), is not based on a model of the real world. The currently accepted description of a "social optimum" is behavior that maximizes an ethically
determined social welfare function (which, like any function of several variables maximized subject to a variable constraint, may be "dictatorial" in the sense of Arrow (1951)). By assuming that it is only "ethical" for a "social welfare" function to be defined over only individual utilities and to possess strictly positive partial derivatives, social optimality comes to imply Pareto optimality (although the converse is generally false.) We reject these metaphysical answers because they are inconsistent with a study based solely on models of the real world. The answers which we do accept are based on a model of the real world. The real world model outlined in the following paragraph suffices to rationalize the use of Pareto optimality as a requirement for social optimality (also see Thompson, 1971). In the model, Pareto optimality is not a requirement for social optimality unless the behavior of at least one individual is Rational in the sense that all behavior which he prefers to his actual behavior are unattainable given the Individual's Technological Facts and the institutional constraints on his behavior. Socially optimal behavior in a society in which everyone is Irrational may be Pareto nonoptimal behavior. Therefore, in any more specific models of the world, such as the competitive model, in which Pareto optimality is assumed to be always required for social optimality, at least one individual must be assumed to be Rational.

Economists are individuals who can produce information relevant to decisions concerning the choice among alternative institutional systems. Assume that an economist (1) costlessly produces whatever information he does produce, (2) can costlessly prevent any other individual from acquiring his information, and (3) is not charitable with his information product. An economist's information is used by individuals who select institutional systems. A "Social Optimum" exists if and only if no economist can profit by selling information to the
institution-selectors. By definition, institution-selectors will personally benefit if they use an economist's information to select a system in which everyone is better off. And, a move from any Pareto nonoptimum to a Pareto optimum can benefit everyone for a certain distribution of utility in the optimum. As a result, assuming that institution-selectors know how to achieve the alternative distributions of benefits in a move from institutions generating an initial Pareto nonoptimum to institutions generating a Pareto optimum, the institution-selectors can always benefit (assuming they are not "locally saturated" in the sense of Koopmans) from information which specifies a move from institutions generating the initial Pareto nonoptimum to institutions generating a Pareto optimum. (An institution-selector may, of course, benefit even more by making some people worse off in moving to the Pareto optimal institutions.) Hence, assuming that institution-selectors are Rational—which means that they behave so as to maximize their benefits under their given constraints—there is positive profit to the economists in providing information on the Pareto optimality of alternative institutional systems.

In the above analysis, if nobody were Rational (or if nobody were locally nonsaturated), nobody would be available to serve as the institution-selector. There would be no reason to assume that there would be positive profits to economists in specifying institutional changes that can benefit everyone. Any goal of Pareto optimality would lose its general behavioral significance. A Social Optimum could easily imply a Pareto nonoptimum. Thus, if Pareto optimality is to be implied by Social Optimality at least one individual must be Rational. (And at least one individual must be locally nonsaturated.) Nevertheless, many individuals can be Irrational (or locally saturated) while Social Optimality requires Pareto Optimality. Hereafter, when we refer to
the Rationality assumption in an economic model, we shall mean the assumption that everyone is Rational, its being understood the discussion itself implies that someone is Rational.

Under the additional assumption that economists cannot produce valuable information on the relative benefits (to institution-selectors) of the alternative distributions of utility characterising the alternative Pareto optima, economists profits to selling any additional information would be nonpositive in any Pareto optimum. All Pareto optima would then be Social Optima so that Pareto optimality would become equivalent to Social Optimality.

c. Actual Behavior, Decisions, and Technological Facts Relevant to a Decision

The Actual Behavior of an individual given his set of Technological Facts--his Technological Information--and his information concerning the constraints which the institutional system imposes upon him--his Institutional Information--is determined by comparisons between all of his feasible Behaviors.¹

There is a Decision for every such comparison to the individual's actual Behavior. For each of these Decisions, only a certain subset of the Social Technology is useful. For example, for the decisions of not producing more wheat with an additional laborer, it is useful to have Technological Facts relevant to the wheat productivity of labor but useless to have Facts relevant to the cloth productivity of capital.

¹We shall not assume a transitive ordering between alternatives because no assumption is made regarding the nature of comparisons of non-optimal behaviors. Intransitivities are entirely consistent with our analysis. The same is true of the decisions comparing Pure Pareto Optima with alternative social behaviors. Thus, there is no necessity of a transitivity assumption in any of our analysis. This is important to note here because we intend to drop the Rationality assumption, and transitivity is often confused with Rationality (even though it is neither necessary nor sufficient for Rationality.) In fact, as Sonnenschein has shown, transitivity is not even required for the existence of an equilibrium in the standard competitive model under the usual convexity assumption of that model.
d. A key assumption on the distribution of Technological Facts

We shall assume (from Section 3c to the end of the paper) that for every Decision, the Social Technology Relevant to the Decision is initially possessed by at least one individual. In other words, for each decision, no two individuals are each endowed with Technological Facts unique to themselves Relevant to the Decision. An individual who possesses all of the Social Technology Relevant to a Decision—we have assumed that there is at least one such individual for every Decision—is said to have Vector Non-Inferior Technological Information Relevant to the Decision. An individual who possesses all of the Technological Facts Relevant to a Decision that are possessed by others and also possesses some additional Technological Facts Relevant to the Decision is said to have Vector Superior Technological Information Relevant to the Decision.

Without the fortuitous, but key, assumption that for every Decision there is someone who is endowed with Vector Non-Inferior Technological Facts Relevant to the Decision, Pure Pareto Optimal Behavior would be generally impossible in any institutional system that this author can imagine. Since, by assumption, there are prohibiting costs of direct communication of Technological Facts, how can a Decision made by any single individual use Technological Facts which are initially unique to other individuals? We shall see below that there are some such Technological Facts are indeed revealed to a decision-maker through the institutional parameters of a general competitive model—and all such Facts are revealed in a correctly described competitive model with a Radner-type Technological Information Structure. Nevertheless, there may exist some yet undiscovered group Decision process in the general technological environment which somehow induces each of the participating individuals to act in such a way that each group Decision is identical to that which would be made by a single decision-maker in possession of the Social Technology Relevant to the Decision.
e. **Definitions of Decision Costs and the Joint Costs of a Decision**

Actual Behavior can differ from initial, reflex behavior only if there are Decisions. A Decision is a comparison between two feasible behaviors in which one is chosen over the other. Decisions entail either zero costs or positive costs, called Decision Costs. An illustration of a costless Decision process is that occurring in a dictatorship in which the dictator possesses the Social Technology, can freely calculate all Pure Pareto Optima, and can communicate and enforce his commands determining Actual Behavior at no real cost. Decision Costs are defined as the losses in selecting and implementing an individual's behavior under a specified Technology in a given institutional system. Under an individual's given Technological Information, these are the losses in making the Decisions implied by a particular behavior of the individual, i.e., in discovering his institutionally feasible behaviors and their effects, computing an optimum, and communicating Institutional Information. The Decision Costs of an Actual Behavior are said to be Effective Decision Costs. All other Decision Costs are Noneffective. Prohibiting Decision Costs are the Noneffective Decision Costs of a behavior that would be at least as beneficial as the Actual Behavior if the Decision Costs of the behavior were zero.

Several forms of Decision Costs are usefully distinguished. A Decision Cost is the sum of Direct and Indirect Decision Costs. A Direct Decision Cost is the value of the real resources spent in selecting and implementing the Decision. An Indirect Decision Cost is the loss resulting under a specified technology from the failure to expend sufficient resource costs of the Decision.

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1 The "value of real resources spent" is merely an intuitive expression for describing the loss in utility to the individual because of the reduction in his set of feasible behaviors.
The cost of an individual's **calculating** his optimal behavior under given Technological and Institutional Information is assumed to be zero throughout this paper.\(^1\) Hence, Decision Costs under an individual's endowed Technological Information will be solely the result of an individual's making a Decision without **freely knowing** the Institutional System, i.e. without freely determining the exact reactions of others to the alternative behaviors of the individual and the exact effects of their behaviors on him.\(^2\)

The Joint Costs of a Decision are the joint Decision Costs to the entire set of individuals whose Decisions are directly affected by an individual's Decision given the union of those individuals' Technological Information and a particular Institutional System. Effective Joint Costs of a Decision imply that Actual Behavior is not a Pure Pareto Optimum. (Effective Decision Costs have no such implication because all such losses may be indirect and may simultaneously represent someone else's gain.)

f. **The definitions of Private Property and a Private Property System**

An Institutional System of Private Property is imposed on a Technological Information Structure by the following procedure: (1) Add an "ownership" subscript, one for each individual \((1, 2, \ldots, N)\), to the quantity index of each Asset--i.e., of each socially perceived source of individual behavioral possibilities and thus benefits--so that each existing unit of each Asset is exclusively "owned"

\[^1\]This is not to be confused with Radner's assumption of the absence of "computational limitations." Such limitations or costs are on the calculation of future acts given certain possible future events. We shall see below that such computational limitations create no problem for a correctly described standard competitive model. The generalization of our results for possibly positive calculation costs is entirely straightforward.

\[^2\]"Knowing" an environmental condition means being certain and accurate about the condition. The reader should notice the asymmetry between the kind of perfections used in describing perfect Institutional Information and perfect Technological Information. The former perfection means having non-statistical, flawless environmental information, while the latter means merely having as much environmental information as anyone else.
by some individual. Thus, if there are $A_L$ units of your work-leisure time in the Social Technology (in a certain period and a certain "state of nature"), then $A_{L1}$, $A_{L2}$, ..., $A_{LN}$ become the number of units of your time "owned by" individuals 1, 2, ..., $N$ respectively, where $\sum A_{Li} = A_L$. (2) Define the Ownership of the units of an Asset to be the ability of an individual, called the Owner of the units, to costlessly prevent any other individual from (a) receiving utility from those units or (b) reducing the utility which he, the Owner, could obtain by using the units along with his other Assets for specified behaviors of the other individuals.

The first part of our definition of private property is apparently standard throughout economics discussions. However, the second part, our definition of ownership, is somewhat more general than that used in the standard competitive model (e.g., Debreu (1959)). The definition of ownership appearing in the standard competitive model has ownership implying control in that if someone individually "owns" an Asset, only he can make Decisions regarding its use. The assumption of "no externalities" in the standard model represents the statement that the real returns to the Assets of any individual are unaffected by the alternative uses of the Assets of other individuals. The only way to alter the real benefits generated by the Assets of others in a standard competitive model is to Transfer "ownership" (i.e., to switch subscripts indicating the control over assets) by the mutual consent of the affected individuals. While our definition of ownership also permits such Voluntary Transfers, our definition precludes neither Involuntary Transfers nor the alteration of the real benefits generated by the Assets of others. Our definition of ownership not only permits an individual to make Decisions
which alter the real benefits of the Assets of others, but these Decisions may be made without the others' consent, as long as compensations are paid. An example of our definition of ownership follows: My Ownership of an hour of your current labor-leisure-time means that I can prevent your current utility from being above a certain, specified value and that I can prevent you (or anyone else) from reducing the utility that I can obtain by employing you so long as your utility is not below the specified value; and if anyone besides me Decides how you are going to spend your hour, either he must obtain my consent or compensation must be due so that my utility will be the same as it was without his Decision.

