A THEORY OF PACKAGE SALES: BUBBLE GUM AND BASEBALL CARDS

by

Robert A. Jones
University of California at Los Angeles

and

Ellen B. Warhit*
California State University at Fullerton

May, 1980
UCLA Economics Department Working Paper #172
Bubble gum and baseball cards are things with which most of us grew up. Children chew gum and flip and trade baseball cards, and, although most of us move on to bigger and better activities, we are inevitably replaced by a younger generation.

The practice of inserting cards in packages of bubble gum began in 1933 (FTC, 1964, p. 838). Topps Chewing Gum, Inc., monopolizes the baseball card industry through exclusive contracts signed with each baseball player. These contracts grant it the sole right to sell baseball cards alone or in conjunction with gum and candy. Topps is also the largest manufacturer of bubble gum, which it markets under the trade name Bazooka. A small number of firms account for the majority of bubble gum sales.

The selling of cards and gum together in a package is a form of tie-in sale -- that is, the purchase of one good is a condition for the purchase of the other. In this case, the two goods are sold in the same fixed proportion to all customers. Could such a marketing strategy be more profitable than selling the two goods separately?

Most writers focus on price discrimination -- the enhancement of profit by extracting consumer surplus -- as a motive for tie-in sales. Their arguments fall into three general categories. Under a "full-line forcing" strategy (Burstein, 1960a, 1960b), the tied sale is equivalent to imposing an excise tax on purchases of the tied good in return for the right to purchase the monopoly good, allowing the firm to approximate a two-part tariff. The argument does not depend on differences among consumers or complementarity between the goods, although the optimal prices are affected by such factors (Warhit, 1980).
The second type of argument utilizes the ability of tie-in arrangements to sort purchasers into groups with different tastes to effect price discrimination (Burstein, 1960b; Adams and Yellen, 1976; Telser, 1979). A firm offering an array of packages distinguishes consumers with different reservation price characteristics by the package they purchase. By pricing the package appropriately, the firm implicitly charges customers different prices for the same components. In this way the firm is able to capture additional consumer surplus.¹

The third argument for tie-in arrangements based on price discrimination exploits the possibility that the reservation price for a package may be more uniform across consumers than the reservation prices of its components (Stigler, 1963; Adams and Yellen, 1976). In such cases, selling the goods as a package allows the firm to set a price that more fully extracts consumer surplus than would be possible if the components were sold separately.

None of these price discrimination arguments appear to explain the tied sale of bubble gum and baseball cards. First, in each analysis, customers were seen as purchasing all of their requirements of both goods from the same firm: Either the firm had a monopoly in both goods or it was assumed able to costlessly police a full-line forcing contract. In the case of bubble gum, however, there are other producers, and the cost of verifying that baseball card buyers purchase only Bazooka bubble gum would be prohibitive. Some buyers of the gum/card package do purchase Double-Bubble. Second, when gum alone can be purchased from other producers, the offering of gum and cards in only one fixed proportion precludes use of the tie-in to segment customers into different groups for purposes of price discrimination.² Third, the application of Stigler's (1963) argument for the tie, based on
A more uniform reservation price for the package than its components, would require a negative correlation between individuals' demands for the two goods -- an implausible relationship for bubble gum and baseball cards.

In this paper we examine package selling in circumstances similar to those prevailing in the markets for bubble gum and baseball cards. We consider a firm that has a monopoly in one good, A, and that is one of several oligopolistic sellers of another good, B. The structure of the B market is assumed to generate both an equilibrium price above marginal production cost and a stable share of industry sales for the firm. The monopolist sells A and B together in a package in order to divert sales of B from other producers to itself. The package sale solves the problem of enforcing the tie-in without incurring monitoring costs. This argument does not depend on complementarity between the goods or on their reservation prices being related across individuals. The optimal ratio of B to A in the package hinges on a tradeoff between the quantity of B sales captured from competitors and the reduction in A sales to customers who place a low value on B. The remainder of the paper verifies this incentive for package selling and examines the nature of the optimal package for a simple but explicit structure of demands and costs.

