Towards a Dynamic Disequilibrium;  
Some Queries Regarding Methodological  
Analogy of Statics and Dynamics in Economics

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Discussion Paper Number 37  
May 1973

A preliminary draft of research outline.
This is to reexamine the theoretical and methodological validity of the structural foundations of existing micro and macro economic theory when the Walrasian Auctioneer is absent in the system. Recent efforts to reformulate disequilibrium models with the incorporations of imperfect information are mainly directed toward a remodification of upper structure of the existing theoretical framework. My analysis shows that these literature concerning the theorization of disequilibrium models within the framework of comparative statics or dynamic apparatus not only reveal theoretical weakness but also contains erroneous misconceptions from the lack of methodological understanding. One needs not point out all the flourishing evidences of currently intensified efforts which attempt to reformulate disequilibrium models from a new perspective with the renewed application of existing economic tools. It may suffice, however, to point out that well materialized arguments presented by many leading economists with the given assumption of imperfect information or the absence of the Walrasian Auctioneer \(^{(1)}\) are mainly focused to the convergence to unique equilibrium price either through theorization of the functional relationship where market prices are postulated to change in proportion to excess demand (Hahn, Uzawa) or through the application of probability distribution on the basis of consumers search behavior (Stigler). On the other hand, large literature concerning disequilibrium analysis focus to the effect of demand uncertainty on the firm, and hence, the element of

\(^{(1)}\) The meaning of imperfect information is used such a diversified way among economists that there is no way to draw an exact boundary of implication. Distinction between imperfect information and the absence of Walrasian Auctioneer may involve more than semantic. However, when the Walrasian Auctioneer is absent, all the facts and information can not be assumed as given at zero cost and without time consumption, and hence the absence of the Walrasian Auctioneer in the system will naturally include the assumption of imperfect information.
uncertainty plays a dominant role as the factor of disequilibrium (Fisher, Baron, and McCall). These literature not only contain theoretical weakness but also involves the methodological mishaps due to the failure of clarifying the doctrinal characteristics related to this problem. Unfortunately, much valuable efforts are wasted without harvesting a fruitful result because no one really questioned the structural and methodological validity of existing micro foundation prior to the remodification of upper theoretical structure. Instead of careful scrutiny of the existing theoretical foundation, economists were and are hastened to modify the theoretical structure in the framework of marshallian partial equilibrium analysis which is, I believe, the mere extention of the Walrasian general equilibrium and modified utilization of partial domain in the given general equilibrium framework.

It is quite tragic to note that Hayek's pioneering and in a sense, prophetic article of 1945. "The Use of Knowledge in Society" never drew any serious attention from contemporary as well as modern economists. In this article, Hayek challenged bravely to the methodological falsity lies in the micro and macro theory which have been guarded so long by economists.

Hayek states, the "practical problem arises precisely because these facts are never so given to a single mind, and because, in consequence, it is necessary that in the solution of the problem knowledge should be used that is dispersed among many people. [Hayek, p. 530].

In spite of Hayek's serious challenge, his assertion has never been either refuted or accepted by economists. It is rather self evident that the current economic theories concerning either equilibrium or disequilibrium analysis are the mere augmentation of Marshallian partial equilibrium analysis and undisclosed stepchild of the Walrasian general equilibrium analysis. Although current micro and macro literature fail to point out, it is quite clear that theories
are the result of the habitual mode of thinking along the Walrasian tradition. In particular, in the existing micro and macro theories, the theorizing economist knowingly or unknowingly replaces the role of the Walrasian Auctioneer in the system. It may hardly require to substantiate the fact that the current economic theories not possess the basic Walrasian characteristics in them but also adopt Marshallian partial equilibrium apparatus. The evidence of the role as the Walrasian Auctioneer is well documented in the current literature of micro as well as macro economics where all the facts and essential knowledge are given for the solution as the crucial assumption to the observing economists. For the pedagogical purpose, it may serve its usefulness to examine the fundamental foundation of micro economics advanced by Marshall which is widely accepted and materialized in the current economics theory in the form of static and comparative statics.