Our definition of ownership should not appear entirely foreign. For one thing, our definition appears to be close, at any rate closer than the standard definition, to the definition revealed by actual common law. For another, our definition of ownership is very close to that implicit in orthodox, informal discussions of the nature of "externalities" (e.g., Meade, Coase, and Ellis and Fellner). These discussions, in contrast to discussions of the standard competitive model, say that there is no "real externality" when an individual alters the real benefits of the assets of others as long as "full" compensations are paid.

We are suggesting here that an appropriately general private property system is characterized by responsibility for reducing or benefiting from the assets of others while the narrow private property system described in the definition of private property used in the standard competitive model is characterized by Voluntary Transfers as the only method whereby an individual

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1 Although Technological Facts are never Transferred because of the basic assumption of Prohibiting Costs of communication of Technological Information, Technological Facts are still Assets themselves because they are still a socially perceived source of individual benefits. Our general definition of Private Property allows an individual to receive direct benefits from the Technological Facts of others even when the Transfer of such Assets is prohibitively Costly.

2 While our definition of a Private Property System prevents the occurrence of "real externalities" (i.e., "non-pecuniary externalities"), it permits any purely "pecuniary externality" (Ellis and Fellner).
can alter the returns on Assets that are not initially his own. In short, responsibility, not "freedom," is the necessary feature of an appropriately general private property system. Indeed, a general private property economy would generate a private dictator, someone forcing all of everybody's behavior, if this someone knew sufficiently better than everyone else what was good for them and if everyone behaved rationally. Only this dictator would have to pay for any losses his decisions created in a Private Property System so that while there is no "freedom" in a conventional sense of the term, there is complete responsibility by the decision maker.

Since we assume that each Asset is owned by a single individual in a Private Property System, we rule out "joint ownership" by a group of individuals and thereby avoid the substantial ambiguity which accompanies this concept. In the familiar special case of the joint ownership of a productive unit, or firm, the conventional assumption (Debreu (1959)) is to exclude Ownership of specific Assets in the firm and replace it with Ownership of a certain fraction of the returns from the assets of the firm. This requires, however, an independent assumption on the behavior of the production unit—conventionally that the firm maximizes profits. Our main objection to this procedure is methodological: It is inappropriate in a model built to evaluate the efficiency of the decisions of individuals in a given Institutional System, to assume that Actual, non-reflex, Behavior (e.g., profit-maximizing Behavior) results without anyone's Decision to produce the Behavior. However, under the narrow Technological Information Structure and the price constancy and non-discrimination assumption of the standard competitive model, any stock holder, having the same Technological Information as any other stockholder, regarding production possibilities and events, and being unable to affect prices to his own benefit, would choose to
make the joint profit-maximizing Decisions under the Social Technology. Therefore any stockholder in a standard competitive model can be made the Decision-maker for the firm. In our analysis, such a stockholder can be deemed the Owner of the Assets of a firm and the other stockholders the Owners of contingent claims representing obligations of the Owner of the Assets of the firm. In this way, a system with Private Property is sufficiently general to include the situation described as joint ownership appearing in the standard competitive model. Nevertheless, the analysis of "joint ownership" occurring in the standard competitive model completely breaks down once we allow interpersonal differences in Technological Information regarding production possibilities (or once we allow individuals to have an effect on prices) in that different stockholders would choose different Decisions for the firm. No such indeterminacy appears in our model. More importantly, as we are about to note, the "firms" that we have been discussing are much different, analytically speaking, from the productive organizations which we observe in the real world.

Certain resources, called the resources of a Private Organization, may be divided up among several separate Owners with the agreed upon Condition on such Ownership that any individual can make Decisions so as to affect the value of some of the firm's resources if and only if there is consent by all of the members of one subset of a prespecified collection of subsets of members of the group. This is analytically descriptive of what exists in partnerships, corporations, consumer cooperatives, clubs, unions, and families.

An Institutional System of private property is not quite an accurate representation of the "laissez faire system" appearing in orthodox economic discussions. In a Laissez Faire System, Private Property does not exist if the costs of establishing and protecting Ownership in some Assets are sufficiently
high that it does not pay anybody to establish or protect his ownership in the Assets. Henceforth, we shall assume that there are zero costs of establishing and protecting the ownership of each asset in a Private Property System so that we make no further distinction between a Private Property System and a Laissez Faire System. An informational equivalent of this assumption is described in subsection h below.

An economy with "rights to" Private Property is an economy containing some individuals who provide joint Private Property protection services to several other individuals. The decisions of such protectors necessarily alter the assets of some individuals without their consent and without payment of compensation. Therefore, these decisions, like the decisions of institution selectors, violate conditions on decisions within a Private Property System. Decisions violating these conditions, which may be called "governmental decisions," are ruled out of our model of decisions within a Private Property System. Such decisions, and economies with rights to Private Property, are not necessary to a Private Property System. Individuals may be capable of acquiring and protecting their own Assets without the benefit of collective protection and supervision. (See Schelling, Kahn.)

8. The Institutional Constraints on behavior in a Private Property System

We assume that the initial asset endowments are such that some behavior is feasible for every individual. This amounts to assuming that the system, in establishing the initial Private Property of other individuals, does not require that an individual perform tasks that are physically impossible. In a Private Property System, any behavior of an individual is feasible if that behavior is within the individual's Social Technology and does not alter (relative to an alternative, feasible behavior of the individual) the Assets of others or the real benefits that other individuals obtain from their given
Assets. The remainder of the feasible behavior of an individual in a Private Property System is determined by including behavior in which the Assets of others, or the real benefits of the Assets of others, are altered (relative to an alternative, feasible behavior of the individual) but Prices are paid. A Price is the amount of an Asset one individual must Transfer to another if the one individual either acquires Ownership of a unit of the other's assets, or benefits from, or reduces the real services of a unit of the other's Assets. Whenever such Transfers for acquisition or compensation are made, i.e. whenever a Price is paid, a Transaction is said to occur.

We shall employ the conventional assumptions that Prices are non-negative and that the Individual's Technology presents him with closed and bounded production possibilities. Consequently, an individual's set of attainable Assets when Prices are set to permit no excess demands is closed and bounded. Adding that the Individual's Technology presents him with closed and bounded behavioral possibilities for a given stock of Assets, an individual's set of feasible behaviors is closed and bounded in a Private Property System.

Behavior in a Private Property System may be Voluntary or Involuntary. Involuntary behavior is said to be achieved by Interpersonal Force. Involuntary behavior by an individual implies that some of the Decisions determining the behavior are made by other individuals and without the consent of the individual. If some of the Decisions determining an individual's behavior are made by other individuals but with the mutual consent of the respective individuals, the behavior is Voluntary, and we say that a Transaction in a Decision Service has occurred. In spite of the fact that consent does not occur when behavior is Forced, there may be an act of neglecting to employ an available Counterforce behavior, a behavior which restores the original
behavior. The difference between the application of Counterforce and the rejection of an Offer of a Decision Service is that in applying Counterforce, one must compensate those whose Assets are affected, i.e. the original Forcers, while no compensations are due when rejecting an offer of a Decision Service.

h. **The Decision Process**

The non-reflex, Actual Behavior of an individual in any Institutional System is the result of Decisions and thus the result of a Decision Process. This special behavioral process can be described as follows: (1) gather information on the nature of the constraints imposed by the Institutional System and combine this with available Technological Information, (2) calculate the resulting set of feasible behaviors and select an Actual Behavior, (3) execute an Actual Behavior, which includes the dissemination of Institutional Information. This taxonomy was used above in defining Decision Costs. In a Private Property System, the first part of the Decision Process resulting in an Individual's Actual Behavior is to gather information concerning the Assets the individual would Own or benefit from under alternative behaviors and combine this with his Technological Information. Then, after making his costless calculations, the individual executes an Actual Behavior, disseminating Institutional Information on what Assets others would Own under alternative behaviors. Each of these parts of the Decision Process are somewhat unique to a Private Property System. For example, in a Non-Private Property System, one does not gather information regarding his Asset Ownership under alternative behaviors because such Ownership does not exist.

The Decision Process of an individual is itself part of his Actual Behavior. In particular, any Voluntary Transaction is the result of a process of gathering and disseminating Offers and Acceptances. That is, in each Voluntary Transaction, there is an Offer disseminated by one of the parties and gathered
by the other and an Acceptance disseminated by the other and gathered by the
former. Such communication of Offers and Acceptances is necessary and suf-
ficient for the "mutual consent" used above in the definition of a Voluntary
Transaction. It is also necessary and sufficient for a Contract.

A Conditional Transaction is a Voluntary Transaction in which Offers or
Acceptances are contingent upon the consent of third parties. An Acceptance
Condition arises when an individual makes an Offer or Acceptance to another
contingent upon a third individual's Accepting a certain, specified Offer.
An Offer Condition arises when an individual makes an Offer or Acceptance
contingent upon the Offers of others. An Offer or Acceptance Condition may
arise because an individual has Voluntarily granted veto power to other in-
dividuals, as he does when he joins an Organization. But Offer or Acceptance
Conditions may also arise when individuals can, at their own option, carry out
a Conditional Transaction Strategy. A special form of Conditional Transaction
Strategy employing Offer Conditions is the Counteroffer Strategy. This
special strategy is to simply beat an individual's Offer, whatever the Offer
is, within certain limits.

Note that an individual's Assets include (1) his ability to commit
himself to a Counteroffer Strategy, even though the Acceptance of his Counter-
offer would make him worse off; (2) his ability to make Acceptances Conditional
upon the Acceptances of others; even though Acceptance would make him better
off even without these Conditions; or (3) simply his contractually granted
ability to block the Voluntary Transactions of members of an Organization.¹

¹The set of Assets is more inclusive than the set of Commodities (i.e.,
real goods or services) in that the former includes all sources of real bene-
fits -- items such as money, the ability to sell assets in the future, the
ability to affect a Transaction between other individuals, and unique Tech-
nological Information. Money, the ability to sell Assets in the future, and
Summarizing, the Institutional Information one gathers and disseminates in making Voluntary Transactions is Institutional Information on the nature called of Offers and Acceptances. All of this Information will be Contract Information.