I. MARKET STRUCTURE AND COSTS

Consider a firm that is the sole producer of good A (baseball cards) and is one of several producers of good B (bubble gum). Let there be N potential buyers of the goods. Assume the demand for the two goods is independent, both in the sense that the demand for one is independent of the price of the other and in the sense that there is no correlation across the population in their demands.
1. The Demand for the Oligopoly Good:

The market for B is an oligopoly. Let $\bar{p}$ be the price of B for all manufacturers and $\alpha$ the share of those sales going to the firm being considered. Assume $\bar{p}$ and $\alpha$ are unaffected by the package arrangement adopted. Oligopolistic competition is interpreted here to mean that $\bar{p}$ exceeds marginal production cost and that $\alpha$ is less than one. Each consumer is characterized by a perfectly inelastic demand for $b$ units of product B at a price of $\bar{p}$ or less. The proportion of the population for which $b \leq x$ is represented by a cumulative distribution function $F(x)$, and the corresponding density function by $f(x) \equiv \frac{dF(x)}{dx}$.

2. The Demand for the Monopoly Good:

The firm has a monopoly in the production of A. Let each consumer be in the market for only one unit of A and have some reservation price $r$ for its purchase. Let $G(y)$ be the cumulative distribution function representing the proportion of the population for which $r \leq y$. Thus, if A is sold by itself at a price $p$, the total quantity of A demanded is $(1 - G(p))N$ where $N$ is the size of the population.

To permit explicit solutions, we further suppose that $G$ is such that this total demand is linear in $p$. That is, the demand for A if sold by itself is

\[
Q_A = N(1 - G(p)) = a - p
\]

where the unit for A is chosen so that the coefficient on $p$ equals 1.

3. The Nature of the Package:

The firm sells A in a package consisting of one unit of A and $\lambda$ units
of B. It sets the price of the package, P, and \( \lambda \), the fixed ratio of B to A. \( \lambda \) can take on any value greater than or equal to 0, with a value of 0 representing a choice not to sell the products as a package.\(^5\)

Let the marginal costs of producing B and A be constant at \( k_B \) and \( k_A \) respectively. We assume no cost advantage to producing and marketing the two goods together, so the marginal cost of the package to the firm is constant at \( k_A + \lambda k_B \). Similarly, we assume that the value a consumer places on the package is simply the sum of the values he places on its components. Given that a consumer's demand for B is inelastic at \( b \) units and that he can purchase B by itself at price \( \bar{p} \), he values the package at \( r + \lambda \bar{p} \) if \( b \) exceeds \( \lambda \), and at \( r + b \bar{p} \) if \( b \) is less than \( \lambda \). All B beyond \( b \) units is valueless and is discarded at no cost to the consumer. Thus the value of the package to a consumer of type \((r,b)\) is \( r + \bar{p} \text{Min}\{b,\lambda\}\).

II. THE OPTIMAL PACKAGE

We first obtain the demand functions for the package and for B alone in order to determine the firm's profit as a function of the variables it chooses, \( P \) and \( \lambda \). Figure 1 depicts the classification of the population into three groups according to their values of \( r \) and \( b \). Individuals buy the package if the value they place on it exceeds \( P \) -- that is, if \( P \leq r + \bar{p} \text{Min}\{b,\lambda\} \). Equivalently, the package is bought only if both \( r \geq P - \bar{p}b \) and \( r \geq P - \bar{p}\lambda \).