Begin with, Marshall acknowledges that in order to develop comparative static apparatus of supply and demand schedules, well defined market concept is vitally significant, because after all aggregate or composite demand and supply schedules are horizontal summations in the market respectively.

Marshall states, "The central point of a market is the public exchange, mart or auction rooms, where the traders agree to meet and transact business. .... But this distinction of locality is not necessary. The traders may be spread over a whole town, or region of country, and yet make a market, if they are, by means of fairs, meeting, published price lists, the post office, or otherwise, in close communication with each other. [A. Marshall, p. 325]. It is perfectly clear that Marshall invited a troublesome concept of market to his theoretical construction. In this modern age, economists may encounter serious difficulties in defining the concept of market such as oil, automobile, turbot fish and etc. Along the Marshallian tradition, economists
adopted Marshallian market concept and proceeded a step ahead along this line by assuming that the exact number of existing consumers for a particular commodity is known to the observing economist.

Furthermore, well accepted common practices of obtaining individual demand and supply schedules are clearly specified by Marshall.

Marshall states, "To obtain complete knowledge of demand for anything, we should have to ascertain how much of it he would be willing to purchase at each of the prices at which it is likely to be offered; and the circumstances of his demand for, say, tea can be best expressed by a list of the prices which he is willing to pay; that is, by his several demand prices for different amounts of it. (This list may be called his demand schedule.) [Marshall, p. 96.] In addition, Marshall introduced several other variables which may effect the demand as well as supply but assumed these variables remain constant throughout the period of conceptual experiment. Marshall states "It is assumed that the general circumstances of the market remain unchanged throughout this period; that there is, for instance, no change in fashion or taste, no new substitute which might affect the demand, no new invention to disturb the supply." [Marshall, p. 342] In the modern context of micro economics, the demand schedule can be expressed in the following function of form, \[ p = f(q, T, C, Y, P_n, R, E, \lambda) \]

where \( p \) = the price of commodity

\( q \) = the quantity demanded by commodity

\( T \) = consumers taste and preferences

\( C \) = the number of consumer under consideration

\( Y \) = consumer's income under consideration

\( P_n \) = the prices of substituted goods where the subscript \( n \) represents the number of goods
R = the range of goods and services available to consumers

E = consumer's expectation

λ = all other unaccounted variables which may affect the demand schedule, such as import or export, weather, and etc.

Holding all the other variables (T, C, Y, P^n, R, E, λ) constant, demand schedule usually lists the different quantities of commodity that consumer under consideration will take opposite the various alternative price of the good. It is significant to point out that the derivation of individual demand schedule is the result of conceptual experiment which hypothesizes the relationship between possible alternative prices of goods and the quantities of it that consumers will take while all the other variables listed above is treated as parameters. In the real world where all the variables are continuously and endlessly changing, any effort to construct an individual demand schedule along this line is simply futile. It is also equally clear that in spite of the essential nature of dynamics which underlies in the micro economic behavior, Marshall dealt with the problems in the framework of stationary statics which is later extended to the comparative static analysis. Elements of time simply does not play any meaningful role in any aspect and travels freely from zero to infinity without affecting real phenomenon disguised in the forms of variables. For the sake of fairness, it should be pointed out that such a derivation of individual demand for that matter, supply schedules based on conceptual experiment may be pedagogically useful device to explain the theoretical element of market behavior in the form of static analysis but may not serve its useful purpose to reveal the real characteristics of micro cosmic individual behavior where dynamics and dispersed partial knowledge are the prevailing facts.