The remainder of the Institutional Information one gathers and disseminates in a Private Property System concerns Involuntary Transactions and Interpersonal Force, where there are no Offers, Acceptances or Contracts. The Forcer merely applies his Force, thus altering the actual Behavior (perhaps through altering the Assets) of the Forcee without any explicit communication (with the possible exception of a costless command). The Decision Process is then described completely by the Decision Process used in protecting the Ownership of Assets. This special Decision Process thus entails no additional Costs of Institutional Information once we are given a Private Property System. Under costless protection of Assets, someone must have the Technological Facts determining the Value of the benefits lost (gained) by the Force through the Force Decision in order that the Forcee be compensated (charged) and the Forcer charged (compensated). This is a basic requirement of a general Private Property System. The existence of such individuals will not be an additional restriction on the distribution of Technological Facts once we have assumed that for every Decision, there is some individual who has the Social Technology Relevant to the Decision. Such individuals are now called Experts with respect to the Decision. However, additional Restrictions on the Social Technology are implied by the assumption of a Private Property

unique Radner-Savage information can be made part of a general description of a standard competitive model (see Thompson (1967), (1972) and Section 3b below). However, the condition of general Technological Information calls for a generalization of the standard competitive model, based upon our general definition of Private Property such as is developed in Section 3c-e, and assets representing the ability to affect a Transaction between other individuals, cannot generally exist in any competitive model (Section 2).
System and therefore costless protection in that Experts must be (a) identified and (b) induced to reveal the correct value of the losses (gains) of the Force. Regarding (a), there must be, in general, a Recognition of Experts by all Individuals. Regarding (b), the value of the Decision must be Computable to individuals besides the Expert. Computability may be achieved by waiting until a future date when the Technological Information of others has expanded to cover that of the current Social Technology Relevant to the Decision. Computability may also be achieved by threatening to devote resources to the production of the Social Technology Relevant to the Decision and then impose severe penalties, including corporal punishment, if an Expert does not correctly reveal the value of the Decision. Once Computability holds, Experts can be induced to reveal themselves so that we can relax the condition of the Recognition of Experts by all Individuals to the condition that for each Decision, each individual knows whether or not he himself is an Expert with respect to the Decision. This will be called the Recognition of Experts condition.

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1 This is the first instance of punitive damages and corporal punishment we have met in our general Private Property System. The private property system assumed in the standard competitive model may appear to use punitive damages and corporal punishment whenever someone alters the Assets of another without Voluntary Transfers of Ownership. However, we may also interpret the punishment and private property system of the standard competitive model as exactly the same as our own because the narrow Technological Information Structure and the Rationality assumption of the standard competitive model obviate Transactions other than those using Voluntary Transfers of Ownership. In the latter interpretation, the restrictions which the Recognition of Experts and Computability place on the Technological Information Structure are not additional restrictions on the Technological Information Structure of the standard competitive model. This is not true in the former interpretation, which describes an unnecessarily harsh system of punishment. Since one can find in the literature no mention of the unnecessarily harsh punishments implied by the standard competitive model and one finds numerous claims of the generality of the private property system which the model describes, the latter interpretation is the most natural. We employ this interpretation throughout the paper. While we are not then generalizing the private property system implicit in the standard competitive model, we are employing a more general definition of a private property system, one which permits interpersonal force.
i. Transaction Costs

The Joint Cost of a Decision in a Private Property System is called a **Transaction Cost**. (The term, "Transaction Costs," is somewhat inappropriate because some Decision Costs in a Private Property System arise from behavior in which no Transaction occurs. However, the term is not completely inappropriate because Decisions yielding such behavior still require the gathering of Information as to whether another individual's Assets are affected by the Behavior, i.e., as to whether a Transaction has occurred.) Those Transaction Costs arising out of a particular Transaction are called the **Costs of the Transaction**. Any Effective Transaction Cost is obviously inconsistent with the achievement of Pure Pareto Optimality, and, since Transaction Costs present the same formal problem as created by transportation costs in multiple markets (the problem of non-convex production or consumption sets), there is no guarantee of the existence of a competitive equilibrium in the presence of Effective Transaction Costs. These transparent results are similar to those recently obtained by Foley and Hahn.

Transaction Costs obviously include the Decision Costs resulting from the Decision Process. That is, Transaction Costs obviously include (a) **Contract Information Costs**, the Direct and Indirect Costs of acquiring and disseminating Contract Information; and (b) the Decision Costs resulting from the Process of making Involuntary Transactions, these latter Costs being zero in a Private Property System. But these Decision Costs do not exhaust the possible Transaction Costs. For Transaction Costs also include any joint Decision Costs to the individuals in a Transaction because of their inter-personal differences in Technological Information. Some examples may be helpful in bringing out the pervasive and deeprooted nature of Transaction
Costs which may result from interpersonal differences in Technological Information (together with positive costs of communicating and producing Technological Information). Suppose a price-taking seller of a car realizes that any buyer will surely have an accident with the car as no one is strong enough to apply the brakes once they get warm. He agrees to sell the car anyway because the costs of communicating this Technological Information are prohibitive and some surplus to him is gained by the sale. The car accident (net of the cost of repairing the brakes) becomes an Indirect Cost of the Transaction to the buyer and to the seller and the buyer jointly. But it is not a loss to the seller. Next suppose there is another car and another car customer, one who realizes that the seller knows how the car will help and hurt him but is only (costlessly) communicating how it will help him, is sufficiently impressed with the helpful points to the car that he would buy it if there was no way the car could hurt him. But he rationally estimates that the untold harmful points are sufficient to counterbalance the helpful points and he does not buy the car. In fact, there were no harmful points, but he could not believe the seller. The loss in consumer and producer surplus to the seller and the customer because he rationally failed to believe the truth is an element of Indirect Transaction to the buyer and seller. For a final example, suppose that after tremendous study, the car seller determines that all cars are going to suddenly become forever worthless beginning next week. He then puts on a massive advertising campaign and sells out his cars at the end of this week. The tremendous study and the advertising campaign are Direct Costs of his Transactions.

However, while any of these Transaction Costs may arise in a standard competitive equilibrium under a generalized Technological Information Structure,
we shall see that such examples cannot occur in a generalized competitive equilibrium in the absence of Unconditional Contract Information Costs under our more general definition of a Private Property System. This will be a central theorem (proposition ii of Section 5).

2. THE CONTRACT INFORMATION STRUCTURES OF ALTERNATIVE ECONOMIC MODELS

   a. The Central Problem

   In the preceding section, we described the unnaturally harsh character of the implicit restrictions that the standard competitive model places on the Technological Information Structure and the definition of Property Rights. We shall maintain these restrictions, along with the assumption of Rationality, throughout this section of the paper in order to isolate the restrictions which the standard competitive model places on the nature of Contract Information.

   In Section 3, we shall eventually maintain only the assumption on the Contract Information Structure. The nature of Transaction Costs will eventually be derived from this assumption and we shall then have substantially relaxed the severely restrictive conditions on a standard competitive model regarding (a) the Technological Information Structure, (b) the definition of the Private Property System and (c) the Rationality of Actual Behavior.

   The basic model of this section thus has each individual voluntarily selecting and implementing his most preferred behavior in a Private Property System with a Technological Information Structure in which each individual has identical Technological Information regarding production possibilities and events and has Vector-Non-Inferior Technological Information regarding his own preferences. But these assumptions on the model do not contain enough data to permit a determination of Actual Behavior for they do not specify the

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1 We also employ the "local nonsaturation" assumption of Koopmans to prevent each individual from keeping assets which are useless to him (but useful to others) because no other assets are useful to him either. This assumption will...
Contract Information Structure. However, once this Information Structure is specified, each individual's rational behavior can be determined and, if each of these behaviors is consistent with the behaviors of others, a descriptive general equilibrium solution (a noncooperative game solution) can be determined. Our problem is to arrive at a standard competitive model by such a specification.

b. The Implications of Naturally Perfect Contract Information

In this subsection, we first note the implications of Naturally (i.e., costlessly) Perfect Contract Information for the nature of Transaction Costs and the costs of the various activities in the Decision Process of this model. Then we characterize the admissible set of general equilibria, or solution set, of this model in terms of individual benefits when there is Naturally Perfect Contract Information.

In the standard environment specified for this section, the additional specification of Naturally Perfect Contract Information of each individual implies identically zero Transaction Costs. For Rational individuals will not incur direct losses in attempting to accumulate Contract Information, and one's set of feasible behaviors given his own Information is, under the special assumption on the Technological Information Structure, identical to his set of feasible behaviors under the Social Technology and the actual set of Institutional Constraints. (An individual may not know another's preferences, but if this is so, then such Technological Facts are not Relevant be unnecessary, however, to the analysis of Section 3 and to our central optimality results there. Thus the assumption of local nonsaturation—which is mathematically and intellectually very similar to the assumption of Rationality—will join the standard Rationality assumption in its exit from the competitive model. As for Rationality: At least one person will have to be locally nonsaturated in order to establish the Pure Pareto Optimality of the generalized competitive equilibrium, but if everyone were locally saturated, Social Optimality would not imply Pareto Optimality.
to his Decisions in the particular Institutional System.) This is not true under a general Technology, and, as our examples in Section 1 show, Perfect Contract Information with only a standard definition of private property permits various forms of Transaction Costs to arise.

Naturally (i.e., costlessly) Perfect Contract Information in a Private Property System implies that every individual costlessly knows the Offers outstanding, the Counteroffer Strategies in existence, and the Acceptances that occur under each of his alternative strategies. And he can costlessly communicate his response to these market possibilities in terms of his own Acceptances, Conditional Acceptances, Offers and Counter-offer Strategies. Given his closed and bounded set of feasible behaviors, each individual has a behavior which is not dominated in preference by any of his other feasible behaviors.

First suppose that a set of such individual behaviors over all individuals is mutually consistent but is not a Pure Pareto Optimum. Then it would pay each individual to accept an Offer made Conditional upon everyone else's Acceptance, by an organization which Offered, in return for everyone's initial Assets, behavioral feasibilities so that his benefits are a certain amount greater than those he received in the initial situation.¹ Thus, any Pareto non-optimal situation is dominated

¹This can be done under orthodox and standard assumptions without requiring the Contract Information of the decision makers in the organization to include Information which implies the preferences of other individuals. In particular, if (a) there are no Collective-type goods, (b) preference relations are continuous, (c) preference sets and feasible production sets are convex, and (d) everyone's wealth is positive, the organization can use the standard Arrow-Debreu economy under an organizationally
by a Pure Pareto Optimum. Now suppose that a set of individual behaviors is mutually consistent and is a Pure Pareto Optimum. Can it be any Pure Pareto Optimum? The answer is clearly negative because some Pure Pareto Optima make an individual worse off than he was within his initial Commodity endowment and the individual need only reject all Transactions to be better-off.