Those that buy the package are further divided into two groups: Those for whom \( b > \lambda \) purchase an additional \((b - \lambda)\) units of B by itself; those for whom \( b < \lambda \) do not purchase any B by itself, being already satiated by the B enclosed in the package. Thus individuals in region I purchase both the package and extra B, individuals in region II purchase just the package, and individuals in regions III and IV purchase just B.
Characteristics \( r \) and \( b \) are assumed independently distributed across the population so that the proportion of individuals with both \( b \leq x \) and \( r \leq y \) is \( F(x)G(y) \). The quantity of packages demanded equals the number of individuals in region I plus the number in region II. Thus for a given \( P \) and \( \lambda \) the demand for the package is

\[
Q_{pkge}^d = N(1 - F(\lambda))(1 - G(\bar{P} - \bar{p}\lambda)) + \int_0^\lambda (1 - G(\bar{P} - \bar{pb})) f(b)db,
\]

where \( f(b) = dF(b)/db \) is the (proportionate) density function for \( b \).

Each individual in region I buys an additional \( (b - \lambda) \) units of \( B \) beyond that contained in the package. Individuals in regions III and IV purchase their entire requirements, \( b \), in the market for \( B \) alone. The industry demand for \( B \) sold separately thus equals the number of individuals in each of these regions weighted by the amount of \( B \) they purchase. Equivalently, the demand for \( B \) by itself equals the total desired consumption of \( B \) at price \( \bar{p} \), minus the amount of \( B \) distributed in packages, plus the amount of \( B \) discarded by package buyers already satiated with \( B \) (i.e., by individuals in region II):

\[
Q_B^d = \int_0^\infty b f(b)db - \lambda Q_{pkge}^d + \int_0^\lambda (\lambda - b)(1 - G(\bar{P} - \bar{pb})) f(b)db.
\]

Substituting our assumed linear form \( (a - y) \) for \( N(1 - G(y)) \) into equation (2) yields

\[
Q_{pkge}^d = (1 - F(\lambda))(a + \bar{p}\lambda - P) + \int_0^\lambda (a + \bar{pb} - P) f(b)db.
\]

Rearranging the terms in (4) and substituting the linear form into (3) simplifies the demand functions to

\[
Q_{pkge}^d = a - P + \lambda \bar{p}H(\lambda)
\]

\[
Q_B^d = \int_0^\infty b f(b)db - \lambda Q_{pkge}^d + \int_0^\lambda (\lambda - b)(a + \bar{pb} - P) f(b)db,
\]
where \( H(\lambda) \equiv 1 - F(\lambda) + \int_0^\lambda (b/\lambda)f(b)db \). \( H(\lambda) \) reflects the average proportion of B sold in packages that is valued by consumers at \( \bar{p} \) rather than at 0.
Including \( \lambda \) units of B with each unit of A thus has the same effect on package sales as reducing the price by \( \lambda \bar{p}H \). We are now ready to examine the firm's profit maximizing price \( P \) and product ratio \( \lambda \).

1. The Profit Maximizing Package Price:

The firm's profit is the sum of profit from package sales and profit from sales of B alone. Since the firm gets a fixed share \( \alpha \) of sales in market B, total profit is

\[
\Pi = (P - k_A - \lambda k_B)Q^d_{pkge} + \alpha(\bar{p} - k_B)Q^d_B.
\]

The first order conditions for a profit maximum are

\[
\frac{\partial \Pi}{\partial P} = a - 2P + \lambda \bar{p}H + \alpha \lambda(\bar{p} - k_B)H + k_A + \lambda k_B = 0
\]

\[
\frac{\partial \Pi}{\partial \lambda} = (P - k_A - \lambda k_B)(1-F)\bar{p} - (a - P + \lambda \bar{p}H)k_B - \alpha(\bar{p} - k_B)(a + 2\bar{p} \lambda - P)(1-F) = 0
\]

where functions \( F \) and \( H \) are evaluated at \( \lambda \).