The derivation of individual supply schedule is essentially identical
with the derivation of demand schedule. The supply schedule which is obtained through conceptual experiment lists the various quantities of it that sellers or producers will place on the market at all possible alternative prices, other things being equal. It is understood that under the short-run model of perfect competition the segment of the marginal cost curve which is above the minimum average variable cost curve is identified as the firm's short-run supply curve. The analytical logic follows that the functional relationship between different prices and the corresponding quantities represent exactly the same relationship which establishes the supply curve. By introducing the individual supply curve through conceptual experiment, Marshall eliminated the time involvement which is essential for production process as Walras did in his fourth edition of *The Elements d'Economie politique pure*. In it, Walras states, "There is still another complication. Production, however, requires a certain lapse of time. We shall resolve the second difficulty purely and simply by ignoring the time element at this point. (L. Walras, p. 242)

In the presentation of aggregate supply and demand schedules which are the horizontal summation of individual demand and supply schedule in the market, Marshall simply ignored the time element by compositing the aggregate demand and supply schedules in a single diagram, which are in reality, the representation of separate dimensions. Through the mind of observing economist, the aggregate demand and supply curves are incorporated in a single dimension so as to reveal the equilibrium market price and equilibrium market quantity. Once, as a static equilibrium model is constructed, a remaining task is that of finding the set of values of the endogeneous variables which will satisfy the equilibrium condition of the model.
Before proceeding toward the solution of simultaneous equations which requires the counting of a number of independent equations and unknowns, including checking of stability condition of the system, it should be, however, warned that the aggregate supply and demand curves are only theoretically conceivable but never observable in reality. Hence, in the strict sense, these supply and demand schedules can never be obtained in reality where all the variables are continuously changing and individual as a consumer or a producer only possesses a partial knowledge. It should be clear now that the observing economist who not only willingly takes the role of the Walrasian auctioneer but also provided a theoretical solution which is by no means relevant with the real market behavior.

Now, a question arises that whether economists can and are able to solve the real problem with the given theoretical framework of Marshall. The answer may be quite painful to economists who had and have devoted so many decades along this line. But it is quite clear that we can not resolve the real problem. Hayek expressed the same opinion,

The problem is thus in no way solved if we can show that all the facts, if there were known to a single mind (as we hypothetically assume to be given to the observing economist), would uniquely determine the solution; . . . To assume all the knowledge to be given to a single mind in the same manner in which we assume it to be given to us as the explaining economists is to assume the problem away and to disregard everything that is important and significant in the real world. [Hayek, p. 530]

The source of such a grave error in the existing theoretical structure stems from the lack of clear understanding of methodological concept which is frequently used without careful scrutiny. Unfortunately, the evidence of such grave error committed in the process of analysis can be traced back to Keynes.
Axel Leijonhufvud's enlightening book, *On Keynesian Economics and the Economics of Keynes* in 1968 and his summary article of it, "Keynes and Keynesian" in 1967 were much praised by many leading economists without clear understanding of its content. In his article, Leijonhufvud states, "Keynes Theory was dynamic. His model was static. The method of trying to analyze dynamic processes with a comparative static apparatus Keynes borrowed from Marshall." (Leijonhufvud, p. 211)

Furthermore, Leijonhufvud points out, "The only thing which Keynes removed from the foundations of classical theory was the deus ex machina --- the auctioneer which is assumed to furnish, without charge, all the information needed to obtain the perfect coordination of the activities of all traders in the present and through the future." [p. 216] Leijonhufvud's two statements quoted above, which carry a powerful incision of the interpretation of Keynes' works may appear harmless but may contain a serious blow in it. The reason of forthcoming warning is especially clear that when auctioneer is removed from the system, Keynes' process-analysis which borrows the long run comparative static method and proceeds in terms of the type of period analysis may not survive at all under a careful scrutiny of these contents of statements. The long run comparative static method which proceeds in terms of type of period analysis and which attempts to show in the form of series of instant pictures, are only feasible and relevant if and only if thre prevail constant functional relationships between aggregates and these aggregates are readily given to the observing economist.