Can the set of behaviors then be any Pareto Optimum in which no individual is worse-off than he is with no Transactions? The answer here is affirmative. In other words, the set of feasible solutions, or the set of undominated sets of individual behaviors, is equivalent to the well-known Imputation Space of N-person Cooperative Game Theory, the set of all solutions (stated in terms of sets of individual benefits) which satisfy both "group rationality" (i.e., Pure Pareto Optimality) and "individual reationality" (i.e., any individual receives at least the benefits that he would receive with only his initial Commodities, e.g. Owen.) To see that no set of individual behaviors dominates a set of individual behaviors generating a point in the Imputation Space under Perfect Contract Information, consider any set of individual behaviors, A, generating a point in the Imputation Space. In any alternative set of behaviors, say B, someone will be worse off than he was in A. This individual, provided that he has sufficient Assets, can always block a

controlled initial commodity distribution. (Note that such an economy prohibits further Conditional Transactions and enforces price-taking behavior.) Furthermore, if these added restrictions on the Technology and the achievability of an Arrow-Debreu economy are not satisfied, the organization can employ the "D-Process" (see Thompson, 1966) which uses uniformly weaker assumptions than the standard competitive model to arrive at a Pareto Optimum without interpersonal preference information. Only the risk-aversion implication of the usual convexity assumption is required with the latter process.
Transaction between other parties which achieves set B. For he can costlessly establish the Counteroffer Strategy of beating any Offer one of these other parties makes to another, thereby preventing the Offer from occurring. Since the set of behaviors is a Pareto Optimum, it does not pay these other parties to "bribe" the blocking individual into withdrawing his Counteroffer commitment (i.e., to purchase the Asset described by the ability to apply the Counteroffer Strategy) and to proceed with his Offer. The familiar game-theoretic assumption of "transferable utility," or the assumption of the ability of each individual to transfer the "wealth" he gains through Transactions, ensures that the individual always has sufficient total Assets to make it feasible for him to carry out the Counteroffer Strategy. For the gains to the other individuals in B cannot then exceed his losses of benefits or wealth and his losses in the event of B cannot exceed his gains in moving from his situation with his endowed Commodities Assets to his situation in A.

The Imputation Space is very large in the sense that, unlike the core and related cooperate solution concepts in N-person bargaining theory (such as Vind's set of "Exchange Equilibrium, a Von Neumann-Morgenstern Stable Set, or Shubik's "Bargaining Set"), all of which are contained in the Imputation Space, the Imputation Space does not converge to a standard competitive equilibrium, or even shrink, when the number of individuals becomes large.

The method of achieving a particular solution out of the set of solutions to the model is apparently a complex dynamic process and we
shall not attempt to analyze it.\(^1\)

c. **The Implications of Costless Unconditional Transactions and Prohibitively Costly Conditional Transactions**

There appears to be an "order-or-magnitude" difference in the Contract Information required under Conditional and Unconditional Transactions. For Conditional Transactions require that an individual detect the occurrence of the Transactions between other parties and Unconditional Transactions do not. We can express this difference most sharply by assuming that the Cost of all Conditional Transactions is Prohibiting while the Cost of all Unconditional Transactions is zero. This assumption on the nature of Contract Information and Transaction Costs serves to remove a great many of the elements of the solution set, the Imputation Space, of the N-person bargaining Model with a zero Cost of all Transactions. Counteroffer Strategies are inadmissible because they are now not worth the effort in communicating the strategy and setting up the facilities to discover the existence of Offers of other individuals to still other individuals. If only Counteroffer Strategies were ruled out by this assumption on the nature of Contract Information Costs, the solution set would be contained in the core (in particular it would be

\(^1\) However, we note that in such a world all of the surpluses of orthodox economic theory are "up for grabs" and will be acquired by those who are relatively proficient at establishing their own Counteroffer Strategies and preventing or breaking down other Counteroffer Strategies during the dynamic adjustment process. Also the process appears to have a "rich-get-richer" property because those with relatively large initial assets, cet. par., have relative ease in establishing Counteroffer Strategies. The process is analytically equivalent to one in which the distribution of all assets is not initially defined but is being established by the interacting individuals anarchistically through conditional commitments to fight wars, commitments which are akin to our Counteroffer Strategies.
Vind's set of "exchange equilibria".) However, Conditional Transactions also include Transactions in which Offers are conditional upon Acceptances of other individuals. Once we rule out these Conditional Transactions, we obviously rule out group transactions in which everyone in the group, although better off with their behavior in the group, must consent to the Transaction. These are the kind of Transactions that are described in generating the core and related solution sets. To maintain such solution sets, one could exclude all Transactions in which Offers are conditional upon the Offers of others but still admit, at no cost, all Transactions in which Offers and Acceptances are conditional upon the Acceptances of others. But this would be a very unnatural procedure. For one thing, there is only little reason to believe that an individual is better at detecting the Acceptances of third parties than he is at detecting their Offers. For another, there is, in group Transactions in which all must consent, an obvious incentive for every individual to bargain and hold out in attempts to acquire for himself the whole of the group's surplus in the group Transaction. These Transaction activities include attempts to convince others that the Technology is such that his benefits from the group Transaction are very small. (When the incentive toward such Transaction activities are present, Force may be used to avoid the resulting Transaction Costs, but Force, even Non-Governmental Force, is not admissible in the standard environment that constrains the economies considered in this section.)

The introduction of Costly Information regarding Conditional Contracts and thus positive Costs of Conditional Transactions renders inapplicable both the arguments employed in the previous subsection and the arguments
employed in the standard analysis of the core to demonstrate the Pure Pareto Optimality of Solution Sets of models of N-person Bargaining under Private Property. This is due to the fact that the existence of Costly Conditional Transactions prevents economy-sized Private Organizations from forming and redetermining behavior whenever the set of individual behaviors is not a Pure Pareto Optimum. This fact, together with the possibility that an individual's behavior may affect Prices, will generally lead to solutions to the model of this subsection which are not Pure Pareto Optima. The standard, single-price monopolist of the price theory texts is a case in point. And even if perfect Price discrimination is possible so that this text book case does not appear, the Price-effects that remain will in general still induce solutions which are not Pure Pareto Optima (Thompson (1969)).

d. The Standard Competitive Model

A special case of a model of N-person Bargaining under Private Property with Prohibiting Costs of all Conditional Transactions and zero Costs of all Unconditional Transactions is the Standard Competitive Model. In particular, the Standard Competitive Model is the special case of this model in which Prices are assumed to be both unaffected by the behavior of any single individual and the same to all individuals. The Pure Pareto Optimality of the equilibrium solutions of the Standard Competitive Model has been established with increasing rigor by Adam Smith, Barone and Marshall, and Arrow, Koopmans and Debreu.

An obvious incompleteness in the Standard Competitive Model relative to the models described above is that its additional specification on price
behavior is not derived (or generally derivable) from a model of N-person Bargaining under Private Property in which the Costs of all Transactions have been specified. In fact, Price constancy and non-discrimination greatly restricts the technology to which the model may apply. For example, when the assumption is relaxed and the nature of Prices is derived from a competitive model of N-person Bargaining under Private Property with Prohibiting Costs of Conditional Transactions and Zero Costs of Unconditional Transactions, the Standard Competitive Model is seen to preclude the existence of collective-type goods, whose competitive Prices can be neither constant, non-discriminatory, nor efficient (Thompson 1968, 1969). Regardless of its implications we shall maintain the restrictive, underived assumption of Price constancy and non-discrimination for the remainder of this paper. This is motivated by a result that we plan to establish in a future paper. Viz, this restrictive Price constancy and non-discrimination [along with (a) the Organization-precluding assumption of Prohibiting costs of Conditional Transactions and (b) the standard, unnatural convexity, continuity, and positive wealth assumptions necessary for the general existence of a Standard Competitive Equilibrium] can be replaced by simply adding an "optimal anti-trust law" without altering the rest of the model. The anticipation of the establishment of this result also explains our lack of assumption in the current paper of the conditions usually assumed in establishing the existence of equilibrium; we just assume existence without these conditions and plan to prove it in a later paper in which an optimal anti-trust law replaces these harsh conditions.

A conventional additional restriction on the Standard Competitive
Model is that there are no Assets other than Commodities (including claims) in a Standard Competitive Model. But this restriction is a result of certain, inessential restrictions on the conventionally described environment and does not imply that the Costs of all Transactions in non-Commodities are positive—-even though there must be positive and prohibiting Costs of Transactions in those non-Commodity Assets representing abilities to block the Transactions of others. The other possible non-Commodity Assets are absent from the Standard Competitive Model because the environment is too narrow to generate any demand for such Assets: Monetary Assets are not present in the conventionally described Model because it is conventionally assumed that the Costs of Transaction in Commodities can be made zero without the aid of an intermediate Asset which generates no real services. (See Thompson, 1972); and there is no demand for Assets which represent the ability to sell Assets in the future when the model has a finite horizon. (See Thompson 1967). However, the Standard Competitive Model as conventionally described can be generalized to include these environmental conditions and the corresponding non-Commodity Assets without producing any alteration in the central results on the existence and Pure Pareto Optimality of the resulting Competitive Equilibria (Thompson, 1972, 1967, respectively).

In summary, we can characterize the Contract Information Structure of the Standard Competitive Model as one in which all Unconditional Offers and Acceptances affecting the Assets of an individual are costlessly known to him and satisfy the Condition of price constancy and nondiscrimina
tion, but it is prohibitively costly to communicate Offers or
Acceptances which are Conditional upon other Offers or Acceptances.  

   e. The Existing Literature on the Information Structure Implied by the Standard Competitive Model

We have seen that the Information Structure of the Standard Competitive Model is one in which (a) each individual has the same Technological Information regarding production possibilities and events and Vector Non-Inferior Technological Information regarding his preferences and (b) each individual costlessly knows all of the Unconditional Offers and Acceptances affecting his Assets that result from behavior implied by Price constancy and nondiscrimination. The requirement that each individual has the same Technological Information regarding production possibilities and events has been noted previously by Thompson (1966) and Hirshleifer.