Equations (8) and (9) implicitly determine the optimal values \( P^* \) and \( \lambda^* \) in terms of the model parameters and the distribution of \( b \) embodied in \( F \) and \( H \). Solving equation (8) for \( P^* \) as a function of \( \lambda \),

\[
P^* = \frac{a + k_A}{2} + \frac{\lambda(k_B + \alpha(\bar{p} - k_B)H + \bar{p}H)}{2}.
\]

The first term, \( (a + k_A)/2 \), is the monopoly price for A if it was not sold in a package. Furthermore, \( P^* \) is an increasing function of \( \lambda, k_B, \alpha, \bar{p} \) and \( H \). Since \( \alpha \leq 1, H \leq 1, \) and \( \bar{p} \) is assumed greater than \( k_B \), it follows that

\[
P^* \leq \frac{a + k_A}{2} + \lambda \bar{p}.
\]
That is, the package is sold for less than the market price of its components were they both sold separately.

All buyers face the same proportion of B to A in the package. Hence one cannot say whether it is the implicit price of A or of B which has been reduced. The pricing strategy may either be interpreted as giving buyers of A the "right" to buy \( \lambda \) units of B at a price below \( \bar{p} \) (a "right" which they are required to exercise), thereby increasing the demand for A and the profit extractable from the monopoly position; or be interpreted as a reduction in the price of A, a "sacrificing" of some monopoly profit from its sale, in order to promote higher implicit sales of B at the prevailing price \( \bar{p} \).

Figure 2 depicts the optimal \( P^* \) for given \( \lambda \) in terms of marginal costs and revenues. Profits lost from displacing separate B sales are treated as a cost of selling the package. The locations of the MC, MR and demand curves depend on \( \lambda \) both directly and through its effect on \( H(\lambda) \). Each \( \lambda \) units of B sold in packages reduces industry sales of B by \( \lambda H \) and this firm's sales by \( \alpha\lambda H \). The marginal cost of selling an additional package (if accomplished by reducing \( P \) rather than by raising \( \lambda \)) is thus the marginal cost of producing the package, \( k_A + \lambda k_B \), plus the profits foregone on displaced sales, \( \alpha(\bar{p} - k_B)H \). Package output is optimally set at \( Q^* \) where \( MC = MR \); \( P^* \) is the price at which that output can be sold given the composition of the package.

2. The Profit Maximizing Product Ratio:

Analysis of the optimal ratio of B to A in the package is complicated by the fact that \( \lambda \) is an argument of the functions \( F \) and \( H \). However the first order condition can be interpreted and the circumstances under which it is profitable to package A and B together examined.
Rewriting equation (9) and identifying its components,

\[
(12) \quad Q_{pkge}^d = \frac{\partial \Pi}{\partial \lambda} |_{\lambda=0} = \frac{(k_B \bar{p})}{1 - \alpha + \alpha (k_B / \bar{p})}
\]

The rational firm thus increases \( \lambda \) until the cost of adding an additional unit of B to the packages being sold just offsets the increase in profit from higher package sales, less the decrease in profit from displacement of B sales.

To establish whether \( \lambda^* > 0 \) -- that is, whether it is profitable to package A with some B rather than by itself -- we evaluate \( \partial \Pi / \partial \lambda \) at \( \lambda = 0 \). From (10), \( P^* = (a + k_A) / 2 \) at \( \lambda = 0 \). Substituting this into (9) gives

\[
(13) \quad \frac{\partial \Pi}{\partial \lambda} |_{\lambda=0} = \frac{1}{2} (a - k_A) \left[ (1 - F(0)) (\bar{p} - \alpha (\bar{p} - k_B)) - k_B \right]
\]

This derivative is positive only if

\[
(14) \quad 1 - F(0) > \frac{(k_B / \bar{p})}{1 - \alpha + \alpha (k_B / \bar{p})}
\]

where \( 1 - F(0) \) is the proportion of A purchasers who also consume B in strictly positive quantities. It is profitable to have \( \lambda > 0 \) only if this proportion is sufficiently large. Notice that (14) cannot be satisfied if either \( \alpha = 1 \) or \( k_B / \bar{p} = 1 \): B must be sold at a price above marginal cost and other firms must share in these sales for there to be an incentive to package A and B together.

