THE INCIDENCE OF A TAX ON
PURE RENT: A NEW (?)
REASON FOR AN OLD ANSWER

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The Incidence of a Tax on Pure Rent: A New (?) Reason For An Old Answer *

In a recent and stimulating article Martin Feldstein reconsiders the Ricardian proposition that a tax on pure rent is fully capitalized in the price of land. As Feldstein points out, the classical proposition that a tax on pure rent is unshifted requires that the supplies of non-land factors of production are unaltered by the introduction of the tax on land rents. This static assumption of fixed factor supplies is inappropriate to the analysis of tax incidence in a dynamic economy in which the supply of capital reflects the economic choice between consumption and saving. In Feldstein's analysis the introduction of a tax on land rents leads to an increased supply of capital. Assuming a fixed supply of labor and land, the increased supply of capital raises the marginal productivity of land and lowers the marginal productivity of capital, thus shifting the tax from land onto capital.

Indeed, Feldstein shows that both the net rental as well as the price of land may actually rise in response to the tax on rents. Feldstein's framework is a two period overlapping generations life-cycle model; each period the savings of the young finance the purchase of claims to land and capital assets. The initial reduction in the value of land arising from the tax on rent permits more savings to be funneled into capital accumulation leading

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to a higher ratio of capital to land and the shifting of the tax.²

While Feldstein's analysis of his model is impeccable, the shifting of the tax on pure rent is not inevitable; it depends on the particular life cycle model posited. This comment demonstrates that within a dynamic life cycle model with bequests a compensated tax on pure rents is not shifted, rather the tax is fully capitalized in the price of land. In the case that the tax is not compensated, the tax will still be fully capitalized in the price of land if preferences are of a certain type. The model presented here differs from that of Feldstein's only in the introduction of a bequest motive. Individuals are assumed to derive utility not from the level of the bequest per se, but rather from the utility of their descendents. Interestingly, Barro uses this overlapping utility model to formally prove another proposition attributed to Ricardo: that the stock of government bonds is not net wealth to society.³ Hence, the validity of the second Ricardian proposition, the neutrality of public debt, implies the validity of the first, that a tax on pure rent is fully capitalized. Unfortunately, Ricardo never directly addressed Feldstein's point that the tax on land rents could affect the supplies of non-land factors of production. Still, it is pleasing to conjure up the image of Ricardo defending his views on the land rent tax by pointing to his bequest model of savings.

²/ Feldstein also presents a model in which shifting occurs as asset holders seek to restore the initial mix of land and capital in their portfolios. This comment does not address the portfolio balance shifting; rather it addresses the inevitability of the former type of shifting.

³/ Barro, R. J., "Are Government Bonds Net Wealth?", Journal of Political Economy, v. 82, November/December 1974. Gerald O'Driscol argues convincingly that although Ricardo was the first to point to the debt neutrality model, Ricardo questioned the empirical validity of the future tax discounting needed to make government bonds truly neutral.
We first discuss the case of a compensated tax on land rents in which the government returns the tax revenues in a lump sum fashion to either the young or old. We then consider tax shifting when the tax revenues are not returned to private individuals, rather they are consumed by the government. Feldstein considers this latter case of an uncompensated tax, however, his result for his model holds for a compensated tax as well.\(^4\) While we shall discuss both cases, we note that the traditional practice in incidence analysis is to consider compensated taxes.\(^5\)

A Life Cycle Model With Bequests

The utility of a representative young individual at time \(t\) is given by:

\[
(1) \quad u_t = U \left( C_{y,t}, C_{o,t}, u_{t+1} \right)
\]

where \(C_{y,t}\) and \(C_{o,t}\) are consumption when young and old of the members of generation \(t\). Individuals are assumed to derive utility from the utility of their children. Population growth is zero. The budget constraint of a representative generation \(t\) individual is given by:

\[
(2) \quad C_{y,t} + \frac{C_{o,t}}{1+r_{t+1}} + \frac{b_{t}}{1+r_{t+1}} = v_t + b_{t-1}
\]

\(^4\) Feldstein errs in his footnote 10 when he states, "If all revenues are paid to the older age group, the traditional results are maintained." We show in footnote 7 below that Feldstein's result (in his model) holds for compensated land taxes whether the tax is lump sum redistributed to the young or the old.

\(^5\) One is generally interested in the effects of switching from one tax to another holding tax revenues and disposable income constant. These substitution effects are captured by looking at compensated tax changes. The incidence of uncompensated tax changes is, on the other hand, quite arbitrary; the final incidence depends on how the government is assumed to spend the tax revenue.
Lifetime consumption and bequests are financed by present lifetime resources which equal labor earnings and inheritances \( b_{t-1} \). Labor (one unit) is inelastically supplied when young and earns a wage \( v_t \). Inheritances, \( b_{t-1} \), are received when young. In the second period individuals are retired; they consume \( C_{o,t} \) and bequeath \( b_t \). \( r \) stands for the interest rate. Throughout our analysis we assume interior solutions for \( C_{y,t} \), \( C_{o,t'} \), and \( b_t \). Letting \( \lambda_t \) be the Lagrangian shadow price of (2), i.e., the marginal utility of lifetime income, we have:

\[
\frac{\partial u_t}{\partial b_{t-1}} = \lambda_t \quad \text{and} \quad \frac{\partial u_{t+1}}{\partial b_t} = \lambda_{t+1}
\]