There are, however, several incorrect claims in the recent, influential literature on economic theory regarding the Information Structure implicit in the Standard Competitive Model. One of these claims, most thoroughly developed by Arrow (1963, 1968) is that the Standard Competitive Model breaks down whenever individuals can control the contingencies whose occurrence results in certain, contracted, deliveries of goods. The standard example is of "moral hazard" in insurance contracts (e.g., health or auto insurance) where an individual who is fully insured will behave

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1These Contract Informational Implications of the Standard Competitive Model are not merely the result of some peculiarity in the Standard Competitive Model developed in this section relative to the standard competitive model developed in the literature (e.g., Debreu). The standard competitive model developed in the literature obviously omits Assets representing the ability to block Transactions between other individuals (otherwise its solution set would generally be the Imputation Space, which is generally longer than the core). So Conditional Contract Information is prohibitively costly in the standard competitive model, and the fundamental theorem of welfare economists—that the standard competitive equilibrium of the literature is a Pure Pareto optimum—obviously implies zero costs of Contract Information concerning the Unconditional Transactions occurring in the model.
as if there were no real cost resulting from his behavior which increases the likelihood of a socially unfortunate event (e.g., a disease or an auto accident). However, in the example it is obvious that an individual will not so behave if someone--viz. the insurer--purchases the insured's services in the prevention of the unfortunate event at the time of the insurance sale to the individual (the timing condition is there to keep markets behaving competitively). The objection to this by Arrow et. al. would (should) be that such service Transactions are very expensive--that the insurer does not know the production possibilities of the insured or what insured is doing. But then the source of the market breakdown is the occurrence of Transaction Costs--the occurrence of those deep-seated Transaction Costs based on interpersonal differences in Technological Information. The example simply does not show that there is a market breakdown whenever individual's can control contingencies whose occurrence results in certain contractual deliveries. The source of this error is perhaps Ch. 7 of Debreu. In this famous chapter, Debreu shows that the conclusions of the Standard Competitive Model can be extended to the case of Commodities in which future deliveries are contingent upon the occurrence of certain events. However, Debreu describes his results as applying when the events are completely exogenous to the behavior of this individuals. Nevertheless, there is nothing in the formal structure Debreu creates which individuals appear to affect the probability of occurrence of events made the basis of contingent-claim Transactions.

A different claim of sufficiency conditions for the breakdown of the Standard Competitive Model is that of Radner. This claim is that the inability (costliness) of an individual to currently compute what he
would do in the future in the event of certain future conditions implies the existence of markets in the future and thus losses in having a disequilibrium because of the resulting lack of current information about Prices in the future. But there is nothing in the Standard Competitive Model that prevents individuals from making future deliveries contingent upon the outcome of future computations. These outcomes are themselves contingencies available for making current contracts so there is no need for markets in the future.\footnote{Empirically speaking, the Costs of these Contingent Claim Transactions can be expected to be high because of the likely natural differences in Technological Information regarding one's calculations. But, once again, it is Effective Transaction Costs, perhaps based upon interpersonal differences in Technological Information such as appeared in our car-seller examples, that would be responsible for the informational breakdown of the Standard Competitive Model if the existence of markets in the future actually did force losses on the economy.}

The following section of this paper will show that once we admit a general Private Property System, no Effective Transaction Costs result from general differences in Technological Information in a generalized competitive equilibrium. The Transaction Costs implied by correct formulations of Arrow's "moral hazard" and Radner's "computational limitations" do not occur in the generalized competitive equilibrium appearing in a general Private Property System.

\footnote{Furthermore, markets in the future do not logically imply disequilibrium and losses due to imperfect information regarding future Prices. In fact, the assumption of perfect information on Prices in an equilibrium of a Standard Competitive Model extends to markets in the future. An equilibrium in a Standard Competitive Model implies that Prices of Assets in the future, under a given set of Technological Facts in the Future, are costlessly known by all. Since individuals have generally different set of Technological Facts in the future, this appears to violate equilibrium conditions in future contingent claims markets. But it does not, as is shown in the following section.}
3. THE GENERALIZATION OF THE STANDARD COMPETITIVE MODEL

a. The Organization of the Argument

We are now ready to relax the restrictive assumptions on the Technological Information Structure, the Private Property System, and Rationality implied by standard economic models. We retain the structure of Contract Information Costs and the Price rules of the Standard Competitive Model. The only restrictions on Technological Information employed will be the restrictions implied by the assumptions of zero costs of protection in a general Private Property System.

We shall proceed in stages. In subsection b we shall simply drop the assumption on Technological Information implicit in the Standard Competitive Model. In c we shall drop the restrictive definition of Private Property, using restrictions on the Technology to assure costless protection. In sections d and e we shall drop all Rationality assumptions. Our model and theorems are thus the result of the progressive relaxation of some assumptions of standard economic models.

b. Generalizing the Technological Information Structure

We now drop the assumptions on the Technological Information Structure which are implicit in the Standard Competitive Model (given our definition of the Social Technology) and which prevent an individual from having any Technological Facts Relevant to Decisions regarding the behavior of other individuals when these other individuals do not also have the Facts. Do the Offers and Acceptances at known Prices which are costlessly communicated to each individual in a Standard Competitive Equilibrium serve to obviate the communication of, or to reveal to an
individual, Technological Information Relevant to his Decisions even though the individual is not endowed with the Information and the Information is not directly communicated?

Many ordinary Commodity Transactions serve to obviate communications. For example, suppose a machine is not owned by an individual who has a Vector Superior set of Technological Facts Relevant to Decisions on the use of a machine over all individuals, cet. par., and each Technological Fact Relevant to Decisions on the use of the machine represents a certain knowledge of a distinct thing the machine can do. The machine will then be sold to the individual with the Vector Superior Set of Technological Facts rather than the original owner's incurring the costs of acquiring this Information. But, as our previous examples indicate, there are also cases in which Vector Non-Inferior Technological Information leads to a relatively pessimistic appraisal of the value of the Commodities or in which Decisions are with respect to the use of non-transferable Commodities. A more reliable ameliorative to the presence of general differences in Technological Information is found in the Prices in the Arrowian contingent claim markets. These markets do much more than the writers on these markets have claimed. It is fairly well-known that the presence of these markets both provide for a Pareto optimal distribution of risk in some context and allows us to currently determine Prices for all Assets and future behavior in a world with an uncertain future. But also these markets are unique in that they communicate Technological Information in the form of a function of the prior probabilities of some generally well-informed individuals. If, for example,
there are a certain set of Technological Facts available on the topic of whether or not a particular kind of house will burn down and only a small set of people initially have all of these Fact, the people who have these Facts, knowing that nobody outside the group has any more Facts will arbitrage the market for fire insurance so that their Facts will largely determine the market odds. Then an individual who does not buy insurance but who is deciding on whether or not to build himself a particular kind of house without knowing the Technological Facts Relevant to Decision concerning house-fires can decide on the basis of a prior probability of a fire destroying his house during a year equal to the ratio of annual insurance premium to payout-in-case-of-destruction-by-fire revealed in the market (minus a little bit if the insurers are risk-aversive and house-fires are not negatively autocorrelated events). The point is that individual prior probabilities on technological events adjust to observed market odds and these odds are determined by individuals who can ordinarily be assumed to have most of the relevant Facts. It is obvious that this is not a perfect mechanism in a general model since (1) several of the arbitragers may have some Facts not possessed by other arbitragers so that, in general, the prior probabilities on events revealed by the market odds are not generally those Resulting from the Social Technology, (2) the market odds differ from prior probabilities by an unknown adjustment factor reflecting attitudes toward risk and (3) the individuals with Inferior Technolgical Information must recognize its inferiority relative to those who are arbitraging the rates. Nevertheless, the mechanism does serve to reduce
the extent and magnitude of the communication problems in a Private Property System. (However, alternative Institutional Systems may also use specialist markets in contingent claims to generate universally agreed upon prior probabilities. See Thompson (1966, 1967a).)

Now imperfection (1) is removed by the assumption, described in Section 1, that for every Decision some individual has a Vector Non-Inferior Set of Technological Facts Relevant to the Decision. But to avoid imperfections (2) and (3) we require the additional Assets which are described in the following subsection.

The above, introductory discussion applies to the special kind of information differences described by differences in prior probabilities on a given set of events rather than to the Savage-Radner-type of information differences, which result from interpersonally different sets of distinguishable events. If these latter kind of differences in Technological Information were the only kind of differences appearing in the economy, objections (1)-(3) would not apply and the use of contingent claim markets would ensure the Pure Pareto Optimality of a Standard Competitive Equilibrium. This is easy to see. Consider any pair of events between which some individual can distinguish. As is usual, there is an Asset representing a claim to a certain amount of a useful good in the first of these two events but nothing in the second. Now the occurrence of the first event would put the value of the conditional claim once the event occurs equal to the value of the useful good to this individual, which is positive. The non-occurrence of this event would put the value of the conditional claim at zero to this individual. The equilibrium Price of the conditional claim given the occurrence of this event is thus positive -- a zero price would
imply an infinite excess demand. However, the Price of the conditional claim if and when the event does not occur is zero, assuming, with Radner, that people who can distinguish two events do not disagree on which of the events has actually occurred. Thus the individual who cannot distinguish between events need only observe the equilibrium prices of used contingent claims to know when one event has occurred and the others haven't. That is, an individual who cannot perceive distinctions that others can perceive can always buy and sell contingent claims just as if he could tell which events actually occur because it is feasible for them to sell their claims to individuals who can perceive such distinctions. The individual who could not distinguish between rainy and non-rainy weather need only look at the market price of used bets on the existing weather in order to know whether or not it is raining. Where Radner went wrong in constructing a Standard Competitive Model was in failing to permit markets in the future to exist. Such markets definitely exist under his type of information differences. It is Effective Transactions, not markets, in the future that need not exist (this still does not say that they do not exist) in the Arrow-Debreu model. There may be no Effective Transactions in the future in the presence of Assets and market Prices in the future.

1 This analysis, as do others in the Standard Competitive model, greatly strains the price constancy assumption. The most extreme strain appears to occur when one individual can perceive a distinction. In a sensible model, such an individual need only register a zero demand at a zero price for a used conditional claim when the conditional event has actually occurred in order to receive a financial gain in the case the individual holds a negative position in such claims. Of course, when the individual must play as if he does not have an effect on the Price, he must express his willingness to buy up the available supply of the contingent claim and more, at a zero price.
Nevertheless, for a general Technological Information Structure, we must go beyond the set of Assets that appears in the Standard Competitive Model in order to establish the Pure Pareto Optimality of an equilibrium in a general competitive model.

c. A General Private Property System with Perfect Information Regarding all Unconditional Contracts

In this subsection, we shall permit a general Private Property System. As noted in Section 1, this means that we are going to add three restrictions on the Technological Information Structure. First, we assume the existence of an Expert for each Decision; for each Decision, there is some individual who has Vector Non-Inferior Technological Facts Relevant to the Decision. Second, we assume the Recognition of Experts so everyone knows whether or not he is an Expert with respect to a Decision. Third, we assume that there is Computability of the value of the effect which one individual's behavior has on the Assets of another. We shall consider the sale of Decision Services in this further-generalized environment and shall wait until the next subsection to introduce conditions generating a positive profit to applying Interpersonal Force.