Hayek argues in his article, "Personal Recollections of Keynes and the Keynesian Revolution"

His [Keynes'] final conceptions rest entirely on the belief that there exist relative simple and constant functional relationships between such "measurable"
aggregates as total demand, investment, or output, and that empirically established values of these presumed constants would enable us to make valid predictions. [Hayek, The Oriental Economist, 1966, p. 79]

The main difficulty remains, however, because these functional relationships may change over the long run and in addition, values of aggregate may not be obtainable with the given assumptions.

It is my firm belief that much confused and wasted efforts of interpreting Keynes’ works by the followers of Keynes could have been avoided if Keynes had presented his dynamic theory on the basis of newly formulated truly dynamic model. There are flourishing evidences that many leading economists use dynamic tool as a merely limited extension of comparative statics even though the opposite is true. Many economists borrowed generously the concept of correspondence principle advanced by Samuelson in order to demonstrate that their theory not only involves comparative static apparatus but also contains dynamic properties. Borrowing from Lange and Patinkin, the implication of the consistency and of the existence of equilibrium in a static and a dynamic system can be illustrated with the following simple equations; the existence of stationary equilibrium can be expressed in mathematical form.

\[(1) \quad D = D(p) \quad \text{demand function}\]

\[(2) \quad S = S(P) \quad \text{supply function}\]

\[(3) \quad \frac{dp}{dt} = f(D-S) \quad \text{market adjusting functions}\]

\[(4) \quad \text{sign} \left( \frac{dp}{dt} \right) = \text{sign} (D-S)\]

In the above equations, D and S represent demand and supply while p and t represent the price of a particular good and time respectively. The equation \((4)\) expresses that if \(D-S > 0\), then \(\frac{dp}{dt} > 0\) which indicates price increases and if \(D-S < 0\), then \(\frac{dp}{dt} < 0\), which indicates price decline.
In a static system, equation (3) replaces equation (5) D=S and thus
(1) D = D(P), (2) S = S(P), (3) D = S, (4) \( \frac{dP}{dt} = 0 \). As long as D \( \neq \) S, the
system will not be in equilibrium. The equations, (1), (2), and (3)
represent a dynamic system in a simple form while equations (1), (2),
and (4) represent the static system. The existence of a solution to
the static system which is the consistency of equations (1), (2), and
(4) is a necessary condition for the existence of stationary solution
for the dynamic system. But this does not constitute a sufficient
condition. If the dynamic system is not convergent, a consistent system
will never be reached even though the static system (1), (2), and (4)
may have the consistency. (2)

The aspect of an analytical context, Walrasian as well as Marshallian
approach methods possesses exclusively the nature of static, but not dynamical
nature. Jevons supports this argument by stating that the equations of
exchange in the Walrasian system are of a statistical, not dynamical, character.
They define a position of equilibrium, but they afford no information as to
the path by which that point was reached. Professor Walras' labored lessons
indicate a way, not the way of descent to equilibrium. (3) Further clari-
ification can be made by examining the stability analysis of Walras, which was
based upon a vital assumption that the rate of price change varies directly
with the amount of excess demand. As a stability condition, the Walrasian
system requires that excess demand equals zero at equilibrium level and a

(2) This analysis is based on the presentation made by O. Lange (Price
Flexibility and Employment, Principia Press, 1945), p. 91, and
Patinkin (Price Flexibility and Full Employment, in Readings in Macro-

(3) F. Edgeworth, "Review of L. Walras," Elements d'economie politique pure
positive excess demand always lead to a rise in price and vice versa. When
the supply curve is positively sloped, an upward shift of demand curve gene-
rates positive excess demand which makes the price rise to a new equili-
 brium. Walras' main objective was to show that in a really competitive
market, equilibrium would be determinate because price would reach their
equilibrium values instantaneously as exchange took place. This is only
possible when every trader has perfect knowledge of alternative trading
opportunities and arbitrage would be carried out without time consumption
on the basis of perfect information provided by an auctioneer at zero
cost. For that matter, the process of tatonnement does not allow the
market to remain at a nonequilibrium set of prices. As far as the
stability condition of the Walrasian system is concerned, a stable system is
one in which the process of tatonnement will succeed in establishing
equilibrium prices while an unstable system is one in which it will not.