Second, consider the case in which the producer of A has zero share of the B market. The displacing of B sales by enclosing it in the package is thus irrelevant to the firm; its only concern is the tradeoff between the additional package sales to those customers who value B and the lost sales to those who
preceeding case where $k_B = 0$ and $\alpha > 0$, the parameters $a$ and $K_A$ have no effect on $\lambda^*$, and the effect of a rise in $\bar{p}$ is reversed.

III. CONCLUSIONS

Our analysis suggests a motive for package selling that does not hinge on price discrimination. Price discrimination arguments generally assume that purchasers of the tying good obtain all their requirements of the tied good from the same firm, either by supposing the firm has a monopoly in both products or by assuming that "full-line forcing" contracts can be economically enforced. By supposing that consumers can and do buy good B elsewhere, the only purchase of B that can be "forced" on buyers of A is that enclosed in the package, and the B enclosed is valued by purchasers uniformly at no more than the price charged elsewhere. To the extent that existing arguments extend to situations where other firms share in the sales of B, our additional assumptions of non-complementarity between the goods and lack of statistical dependence in their demand across the population preclude an incentive to package based on extraction of greater consumer surplus.

Instead, the firm in our analysis uses the package sale to transfer profits from other B producers to itself. This interpretation is supported by noting that the incentive to package disappears ($\lambda^* = 0$) if there are no other firms to transfer profits from ($\alpha = 1$), or if there is no potential profit to transfer ($\bar{p} = k_B$). In the special cases examined, the value of B enclosed varied directly with the price charged for separate purchase and inversely with the firm's share of those sales. The firm would prefer an arrangement in which buyers of its monopoly product simply agreed to purchase all their requirements of the other good from it -- an arrangement that buyers would be indifferent to if B is homogenous and uniformly priced. But when verifying
do not (but who now pay a higher price for A). Such a firm is either an insignificant participant in the B market or considers the package as an alternative to incurring start-up costs of establishing a significant share in the B market. Setting $\alpha = 0$, substituting $P^*$ from (10) into (9), and factoring the resulting expression gives

$$\delta \Pi/\delta \lambda = \lambda [1 - F(1 - F)](a - k_A) + \lambda \{H - k_B\} = 0$$

The second order condition for a maximum with respect to $\lambda$ is satisfied only when the first factor equals 0. Hence $(1 - F(\lambda)) = 0$ and the optimal ratio of B to A is given by

$$\lambda^* = F^{-1}(1 - \frac{k_B}{\bar{p}})$$

$F^{-1}$ denotes the inverse of the distribution function $F$. Figure 3 depicts this optimal $\lambda$ for a hypothetical distribution of $b$ across the population. The firm increases the ratio of B to A until the proportion of the population that consumes B in a higher ratio just equals $k_B/\bar{p}$. This solution can also be visualized in terms of Figure 2. Increasing $\lambda$ raises the demand curve at a rate $\partial P_{\text{pkg}}^d / \partial \lambda = \bar{p} \partial (\lambda H) / \partial \lambda = \bar{p}(1 - F)$ while it raises the marginal cost curve at a rate $\partial MC / \partial \lambda = k_B$ when $\alpha = 0$. (18) provides the value of $\lambda$ for which the intercepts of these two curves are as far apart as possible.

Since $F$ and hence $F^{-1}$ are increasing functions, it follows immediately that

$$\frac{d\lambda^*}{dk_B} < 0 \quad \text{and} \quad \frac{d\lambda^*}{d\bar{p}} > 0$$

The more costly is B to produce, the less is enclosed in the package; the higher its price in the other market, the more is enclosed. In contrast with the
compliance with such a contract is too costly, as might plausibly be the case with items such as baseball cards and bubble gum, commodity bundling is an inexpensive way of partially enforcing such a tie.