A Decision Service is an Asset supplied to an individual in which the supplier of the Service selects a Decision for an individual, receives from the individual the difference between the benefits the individual ostensibly receives under the selection and the benefits that the individual would have received without his service and pays the individual a non-negative, lump-sum for his consent in the Transaction. A Decision Service never makes the receiver worse-off. It makes the supplier better-off only if he increases the ostensible benefits to the receiver of the
Decision Service, benefits collected by the supplier of the Decision Service. Since Ownership is divorced from control when such services are provided, we necessarily use our general definition of a Private Property system specified in Section 1. If we still, however, consider only Voluntary Transactions. A supplier of a Decision Service initially has an Asset in the form of Technological Information useful to others. This Asset is transformed by the Decision Service into an Asset of the receiver called a Decision Service. The Asset Transaction when a Decision Service is supplied is called a Decision Service Transaction.

Examples of Assets similar to Decision Services in the real world are business and personal management services and retail product guarantees. Company Managers typically make Decisions on the use of capital inputs for the owners of the capital and take options on the company's common stock with correspondingly low salaries. Personal management services (e.g., tax computing services, legal services provided by auto liability insurance companies, and health clinic services) typically compensate a client for losses (due to miscomputation of taxes, to faulty legal defense, or to failure to cure an illness) which its services can influence. Manufacturers of retail goods typically make Decisions regarding the quality of a particular good among a given category of goods sold to a customer and freely repair or refund the purchase price (in exchange for the used good) if the product is found to yield benefits below an amount which justifies its purchase.

The actual situations in which such activities arise in the real world indicate the theoretical usefulness of Decision Services in a generalized competitive model in which some individuals have Technological
Information Relevant to Decisions regarding the behavior of others: First, business management services are used in situations where the owners of capital (who are the economy's well-to-do optimists about the future productivity of the capital or those most prepared to accept the unavoidable risks of the business) are not the economy's specialists in the techniques for using the capital to produce goods. These latter individuals are the suppliers of business management services. By giving these suppliers a marginal financial reward close to what would be the owners' if they hired no managers, the owners are inducing the managers to use their specialized Technological Information as would the owners if only they had the Technological Information. Second, personal Decision Services are employed in situations in which the Decision makers (tax accountants, lawyers, doctors) have much more Technological Information Relevant to the Decision than the receivers (tax payers, victims of auto liability suits, the sick). All such specialist-decision-makers can, in the absence of competitively supplied Decision Services, fairly easily convince the receivers that they are in deep trouble and can be saved only at great cost. Third, many manufacturers and retailers have many more Facts about the physical characteristics of their own outputs than new buyers and it would be very expensive if the customers worked before purchase to discover the characteristics or if the only companies for the customer to choose from without working hard to find out about the good were a few companies who work to establish a brand name.

Suppose now that there is a Standard Competitive Equilibrium in a world in which some individuals have Technological Facts Relevant to a Decision concerning the Behavior of individual A that A does not have. Decision Services are inadmissible because their creation requires Assets,
Technological Facts, inadmissible in a Standard Competitive Model. This equilibrium is not a Pure Pareto Optimum. Earlier examples (those with rich dreamers and informed car sellers) are sufficient to show the non-optimality of such equilibria. In this context, we drop the artificial prohibition against Decision Service Transactions. By our first restriction on the Technological Information Structure, at least one individual other than A, say individual B is an Expert with respect to the Decision. A receiver of a Decision Service is never worse-off, for he is compensated for any losses under the current Technology. We need only show that individual B can profit by offering, and hence selling, a Decision Service to Individual A. With a zero payment from individual A to individual B, A would be better-off if he had the Technological Information by an amount equivalent to the amount he is better-off under the Decision that the extra Information implies. This is because no one has any direct utility for the Technological Information, and, if the Decision Service does not cover all of A's Decisions, A himself may adjust these other Decisions, perhaps by purchasing other Decision Services. Such Decisions are dependent upon the purchased Decision but not upon the Technological Facts Relevant to the Decision. Because he is Rational, individual A would be better off if he had the superior Technological Facts. Hence, still ignoring payments from B to A, A pays B a positive amount of wealth if B makes the Decision. Under the assumptions of (1) Computability and (2) no Costs of Unconditional Transactions, there are (1) no costs of B's knowing that A can use his service and (2) no costs of executing the Offers and Acceptances.

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1As our empirical examples indicate, it is not necessary the settlement of a Decision Service Transaction be made at the time of the provision of the Service. The benefits received by the recipient of the Decision Service over his benefits without the Service, applicable to the period in which the Decision Service is sold, may be computed (and computable) only at some future date.
Prices, or lump sums that B must pay to A for the right to supply him the Decision Service, are given and non-negotiable. B is willing to pay a higher price for this right than someone who has information inferior to his but superior to A's for that someone recognizes the inferiority of his information. An Equilibrium Price for receiving a Decision Service in a General Competitive Model is one that the supplier, like individual B, is willing to pay and no one else is willing to exceed. The Rationality of Actual Behavior assures us that the profits are taken by the suppliers or receivers of Decisions Services. Now consider an equilibrium so that no behavior which will increase profits is feasible. The absence of sources of additional profits implies that no Decisions are made without all of the Technological Facts Relevant to the Decision that are contained in the Social Technology. And, because of the incentive system built into a Decision Service Transaction, the Decision Makers make the same Decisions that would be made by those who receive the Decision Services if only the latter had the Information. Hence we can see that our competitive equilibrium with Decision Services is equivalent to a Standard Competitive Equilibrium in which everyone has the Social Technology. This, of course, is a Pure Pareto Optimum.

We can now see what happens to the Transaction Costs appearing in the informed car seller examples once we introduce our market in Decision Services. When the car seller knows the brakes are bad, he benefits more by Deciding for the individual that he fix the brakes and collecting from him slightly less than the current cost, minus the cost of brake repairs, of his having an accident. In the case of the car seller's not being able to convince the healthy skeptic, the car seller merely takes responsibility for any harmful effects of the car (there are none) and receives the consumer surplus of the buyer minus a sufficiently small lump sum.
In the case of the car seller who researches the future use of cars and advertises in order to dump them in a hurry, his gain is apparently through acquiring superior information regarding future Prices, which is impossible in even a Standard Competitive Equilibrium because these Prices, contingent upon different future Technological Facts, are known by all. Thus, the example appears to describe a Transaction Cost due to a disequilibrium.\footnote{If, however, the car seller acquires Technological Facts relevant to the existence of future Technological Facts, then the assumption of known future Prices for given future Technological Facts is not violated. Another basic assumption is violated instead. Since the new Facts have an effect on Prices once the producer sells decision services to take full advantage of his Information, the producer will substantially overproduce such Facts because of the redistribution of gain involved (see Thompson (1965) and Hirshleifer). This argument applies to all economic activities, not to just the production of Facts and is ruled out by the assumption that individuals have no effect on Prices.

A case of Fact production that is consistent with the Standard Competitive Model is the case in which the produced Technological Information is directly useful to only one individual. This makes the Fact indistinguishable, in all characteristics relevant to theory, from an ordinary Commodity.}

It is also easy to see how the non-optimalities resulting from the Technological Information differences in our other examples of Standard Competitive Models disappear once we introduce our market in Decision Services. In the rich dreamer example, the informed chicken farmers, unfortunately for them as a group, compete with one another by offering Decision Services to the rich to stop them from paying higher Prices for and increasing the breeding of chickens. They receive the losses that the rich would have made minus lump sums sufficiently large to reflect the competition among the chicken farmers. The individual who cannot tell that it is raining—he is now Irrational because he must be able to observe the zero market Prices of claims contingent upon the absence of current rainfall—has another individual Decide for him that he should go inside. This other

\footnote{If, however, the car seller acquires Technological Facts relevant to the existence of future Technological Facts, then the assumption of known future Prices for given future Technological Facts is not violated. Another basic assumption is violated instead. Since the new Facts have an effect on Prices once the producer sells decision services to take full advantage of his Information, the producer will substantially overproduce such Facts because of the redistribution of gain involved (see Thompson (1965) and Hirshleifer). This argument applies to all economic activities, not to just the production of Facts and is ruled out by the assumption that individuals have no effect on Prices.

A case of Fact production that is consistent with the Standard Competitive Model is the case in which the produced Technological Information is directly useful to only one individual. This makes the Fact indistinguishable, in all characteristics relevant to theory, from an ordinary Commodity.}
individual does so because he receives an amount of wealth equal to the
cost of the individual's catching pneumonia minus a lump sum payment,
which is close to his revenue because others can also provide the Service.
This example anticipates some of our analysis of Irrationality appearing
in the following subsection.

Other examples of the model in which business managers, lawyers
and doctors provide Decision Services and create a Technological Information
Structure consistent with a Standard Competitive Model are easy to con-
struct from the empirical examples of these activities mentioned above.

The benefits received by the suppliers of a Decision Service may
be benefits applicable to a future Technology rather than benefits taken
with respect to the current Social Technology (but perhaps computed at
some future date.) This possibility occurs in all of our empirical examples
of Decision Services. The supplier of the Decision Service can costlessly
insure against variation in future Technological facts or benefits (which
may require providing Decision Services to the insurer) in the same way
as the recipient could if the current Social Technology were the basis
of computing benefits. Thus, no differences between payments determined
by benefits under current and future Social Technologies exist. That is:
If a supplier of a Decision Service in one of our examples wants to convert
the uncertain future returns from his Service into a certain current return,
thereby paying someone a risk-premium to bear the risk, he will receive
the same return as he would under a payment determined by the current
Social Technology, for in the latter the recipient of the Decision Service
passes the insurance cost onto the supplier of the Decision Service anyway.
(And if the supplier in our more formal exercise wants to bear the risk of
variations in Social Technologies, he may sell such insurance to the
recipients of the Decision Services, thereby receiving the benefits from
his Decision Services applicable to a Future Technology, such as described in the empirical examples.)

There is a sense in which the Standard Competitive Model generalized to contain Decision Services is more restrictive than the Standard Competitive Model. Since no Decision Services are in demand in a Standard Competitive Model (due to its narrow Technological Information Structure), there is no need in that model to retain any assumption of Perfect Information regarding Decision Service Contracts. Accepting this, we shall drop the assumption of Perfect Contract Information regarding Decision Services for the remainder of the paper.

d. A General Private Property System with Imperfect Contract Information

We now allow positive costs of communicating Offers and Acceptances in the Decision Service market. Then Effective Costs of Unconditional Transactions can arise either as Direct Costs of Transactions or as Indirect Costs in that an individual will make Voluntary Decisions that are not the Decisions he would make if he had another's Technological Information. All of our previous examples of equilibrium losses due to Inferior Technological Information would be possible. This admits Informational Irrationality in that an individual makes decision which do not maximize his utility under the Given Social Technology because the Costs of acquiring Institutional Information prohibit him from making Decisions under Vector Non-Inferior Technological information. But we still assume the absence of Behavioral Irrationality in that an individual, when left alone, makes the decisions which maximize his utility given his own, perhaps Inferior, Technological Information.

