STEADY-STATE STABILITY OF MONETARY POLICY WITH
A FIXED FISCAL POLICY: AN EXPOSITION

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In moving from the macrostatics to the macrodynamics course, good
graduate students are often troubled by the implications of the Blinder and
Solow (1973, 1974), Tobin and Buiter (1976) and Steindl (1974) analyses of the
stationary state for the stability of the steady-state equilibria when it is
assumed that (base) money growth is increased while the government-spending-
to-income ratio and the tax rate are fixed so that government borrowing is
adjusted passively via open-market operations.¹ This note illustrates
conditions under which such "neutral fiscal policy" is consistent with
exogenous choice in monetary policy.

Consider the government's budget identity:

\[(1) \quad \Delta B + \Delta D = R \cdot D + G - T\]

where \(\Delta X = \frac{dX}{dt}\) for any \(X\), \(B\) is nominal base money, \(D\) is nominal government
debt, \(R\) is the after-tax nominal interest rate, \(G\) is nominal government
spending, and \(T\) is the yield from a proportionate tax on net national product
\((Y)\). Since we are dealing with after-tax interest rates,² we can work with
the simple Fisher equation

\[(2) \quad R = r + \tau P\]
where $r$ is the real after-tax yield on government bonds, $\Gamma X \equiv d \log X/dt$ for any $X > 0$, and $P$ is the price level. Assume that the income elasticity of money demand is unity so that

(3) \[ \Gamma B = \Gamma P + \Gamma y \equiv \Gamma Y \]

where $y$ is real NNP.

Now, divide identity (1) by $Y$ and rearrange terms to obtain:

(4) \[ \Gamma B \cdot \beta + (\Gamma D - R)\delta \equiv \gamma - \tau \]

where $\beta \equiv B/Y$, $\delta \equiv D/Y$, $\gamma \equiv G/Y$, and $\tau \equiv T/Y$. That is, with all variables measured as a fraction of NNP, the excess of spending over taxes on value added must be financed either by base-money creation or by increasing debt at a faster rate than the after-tax interest rate. In steady-state, $\beta$ and $\delta$ are constants so $\Gamma B \equiv \Gamma Y \equiv \Gamma D$. In view of equations (2) and (3) this gives us

(5) \[ \delta \equiv \frac{\gamma - \tau - \Gamma B \cdot \beta}{\Gamma y - r} \]

where $\delta$ is the steady-state value of $\delta$ which is consistent with given values of $\gamma$, $\tau$, $\Gamma B$, and hence $\beta = f(\Gamma B)$.\(^3\) Note that $\gamma - \tau - \Gamma B \cdot \beta$ measures the excess of government expenditures over value-added and (loosely) inflation taxation; this amount must be financed by net borrowing in excess of the amount needed to make interest payments.\(^4\)

It is important that $\Gamma y$ exceed $r$ in order for the government to find it attractive to maintain an outstanding stock of government debt. If $r$ exceeded $\Gamma y$, then any positive excess of $\gamma$ over $\tau + \Gamma B \cdot \beta$ would indeed cause $\delta$ to grow
without limit, a possibility which has much concerned Sargent and Wallace (1981). While most "unadulterated" monetarist and other economists would agree that the average real return to capital would equal or exceed $\Gamma y$, certainly there is considerable evidence that nonpecuniary services and a negative correlation with the market return lowers the before-tax return on government bonds to the point that the after-tax real yield $r$ — the one relevant to the government budget restraint — is indeed less than $\Gamma y$.