Optimal choice of \( b_t \) entails:

\[
\frac{\partial u_t}{\partial b_{t-1}} \frac{\partial u_{t+1}}{\partial b_t} = \frac{\lambda_t}{1 + r_{t+1}}
\]

In the steady state \( \lambda_{t+1} = \lambda_t \) and (3) and (4) imply (we drop time subscripts to indicate steady state values):

\[
r = \frac{1}{\frac{\partial u_t}{\partial b_{t-1}}} - 1
\]

Another steady state condition results from the optimal choice of \( C_y \) and \( C_o \):

\[
\frac{\partial u}{\partial C_y} = (1+r) \frac{\partial u}{\partial C_o}
\]

In this economy both land, \( T \), and labor, \( L \), are taken as fixed; the only variable factor is capital, \( K \). Taking the production function \( X \) as depending on \( T, L, \) and \( K \), the interest rate \( r \) equals the marginal product of capital.

\[
r = \frac{\partial X}{\partial K} (L, T, K)
\]
\[ \frac{\partial x}{\partial t} \] the marginal product of land equals gross land rents. The price of land \( P \) is determined such that the net return to holding land equals the net return from holding capital. We have:

(8) \[ P = \frac{1}{r} \frac{\partial x}{\partial t} \]

The final equation of the model is the asset clearing equation. In the steady state the young use their savings to purchase the economy's assets. Letting \( n \) be the number of young people (or old people since there is zero population growth):

(9) \[ n(w + b - C_y) = K + PT \]

We also rewrite equation (2) for the steady state dropping time subscripts:

(10) \[ C_y + \frac{C_0}{1+r} + \frac{b}{1+r} = w + b \]

From the production function \( w \) may be written as a function of \( r \). In addition, from (1) the steady state level of utility, \( u \), may be written as a function of \( C_y \) and \( C_o \). As a result the steady state derivatives of the utility function depend only on the steady state values of \( C_y \) and \( C_o \). Under standard regularity conditions, the six equations (5), (6), (7), (8), (9) and (10) determine the six steady state values of \( r', P', K', C'_y, C'_o \) and \( b' \).

**The Incidence of a Compensated Tax on Land Rents**

We now demonstrate that a compensated tax on land rents is fully capitalized in the price of land regardless of whether the compensation is

\[^6\text{The supplies of land and labor are fixed; hence the wage, } w, \text{ may be written solely as a function of the amount of capital, } K, \text{ by equating the wage to the marginal product of labor. Similarly, } r \text{ is a function solely of } K. \text{ These two functions may be used to eliminate } K, \text{ leaving } w \text{ as a function of } r. \text{ In the case that the supply of labor is not fixed but depends on } w \text{ and } r, \text{ we can write } L = L(w,r) \text{ and substitute this expression for } L \text{ into the marginal productivity equations defining } w \text{ and } r. \text{ We again obtain two equations in three unknowns, } w, r, \text{ and } K \text{ and can eliminate } K \text{ to write } w \text{ as a function of } r \text{ alone.}\]
given to the young or old generations. The compensated tax reduces the
value of land and the amount of bequests but does not change the steady
state levels of \( r', K', c'_y, c'_o \) or steady state utility. With a tax on land
rents at rate \( \theta \) equation (8) is modified as follows:

\[
(8') \quad p = \frac{(1-\theta)}{r} \frac{\partial x}{\partial t}
\]

First, take the case of lump sum compensation to the old of the tax
revenues \( \frac{\theta x}{\partial t} \). Let \( t \) be the land holdings per old person; the new steady
state budget constraint is then:

\[
(11) \quad c_y + \frac{c_o}{1+r} + \frac{b}{1+r} = w + b' + \frac{\theta x}{\partial t} t
\]

Suppose now that the price of land falls by exactly the amount of the
capitalized rents, i.e.,

\[
(12) \quad p^* = p' - \frac{\theta x}{\partial t} \frac{\theta x}{r}, \quad \text{where } p^* \text{ is the new steady state}
\]

price of land after the tax is imposed and \( p' \) is the steady state price of land before
the tax is imposed. Also, suppose that the level of bequests falls by
exactly the capital loss on the land:

\[
(13) \quad b^* = b' - \frac{\theta x}{\partial t} \frac{\theta x}{r},
\]

where \( b^* \) is the new steady state bequest and \( b' \) is the pre-tax steady state
bequest. Substituting from (13) for \( b^* \) into (11), the tax terms cancel and we
arrive at the original equation (10) which holds with no tax. Similarly,
substituting for \( b^* \) and \( p^* \) in equation (9) yields the no tax form of
equation (9). Hence the values of \( p^* \) and \( b^* \) given in (12) and (13) and the
old values of \( r', K', c'_y, c'_o \) satisfy equations (5), (6), (7), (8'), (9)
and (11). Since \( K \) is unchanged, the gross marginal product of land is fixed,
and net land rents fall by exactly the amount of the tax.
If tax revenues are lump sum redistributed to the young, (11) becomes:

\[(14) \quad C_y + \frac{C_o}{1+r} + \frac{b}{1+r} = w + b + \theta \frac{3X}{T} \]  