The welfare implications of the package arrangement, as opposed to the separate sale of both goods, are ambiguous. From the assumed inelastic demands for B it follows that its consumption is unaffected by the manner in which A is sold. However the total production of B rises by the amount discarded by satiated package buyers, resulting in a social loss if B is costly to produce. The quantity of A produced and consumed is higher than the monopoly level were it sold alone, suggesting an efficiency gain from this source. But an additional problem arises. Some individuals buy the optimally priced package who would not buy A at the simple monopoly price; however, some may not buy the package, though they value A above the monopoly price, because they place a low value on B. The A that is produced is thus not necessarily consumed by those who value it most highly.\(^8\)

We did not address the issue of how \(\overline{p}\) and \(\alpha\) are determined, or why they should be unaltered by the package arrangement.\(^9\) Indeed, interpreting the package as a strategic device for increasing one firm's share of profitable B production invites the question of how other producers would respond to it and how their response would affect \(\overline{p}\) and \(\alpha\). Resolution of such issues awaits a theory of the strategic interaction of firms with asymmetric marketing alternatives.
REFERENCES


FOOTNOTES

*Assistant Professors at University of California, Los Angeles and California State University, Fullerton respectively. We are indebted to Stewart Long, who first suggested this research topic, for helpful comments and insights.

1In the familiar IBM punch card and tabulating machine tie-in (International Business Machines v. United States, 298 U.S. 131 (1935)), the packages consist of different mixtures of machines and cards. Because cards were priced substantially above cost, high intensity users paid a higher implicit price for the machine. Adams and Yellen (1976) provide examples with a finite number of package offerings (e.g., complete dinners versus a la carte selections). The variation in implicit component prices with the package chosen is then more apparent.

2Topps baseball cards are occasionally sold alone. However, their distribution in this form is sufficiently limited that we ignore it in our analysis.

3Burstein (1960b, p. 67, n. 21) points out the incentive for tie-ins in such circumstances, assuming that full-line forcing contracts can be costlessly enforced.

4A consumer who demands n units of A and b units of B may be viewed as n consumers, each demanding one unit of A and b/n units of B. That is, as long as the consumer has the same reservation price for each unit of A he might purchase, there is no further loss of generality in assuming that he demands only one unit.

5Ideally, the firm would offer a variety of packages with different ratios of B to A, possibly including one with no B at all, such as in Telser (1979). We are implicitly assuming that the additional profit from offering more than one type of package would be outweighed by unspecified additional costs of producing and distributing a variety of packages.

6$\lambda^*$ does not equal 0 at $a = k_A$. Instead, it takes on the value which makes the denominator in (15) equal to 0: i.e., $H(\lambda) = 4\lambda/(1+\alpha)^2$. For small values of $\alpha$, however, this could imply that $P^*/\lambda < \bar{p}$ -- i.e., that it is cheaper to acquire B by buying packages and discarding A than by purchasing B by itself. Obviously the market for B by itself would disappear and our specification of the demand functions is inappropriate for parameter values in that range.

7Substituting $P^*(\lambda)$ from (10) into (9) with $k_B = 0$ and differentiating the result with respect to $\lambda$ again: $d^2 \Pi(P^*(\lambda), \lambda)/d\lambda^2 = [(1+a)^2(1-F) - 4\alpha]\bar{p}(1-F)/2$. This derivative and hence $(1+a)^2(1-F) - 4\alpha$ must be negative for the second order condition for an interior maximum to be satisfied. This fact is used to sign some of the derivatives in (16). We also assume that $a - k_A > 0$ and make use of the fact that $d(\lambda \bar{H})/d\lambda = 1-F$.

8This additional allocative inefficiency is pointed out by Adams and Yellen (1976).
FOOTNOTES, continued.

9 A prevailing price for B above marginal cost could reflect forces other than oligopolistic competition, of course. The price of B sold alone could be maintained by price regulations or by "fair trade" laws and the same analysis applies.
\[ F(x) \equiv \Pr\{b \leq x\} \]

Proportion of population that consumes less than \( x \) units of good B

FIGURE 3