So far, we have restricted the Model of N-Person Bargaining under Private Property to Voluntary Behavior. We also drop this assumption. Thus, one individual can alter the Assets of another without any consent --
i.e., he may apply Interpersonal Force. Since, in a Private Property System, the Forceree receives any benefits that he would have had without the Force, and since the Forcer receives any benefits that he creates for the Forceree because of the Force, the marginal reward structure to the Forcer is the same as it was to a supplier of Decision Services in the above subsection. Nevertheless, no lump sum payment to the Forcer is required as it was to the recipient of the Decision Service.

Now suppose that, due to the relaxation of assumptions, there is a set of Decisions by individuals such that some individual is, in fact, using Inferior Technological Information in making a Decision. An Expert with respect to the Decision then does not provide a Decision Service because there are Prohibiting Costs of communicating Offers and Acceptances. No Offers and Acceptances are used in an Interpersonal Force Transaction. Therefore, when such a Transaction achieves the same behavior as a costly Decision Service Transaction, we can assume there are no Transaction Costs of the Interpersonal Force Transaction given our assumptions on the rest of the Model. As a result, profits in the application of Interpersonal Force exist to an individual with Vector Non-Inferior Technological Information. The profits arise in Forcing the individual with the Inferior Technological Information to behave as if he were an Expert. The benefits to the Forceree are then paid to the Forcer. A Forcer may make profit if he has Technological Information superior to the Forceree but inferior to an Expert with respect to the Decision. However, an Expert with respect to the Decision may still profit in Forcing behavior further so that Decisions are finally made under Non-Inferior Technological Information. Only the ultimate Forcer will then have to share the total Forcing profit with the intermediate Forcer. Since no Effective Transaction Costs exist, this unnecessary intermediate Force does not alter the Pure Pareto Optimality of the Competitive Equilibrium. (To obtain distributional determinacy,
we may assume that Private Property in the ability to apply Force exists so that a Forcer must pay someone a lump sum equal to the profit of the most effective alternative Forcer. If this someone is the Forcee, we have the equivalent to a result obtained above under zero Costs of Decision Service Transactions.) Once a Standard Competitive Equilibrium is achieved, there is no further profit to Force, as applying it will only lead to losses due to Decisions other than those made with Vector Non-Inferior Information. The reason profits exist in Interpersonal Force and not in the provision of Decision Services is that there are positive costs of communicating Offers or Acceptances in the market for Decision Services and no such costs are necessary in the case of Interpersonal Force. Examples of this cost situation are that an individual may be away from his house while it is on fire. Or an individual may be unconscious after an accident which would be fatal without an immediate blood transfusion, while a potential property saver (such as a fireman or a doctor) may be standing-by. Since the costs of communicating Offers and Acceptances are Prohibiting in these cases, the potential saver cannot acquire consent and hence does not save the property in a Private Property System without Force. Other kinds of examples exist in which communication of Offers and Acceptances of Decision Services is costly merely because of the poor quality of understanding Social Institutional Information of one of the individuals in a Decision Service Agreement. For example, young children may simply not understand that they will be better-off (as recipients of a Decision Service) if they do not run into the street. For another, any form of communication may be precluded by language barriers as in the case of communicating with foreigners, newborn babies, or pets.
Suppose that we permitted an imperfection in the Private Property System by assuming positive costs of applying Counterforce. This is a very natural imperfection to assume once we have assumed imperfections in communicating Offers and Acceptances in the market for Decision Services. For instance, the above examples of communication costs in the Decision Service market are apparently also examples of costs of applying Counterforce. In any case, since the Forcees in the above model are never worse-off because of the Force, the presence of this imperfection has no effect on Actual Behavior in the above model. However, Forcees could be made worse-off by uncountered Force in a model that permitted bankruptcy; and in such a model positive costs of Counterforce Transactions would generally lead to non-optimalities (Section IV).

e. Dropping the Rationality Assumption

Let us now simply drop the assumption of Behavioral Rationality (called simply Rationality in Sections 1 and 2). Behavioral Irrationality is a situation in which an individual's actual decisions do not correspond to his optimal decisions even though he knows both. His choices do not correspond to his preferences. His body does not cooperate with his mind. Having freely available Vector Non-Inferior Technological Information is sufficient to determine optimal individual Decisions but is obviously not sufficient to induce an equivalence between actual and optimal decisions. Suppose that the costs of all Counterforce Transactions are Positive. A non-optimality immediately now infects the resulting set of possible competitive equilibria. This arises because someone may now employ Interpersonal Force in a Behaviorally Irrational way with no institution available to stop him. One pathetic example will suffice: Someone burns down another's house with no one around to stop him; he pays the huge Price and is much worse off. He preferred not to burn it down but he did it any way.
His Actual Behavior is obviously inconsistent with Pure Pareto Optimality. The loss is an Indirect Cost of Transactions in Forcing an individual who is himself Forcing the behavior of another individual. If all Force were simply prevented by sufficient corporal punishment, assuming it is possible to do so, these might be a net social improvement in some sense even though the advantages of Force indicated in the above subsection would thereby be precluded. In any case, it is clear that Interpersonal Force does not generally lead to Pure Pareto Optimality once we add Behavioral Irrationality to Informational Irrationality when there is costly communication of Counterforce behavior. Hence, we maintain the assumption of a Private Property System in this subsection so that there are no costs of Counterforce behavior, although doing so substantially reduces the imperfections in Contract Information and Informational Irrationality which Interpersonal Force was so useful in overcoming in the above subsection.

Behavioral Irrationality by an individual with Vector Superior Technological Information regarding his Irrational Decision is another source of nonoptimality which our assumptions must somehow eliminate. Such a Behavioral Irrationality implies an Informational Irrationality of all other individuals. Our analysis has not precluded individuals from Forcing others to make a Decision even though they do not currently know all of the characteristics of the Decision. Furthermore, the informational basis of the Computability assumption allows us to assume that any individual with Inferior Technological Information can Force an individual with Vector Non-Inferior Technological Information Relevant to a Decision to make a Decision that the Forcer knows but the Forcee doesn't currently know. This is because the Forcee either will know the Decision or would know it if he devoted available resources to a current Decision. We set
this possibility with an example. Consider a woman who can't stop biting her fingernails even though she wants to and won't, for some unexplained reason, hire someone to stop her. Now if someone who does not costlessly know what she is doing but still commands the woman to simply "behave right," and she does not apply the available Counterforce measure, she must, to avoid severe Corporal punishment once the Forcer finds out what she was doing, stop biting her nails.

We are now ready to proceed with our optimality demonstration.

First, suppose that every individual is Behaviorally Rational in his own Interpersonal (including Counterforce) Force behavior. Then the results of subsection d and the above immediately apply so that Interpersonal Force removes any Irrationality. But we cannot retain the assumption of Behavioral Rationality with respect to Force behavior. Interpersonal Force may be applied Irrationally by everyone except one individual.

Now suppose that some individual Irrationally applies Counterforce to the Force Decision of a Rational Individual. The difference between applying Counterforce and simply Rejecting an Offer of a Decision Service is that Counterforce, being a form of Force, requires the Counterforcer to pay for any real losses his Actual Behavior inflicts on others. Therefore, the Force-Counterforce procedure nets the Rational Forcer the same returns as if he were not Counterforced and the Irrational Counterforcer the negative of these returns, leaving the individual to whom the Rational Force was directed with his original, Irrational Decision. But the Behaviorally Rational Forcer will apply the same Force once again, for his profit opportunity is not gone. In this way he makes twice the return he would have if he did not initially meet with Counterforce. If Counterforce is again met, the same argument applies. But there is only so much wealth in the economy. Sooner or later, everybody besides the
Rational Forcer will either decide not to apply the Counterforce or will lose all of his wealth to the Forcer. An individual in the latter group can no longer afford the luxury of Irrational Counterforce. If the original Forcer loses his Behavioral Rationality with respect to Force decisions before this happens, some Behaviorally Rational Individuals remain by assumption so that he will simply be replaced in the remaining iterations. Once each Counterforcer has lost his habit or his wealth, Counterforce ceases and the Behavioral Irrationality of the initially Irrational is removed. (Force yielding a given negative profit is not feasible behavior for an individual whose wealth is sufficiently close to zero.)

As we have just indicated, if there is profit from Force but some individuals are Behaviorally Irrational in their Force decisions and do not take the profit opportunity, others will take the opportunity so that there would be no Behavioral Irrationality. As long as someone is Behaviorally Rational, Interpersonal Force will remove the Behavioral Irrationalities of others.

Finally, if any individual Irrationally imposes Interpersonal Force, he will be Counterforced by someone who is Rational in his Interpersonal Force decisions and will lose wealth until he quits as we saw above in the case of the Irrational Counterforcer.

Hence we arrive at a Standard Competitive Equilibrium that would result if everyone were Behaviorally Rational and everyone had the Social Technology. This, again, is a Pure Pareto Optimum.

It is obviously straightforward to extend this result on the optimality of a solution with Irrationality but Interpersonal Force
to the solution set (the Imputation Space) of the N-Person Bargaining Model of section 2 containing Perfect Contract Information and also to the solution set (contained in the Core) of the N-Person Bargaining Model with Perfect Contract Information except for Prohibiting Costs of Counteroffer Strategies.
4. AN EXTENSION AND APPLICATION OF THE MODEL TO THE PHENOMENON OF BANKRUPTCY

a. The Case of Perfect Information regarding Unconditional Contracts

The immediate problem which the phenomenon of Bankruptcy creates for economic theory is that it implies that an individual does not pay the market Price for a Commodity he has received and that another does not receive the market Price for the Commodity he has sold. However, suppose there is certainty with respect to the future Technological Facts which determine the future payment for a currently delivered Commodity. Then it can be determined at the time of the original agreement whether or not there will be a full payment. If there is not full payment, then there is no Transaction. If there is full payment, there is no problem. Effective Bankruptcy therefore implies uncertainty with respect to the conditions under which a future delivery of Commodities will be made by a particular individual. Prices then are not to be understood as certainty payments but rather as contingent claims. Once this is recognized, the occurrence of Bankruptcy does not violate any budget constraint. Thus, the Price an individual pays for a delivered Commodity which is not paid for instantaneously is a contingent claim—a claim to a certain amount of wealth if certain Technological Facts occur and, say, zero otherwise. This alone insures that the Standard Competitive Model (Debreu, Chapter 7) applies to the case of Bankruptcy. It also implies that our General Competitive Equilibrium with interpersonal differences in Information regarding production possibilities and events applies to the case of Bankruptcy. Let us see how.