In this case it is easy to verify that the steady state bequest falls by the amount of the capital loss plus the tax on land rents:

\[(15) \quad b^* = b' - \frac{(1+r)}{r} \frac{3X}{T} \]  

Equation (9) is now:

\[(16) \quad n(w + \frac{3X}{T} + b - C_y) = K + PT \]

Again, steady state consumption and welfare and the steady state capital stock are not affected; the value of \(b^*\) in (15) and \(P^*\) in (12) plus the initial values of \(r', K', C'_y\), and \(C'_o\) satisfy the equations (5), (6), (7), (8'), (14) and (16).

Unlike Feldstein's findings, a compensated land rent tax is fully capitalized in the price of land in an overlapping utility model with operative bequests.\(^7\)

The Incidence of an Uncompensated Tax on Land Rents

For particular utility functions an uncompensated tax is also fully capitalized into the price of land. A sufficient condition for full capitalization is that the marginal utility of the utility of one's descendents be independent in the steady state of the level of \(C_y\) and \(C_o\). Take the

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\(^7\) Feldstein's results also hold for his model when tax revenues are lump sum redistributed to the young or the old. This is clear from equation (9). In Feldstein's model \(b = 0\); if we add the present value of lump sum transfers received when young or old to the right hand side of (10), the pre-tax values of \(C_y\) and \(C_o\) will not satisfy the new equation for the pre-tax values of \(w_y\) and \(r_o\). One can easily show that tax shifting is greater in Feldstein's model in the case that the tax is lump sum rebated to the young than in the case that the old receive the compensation. Shifting, however, occurs in both cases.

While we have proved this proposition for the steady state, one can demonstrate that full capitalization of a compensated land rent tax occurs as well for economies which have not yet converged to a steady state, i.e., the price of land along the path is reduced by the present value of future land rent taxes.
logarithmic function with rate of time preference $\rho$ as an example.

\begin{equation}
    u_t = \log C_{y,t} + \log C_{o,t} \frac{t+1}{1+\rho} + \frac{u_{t+1}}{1+\rho}
\end{equation}

Here $\frac{\partial u_t}{\partial u_{t+1}} = \frac{1}{1+\rho}$ and from (5), $r = \rho$ in the steady state. For this utility function, $r$ is a fixed value in the steady state independent of the value of the tax on land rents. Since land and labor are inelastically supplied and $r$ is fixed in the long run, the supply of capital is determined in the long run by (6) and, unlike Feldstein's result, does not increase as the result of the tax on rents. From (8') the price of land falls by exactly the capitalized land taxes because the gross marginal product of land as well as $r$ are fixed. Given the fixed price of second period consumption, 1/1+r, $C_y$ will move with $C_o$ if the two goods are normal; from (9) and (10), since $w$ and $r$ are fixed, the lower price of land $P$ is associated with a reduction in the steady state bequest $b$ and the steady state consumption of $C_y$ and $C_o$. Although the land tax is not "shifted" in the sense that it is fully capitalized into the price of land, the tax is shifted in the sense that future generations have a lower level of utility than would occur in the absence of the tax; the first generation of land owners respond to the capital loss on their holdings of land by cutting their bequests which leads to a reduction in steady state consumption. The transition to the new steady state is immediate for this separable utility function. When the tax is announced the initial generation of old people will maximize $\log C_{o,t} + u_{t+1}$ subject to $C_{o,t} + b_t = \tilde{K}(1+r) + P\tilde{T} + (1-\theta)\frac{\partial x}{\partial t}$. $\tilde{K}$ is capital holdings per old person. If $P$ falls immediately to its new steady state level, then the initial generation of old people will make the same choice between $C_{o,t}$ and $b_t$ that will be made in the new steady state.
If all agents believe the transition to the new steady state is immediate, \( P \) will fall at once by the amount of the capitalized land rent taxes, and consumption and bequest choices of all agents will maintain the new steady state.

If \( \frac{\partial u_t}{\partial u_{t+1}} \) is not constant, but decreases with \( u_{t+1} \) then the lower steady state level of utility arising from the uncompensated tax will involve a higher value of \( \frac{\partial u_t}{\partial u_{t+1}} \) and from (5) and (7) a lower level of \( r \) and a higher level of \( K \). In this case part of the tax on rents will be "shifted" onto capital.

To summarize, we have demonstrated that a compensated tax on pure land rents will not be shifted in the long run in a life cycle model with bequests. An uncompensated tax may or may not be shifted depending on the utility function. Michael Darby has presented evidence that savings for bequests is the dominant motive for saving in the U.S. economy.\(^{8}\) Darby finds that over 70% of savings represents savings for bequests rather than life cycle savings for retirement. Thus both economic theory and empirical evidence support David Ricardo's view of the incidence of a tax on rent.

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