Let us examine the behavior of $\delta$ away from its steady-state value $\bar{\delta}$. Differentiating the definition $\log \delta \equiv \log D - \log Y$ with respect to time:

\begin{equation}
\Gamma \delta \equiv \frac{1}{\delta} \left( \frac{\Delta D}{Y} - \delta \cdot \Gamma y \right)
\end{equation}

\begin{equation}
\Gamma \delta \equiv \frac{1}{\delta} \left( R \delta + \gamma - \tau - \Gamma B \cdot \beta - \delta \cdot \Gamma y \right)
\end{equation}

Assuming full employment and perfect foresight to simplify the dynamics,

\begin{equation}
\Gamma \delta = \frac{1}{\delta} \left[ (r - \Gamma y)\delta + (\gamma - \tau - \Gamma B \cdot \beta) \right]
\end{equation}

\begin{equation}
\Gamma \delta = \frac{\Gamma y - r}{\delta} \left[ \frac{\gamma - \tau - \Gamma B \cdot \beta}{\Gamma y - r} - \delta \right]
\end{equation}

Which in view of equation (5) simplifies to

\begin{equation}
\Gamma \delta = \frac{\Gamma y - r}{\delta} (\bar{\delta} - \delta)
\end{equation}

The difference between the steady-state and actual debt-income ratio is thus eliminated at the rate $(\Gamma y - r)/\delta$ which is indeed positive in the case in which the government can profitably issue debt.\textsuperscript{5} Equation (10) implies that
passive changes in debt as implied by constant values of $\gamma$ and $\tau$ will cause $\delta$ to converge to the new $\bar{\delta}$ after a change in $\Gamma B$.

Consider the following simple example:

$$
\begin{align*}
\gamma &= 0.22 \\
\Gamma y &= 0.04/\text{year} \\
\Gamma B &= 0.10/\text{year} \\
\tau &= 0.18 \\
r &= 0.02/\text{year} \\
\beta &= 0.10 \text{ year}
\end{align*}
$$

Therefore, the steady-state debt-income ratio is

$$
\bar{\delta} = \frac{0.22 - 0.18 - (0.10/\text{year})(0.10 \text{ year})}{(0.04/\text{year}) - (0.02/\text{year})} = \frac{0.03}{0.02/\text{year}}
$$

$$
\bar{\delta} = 1.5 \text{ year}
$$

Suppose that the Fed decided to increase money growth to $\Gamma B' = 0.20/\text{year}$ and that this induced $\beta$ to fall to $\beta' = 0.09$ year. Then

$$
\bar{\delta}' = \frac{0.04 - (0.20/\text{year})(0.09 \text{ year})}{0.02/\text{year}} = 1.1 \text{ year}
$$

When this policy is initiated, the growth rate of the debt-income ratio would be

$$
\Gamma \delta' = \frac{0.02/\text{year}}{1.5 \text{ year}} (1.1 \text{ year} - 1.5 \text{ year})
$$

$$
\Gamma \delta' = -0.0053/\text{year}
$$

That is, over the first year of the new policy $\delta$ would fall by approximately (1 year) (-0.0053/year) (1.5 year) = -0.0080 year to 1.492 year. The rate of decline would decrease as $\delta$ asymptotically approached $\bar{\delta}' = 1.1$ year. Thus,
the government-budget identity does not pose any problems for the existence or
stability of the steady-state equilibrium as money growth is varied
exogenously with fiscal policy fixed. Similarly, government spending or tax
rates can be varied exogenously with the other fiscal variable and monetary
policy held unchanged. In this way the standard macroeconomic practice of
varying fiscal or monetary instruments with government borrowing adjusting
passively is shown to be consistent with a stable steady-state equilibrium.
REFERENCES


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1This paper does not attempt to comment on the relevance of the balanced-budget condition within the stationary state. See, however, Fischer (1976) and Auerbach and Rutner (1977) on this point.

2See Darby (1975).

3The assumption above equation (3) implies that velocity (or fluidity) has a 0 growth rate in any steady-state, but higher values of \( \Gamma_B \) imply higher values of \( \Gamma_P \) and \( R \) and hence a lower value of \( B \).

4An alternate term for net borrowing \((\Gamma y - r)D\) would be "negative capital service."

5At least for the U.S., the before-tax real yield on government debt approximates the growth rate of real income so that the after-tax real yield a fortiori is less than \( \Gamma y \).

6Note however that absent perfect foresight or a prior refunding into indexed bonds of long-term bonds — see Darby and Lothian (1983) — this adjustment will be much faster as the real value of the existing bonds and debt service drops.