The mode of approach method along the Walrasian including Marshallian
tradition is well documented in the works of Patinkin although he argues
that his theory is dynamic.

For the sake of clarification, it is worthwhile to visit briefly
the Patinkins' model.\(^4\) His model consists of four markets, namely,
Labor, Commodities, Bonds and Money. For each market, there are three
equations: a demand equation, a supply equation, and an equilibrium
equation. For each market there are also three variables: the amount
demanded, the amount supplied, and the price of goods in question. The
price of money is, by definition, unit. Hence, there is a total of only

eleven variables to be determined. On the other hand, by Walras' law, there are only eleven independent equations.

By substituting these demand and supply equations into their respective equilibrium equations, the foregoing system can be reduced to the following one:

Condition for Equilibrium

(1) \( Q \left( \frac{M}{p}, K_0 \right) = R \left( \frac{W}{p} \right) \) ..................... Labor services

(2) \( F \left( Y_0, r, \frac{M_o}{p} \right) = Y_0 \) ......................... Commodities

(3) \( r_p \cdot H \left( Y_0, \frac{1}{r}, \frac{M_o^H}{p} \right) = r_p \cdot J \left( Y_0, \frac{1}{r}, \frac{M_o^F}{p} \right) \) ....... Bonds

(4) \( p \cdot L \left( Y_0, r, \frac{M_o}{p} \right) = M_o \) ......................... Money

Here, the level of output is assumed to be fixed at \( Y_0 \). By Walras' Law only three of these equations are independent. Correspondingly, there are only three unknown variables to be determined: the money wage rate, the price level, and the rate of interest. Thus, Patinkin states that, "We then take this equality between the number of equations and unknowns as justifying the reasonableness of the assumption that this system of equations does have a solution."

(5) Along the Walras fashion, Patinkin states that the equality between the number of equations and unknowns provides a justification for the reasonableness of the assumption that the system of equations does have a solution. It is sufficient to point out how vividly the mode of Walrasian theoretical essence is still alive in the contemporary economics. It is rather self-evident that as long as dynamic apparatus used along the above context, dynamics may not be able to reveal its true characteristics of it.

(5) ibid., p. 229.
As Kenneth Boulding points out, simple equilibrium system is a limiting case of a dynamic system. Boulding states in his article, "General System Theory," "Simple equilibrium systems really fall into the dynamic category, as every equilibrium system must be considered as a limiting case of a dynamic system, and its stability cannot be determined except from the properties of its parental dynamic system." [p. 6] Boulding continues, "In comparative statics we compare two equilibrium positions of the system under different values for the basic parameters. These equilibrium positions are usually expressed as the solution of a set of simultaneous equations. The method of comparative statics is to compare the solutions when the parameters of the equations are changed. Most simple mechanical problems are solved in this way. In true dynamics on the other hand, we exhibit the system as a set of difference or differential equations which are then solved in the form of an explicit function of each variable with time. Such a system may reach a position of stationary equilibrium or it may not--there are plenty of examples of explosive dynamic systems, a very simple one being the growth of a sum at compound interest." [K. Boulding, p. 7]

Furthermore, when the Walrasian auctioneer is absent in the system, the gathering processes of information itself involve dynamic nature where the existing facts are dispersed partial knowledge possessed by individual being and continuously changing variables. From this aspect, it is safe to assert that economists never properly conceived economic dynamics. Knight states, "...no science of economic dynamics exist ... In actual usage economic dynamics, or dynamic economics, has become merely a critical and negative term to refer to the limitations of static analysis or more exactly
to any particular objection to any other author's use of equilibrium concept." [F. Knight, On the History and Method of Economics, p. 179]

It is rather a painful fact to admit it but it is a starting point toward the new development of dynamics. The following model is the result of my effort to implement the truly dynamic nature.
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