There are several violations of Pure Pareto Optimality created by the possibility of Bankruptcy once we generalize the Technological Information Structure of the Standard Competitive Model without generalizing its private
property system. One is that debtors who know there is a higher chance of their going Bankrupt than others think will over-buy and over-use Assets because they pay an artificially low cost for them due to the high probability of their not having to pay. A second violation occurs when debtors who know they are not going to go Bankrupt have to pay an artificially high cost for their assets to reflect the risk that the undiscriminating lender puts on the typical borrower's becoming a Bankrupt. Third, borrowers devote real resources to demonstrate the low probability of default when the borrower may have been willing to borrow at the high rate for his possibly indivisible investment. Finally, lenders devote real resources to detect risky investors when the risky investors would have borrowed no more at the lower rates. These Behaviorally Rational activities all create Indirect or Direct, wasteful Transaction Costs to pairs of Individuals represented by the borrowers and lenders, thus violate Pure Pareto optimality in a Standard Competitive Model with a general Technological Information Structure. But all of these violations are based upon the absence of Decision Service (or Force) Transactions. An individual with Vector Superior Technological Information Relevant to Decisions regarding his own Bankruptcy will, under zero Costs of Decision Service Transactions, sell Decision Services to those from whom he buys current Assets in exchange for future Assets contingent upon future Technological Facts. The reason in the first case is that he will thereby collect the losses that he would otherwise have inflicted on his creditors without having to buy more than is jointly efficient to himself and his creditors. The Computability assumption is obviously necessary here because without Computability the potential bankrupt would merely misrepresent the optimal Decision so he can overborrow. That is, the potential debtor will sell a non-optimal Decision Service unless he is faced with eventual detection as well
as punitive damages or corporal punishment for so doing. (For an analysis of Decision Service and Force Markets without the Computability assumption, see Thompson (1970).) A similar argument for the profitability of selling Decision Services applies in the fourth case except the joint savings is not in terms of a reduction of the amount of joint asset purchases but rather in the form of the saved costs of the potential creditors acquiring Technological Information equal to that of the potential debtors. The second and third cases and the mixtures are handled in analogous ways.

b. The Case of Imperfect Institutional Information in applying Counterforce

Examples given above are cases in which one’s house is burning down in his absence or in which he has had an accident, is unconscious, and needs an immediate blood transfusion. These are cases in which there are Prohibiting Costs of communicating Acceptances of Decision Services. Such communication costs generate not only positive profit to Force but also Prohibiting Costs of Counterforce. As the argument of Section 3e has indicated, if we permit such Imperfections in Institutional Information and Behavioral Irrationality with respect to Force decisions, Pareto nonoptimality of an equilibrium generally results. We will find another case of nonoptimalities resulting from imperfections in applying Counterforce in this subsection. But in order to isolate the problem we wish to discuss here, we continue to assume Behavioral Rationality in what follows.

So far in our analysis, a Behaviorally Rational individual does not bother to impose Force which might be countered unless there are positive costs of market communication in the Decision Service Market. By introducing the possibility of Bankruptcy, however, our analysis will extend (unfortunately) to cases far beyond market communication costs in the buying of Decision Services.
Any argument for nonoptimality can proceed by example. Suppose a poor individual with no particular Technological Information advantage forces everyone in society to carry an umbrella tomorrow and that there is a known 10 percent chance of rain. If it rains, the individual will get rich from his collection of benefits from the large group of individuals who otherwise would not have carried umbrellas. But he can just go Bankrupt if it doesn't rain. So his losses are much less than the social losses and he finds his Force Decision rational to him even though it is inefficient to society for all of those umbrellas to be carried around given such a low probability of rain. We note, however, that if the individuals who are being forced to carry umbrellas had no costs of some form of communication with the Forcer, they would apply Counterforce so as to make the original Forcer's profit negative and remove the inefficient behavior from the economy. We call Force imposed when the Costs of Counterforce are Prohibiting an act of Unilateral Force. Another example of Unilateral Force would be that of some poor individual's Forcing a blood transfusion upon any unconscious body he can find, becoming rich if the person needs it but just going Bankrupt if the person suffers badly as a result.¹ Other examples of Unilateral Force are cases in which the Forcer receives some benefits before the time at which he must pay for any losses that his Force inflicts upon others. An owner of a manufacturing plant who Forces on others a form of pollution

¹In both of these examples, if it were possible to scale down the payments that a Unilateral Forcer receives in case the beneficial events to the Forcee occur by an amount equal to his gains from going Bankrupt instead of paying off in case the undesirable events occur, the problem disappears as then the Forcee is receiving the value of his Force to the others despite his possible Bankruptcy. But many cases, for example the next example in the text, exist in which such scaling down of benefits to Forcers implies greater restrictions on the Technological Information Structure than exists in the model developed to this point.
which has certain and catastrophic effects that show up to others only after he has used up his plant is one example. The manufacturer receives the benefits of his production but does not pay the full cost because when the catastrophe occurs, he will just go Bankrupt.

In another interesting case, which contains what we usually think of as crime, there is a state-claim penalty. If a certain event occurs, then the Forcer must pay a penalty equal to a multiple of damages he Unilaterally Forces upon others; otherwise he must pay no penalty for inflicting the same damages, where the multiple—under no Bankruptcy—is sufficient for all the Unilateral Forcers to compensate all of their Forcees. Without Bankruptcy, Pure Pareto Optimality exists and the Unilateral Forces are paid actual compensation.

The Unilateral Forcers may even get a net benefit from the excitement of the behavior. But if their debts in the game may Bankrupt the Unilateral Forcers so that they cannot pay their fines, then: It is not clear what the fines should be; the Unilateral Forcees should not generally be compensated because they should be encouraged to protect their Assets and thereby discourage the overproduction of Unilateral Force; and the principles determining the optimal state-claim odds are similarly altered.¹

These examples show that the cases that break down the Pure Pareto Optimality of a Competitive Equilibrium under Rationality and Imperfect

1In the absence of the Bankruptcy of the Unilateral Forcers, optimal state-claim odds, or probabilities of "catching the criminal," are those which result from the devotion of no resources to the activity of catching criminals. If, however, caught criminals might go Bankrupt, higher odds and thus lower penalties but positive resources devoted to catching criminals might improve the system through reducing its Bankruptcies and thus reducing its over production of crime.
Institutional Information in applying Counterforce are all cases in which the Unilateral Forcer does not adjust the costs of his Force for the damages he inflicts upon others because of the possibility of his avoiding the costs in Bankruptcy. These examples also serve to show that the inefficiency resulting from the possibility of Bankruptcy with respect to debts incurred in applying Unilateral Interpersonal Force goes far beyond the problems that result from costly communication in the Decision Service Market. Indeed, the Transaction Costs of Counterforce may be positive simply because the Forcer intentionally makes market communication with him very difficult (burglary is an example). For Counterforce generally makes a potentially Bankrupt Forcer worse off and Force-without-Counterforce makes him better off because of the possibility of Bankruptcy.

These cases of nonoptimality obviously represent violations of the Private Property System in that an individual does not compensate losers for the real damages his behavior inflicts upon them.

What is most interesting is that such cases of Bankruptcy due to debts incurred in applying Unilateral Force are apparently unavoidable. Decision Costs to Society and a challenge to economics because non-redistribu-
tional tax-subsidy policies are not available to avoid the problem of the over-Forcing and related over-Bankruptcy. This is because Bankrupts cannot pay taxes and the Unilateral Forcers that create the problem necessarily avoid communication at the time of the Force. It may then be Pareto optimal (but not Purely Pareto Optimal) to impose corporal punishment to have the potential Bankrupts suffer an amount sufficient for their losses from Force to equal the losses their Force imposes on others. But such punishment to reflect social costs does not generally remove all Rational (and efficient) Force behavior and it is an obvious violation of Pure Pareto Optimality to so punish someone. Therefore, it may be better--given a sufficient degree of
insensitivity of crime to corporal punishment—to simply permit the over-Force behavior and abolish the corporal punishment. But, in either case, it is generally efficient (although still not Purely Pareto Optimal) to have taxes and subsidies on a myriad of activities in order to create the side effect of reducing the Decision Costs to Society created by the possibility of Bankruptcy due to debts incurred in applying Unilateral Interpersonal Force. Furthermore, expenditures for crime prevention and criminal-catchign, which violate Pure Pareto Optimality and are inefficient in a model with zero costs of applying counterforce, may be justified by noting its effect in reducing the frequency of occurrence of Bankruptcy due to debts incurred in applying Unilateral Interpersonal Force.

The only apparent analytical escape from these realistic problems of "second best" optima in the face of positive costs of Counterforce Transactions is to (a) assume that the Optimum does not require Unilateral Force behavior in which the Forcer may go Bankrupt due to debts incurred in applying Unilateral Force, and (b) apply corporal punishment to individuals who go Bankrupt with respect to debts incurred in applying Unilateral Force sufficient to prevent much Bankruptcies from ever occurring.
5. SUMMARY OF OPTIMALITY RESULTS

Our central optimality results are now summarized.

(i) An equilibrium in a model of N-Person Bargaining in a general Private Property System is a Pure Pareto Optimum if (a) Prices are constant and non-discriminatory, (b) there are no Costs of the Transactions occurring in a Standard Competitive Model, (c) for every Decision there is an Expert (an individual who has Vector Non-Inferior Technological Information Relevant to the Decision) and (d) Pareto Optimality is implied by Social Optimality. The proposition contains no Rationality Assumption.

(ii) An equilibrium in a model of N-Person Bargaining in a general Private Property System except for the existence of positive Costs of Counterforce Transactions is a Pure Pareto Optimum if (a) Prices are constant and non-discriminatory, (b) there are no Costs of Unconditional Contract Information, (c) for every Decision there is an Expert, (d) everyone is Behaviorally Rational, and (e) the Optimum does not require, and the Institutional System does not permit, Bankruptcy due to debts incurred in applying Unilateral Interpersonal Force.

Proposition (i) stated in terms of explicit information and as a generalization of the Standard Competitive Model, is:

(i') The equilibrium of a Standard Competitive Model generalized to permit any Technological Information Structure, Irrationality, Interpersonal Force, and Bankruptcy is a Pure Pareto Optimum if: (1) the Technological Information Structure is restricted only in that (a) for every Decision, there is an Expert (b) Experts are recognized, and (c) there is Computability of the benefits (losses) which one individual Forces on another; and (2) there are no Costs of Contract Information regarding the Transactions appearing in a Standard Competitive Model, and (3) someone is Behaviorally Rational.
REFERENCES


