THE SOCIAL WELFARE CONSEQUENCES
OF LIMITING NONEXCLUDABILITY:
AN ANALYTIC APPROACH

by

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Abstract

Previous authors who have considered partially nonexcludable goods have claimed that an increase in the height of government barriers limiting nonexcludability will have the following two effects. First, it will decrease the social welfare loss due to underproduction. Second, it will increase the social welfare loss due to underutilization. In this paper we investigate these claims in a formal setting by analyzing a model in which consumers only vary in terms of their costs of obtaining a reproduction. Our analysis provides partial support to the first claim of these previous authors, while giving little or no support to the second claim.
There are many commodities other than public goods which are characterized by aspects of nonexcludability. For example, any recording is partially nonexcludable if copies of the recording can be made by individuals other than the original producer. In this paper we look at markets of this type, paying special attention to the effects of government barriers which limit nonexcludability, e.g., copyright laws.

Consider for instance the production and marketing of a computer software package in the form of computer discs. Given that the technology for copying computer discs is available to the public, some agents will be able to get access to the software package without paying the producing agent. This potential for free riding implies that the quality of the package provided via the market will be below the socially optimal quality. This is what has come to be known in the literature as the social welfare loss due to underproduction. This loss is analogous to the loss which occurs in the classic public goods case (see e.g., Samuelson 1954). Unlike free riding in the case of a pure public good, however, free riding here entails the expenditure of resources to gain access to the software package (e.g., one has to borrow the package from some other agent, gain access to the copying technology and purchase blank discs). This, combined with the fact that the producer's price will typically exceed his marginal cost of production because of the market power he possesses, leads to a second social welfare loss which is known in the literature as the social welfare loss due to underutilization. This social welfare loss has two components. The first is the loss due to consumers who would be willing to pay the marginal cost of production, but who do not consume the good. The second component is the loss due to consumers who expend more real resources in copying than would be incurred by the producer if these consumers purchased the software package
from the producer.

Economists who have previously considered this type of market have claimed that an increase in the height of government barriers limiting nonexcludability will have the following two effects. First, it will decrease the social welfare loss due to underproduction. Second, it will increase the social welfare loss due to underutilization (see e.g., Hirshleifer and Riley 1979). In order to investigate the validity of these claims in a formal setting, we here analyze a model in which consumers only differ in terms of their costs of obtaining a reproduction. We first demonstrate that, given a restriction on the distribution of reproduction costs in the population, an increase in the height of government barriers limiting nonexcludability does indeed decrease the social welfare loss due to underproduction. However, we then show through the analysis of a specific example that, when this restriction is not satisfied, an increase in the height of these barriers can actually lead to an increase in the social welfare loss due to underproduction. Finally, as opposed to the partial support given to the claims of previous authors above, we derive a Proposition more consistent with the social welfare loss due to underutilization decreasing, rather than increasing, when barriers limiting nonexcludability are raised. The reason our results on this aspect of the issue so strongly contradict the claims of previous authors is that our consumers do not vary in terms of their valuations of the partially nonexcludable good. We conjecture, however, that even in a world where consumers vary in terms of their valuations of the partially nonexcludable good, the basic point we want to demonstrate would be valid. That is, it would still be the case that an increase in the height of government barriers limiting nonexcludability could lead to a decrease in the social welfare loss due to underutilization.
The outline for this paper is as follows. Section I sets forth a model of an economy containing a partially nonexcludable good, wherein consumers only differ in terms of their costs of obtaining a reproduction. Section II analyzes this model, paying special attention to the social welfare consequences of the government varying the height of barriers limiting nonexcludability. Section III presents some concluding remarks.

I. The Model

Individuals in this model derive utility from the consumption of two goods. The first good is fully excludable and is acquired via a perfectly competitive market. This good is referred to as B, and for ease of exposition we normalize the price of B to one. $b_i$ will denote the number of units of B consumed by individual i. It is assumed that good B is perfectly divisible, and that it exhibits constant marginal utility. The constant marginal utility assumption simplifies the analysis, while leaving the qualitative nature of the results unchanged. The second good is a partially nonexcludable good produced by a monopolist, and is referred to as good M. M is indivisible — in particular, each individual can consume either zero units or one unit of M. The utility derived from the consumption of M depends on the quality of the unit consumed, where the quality of the unit consumed by individual i is denoted $Q_i$.

There are two ways that a consumer can acquire a unit of M. The first method is for the consumer to purchase a unit directly from the monopolist. The second method is for the consumer to borrow a unit from some other agent who has in one way or another acquired a unit of M, and then make a reproduction. We refer to the monopolist as the primary source for obtaining M, and agents who lend out M as secondary sources. Agents who
lend out units of $M$ incur no costs in the lending process. If consumer $i$ acquires a unit of $M$ by borrowing a unit from a secondary source and copying it—a process we will refer to as going through the secondary market—then he incurs the following costs. He incurs a money expenditure of making a reproduction, which we will denote as $c$. He also incurs a disutility $z_i(1+H)$, where $H$ denotes the height of the government barriers limiting nonexcludability. One interpretation for this disutility is that it is due to the time the consumer expends in making a reproduction. Finally, each consumer $i$'s value for $z$ is unobservable to everyone but himself.

The following utility function, which incorporates the properties just discussed, is adopted (note: $L_i = 1$ if consumer $i$ obtains $M$ by going through the secondary market and equals zero otherwise).

\[(1) \quad U_i = b_i + \nu_i - L_i z_i(1+H),\]

where all consumers have the same valuation for quality, this valuation being denoted as $\nu$. The interpretation of (1) is straightforward. $b_i$ represents the utility derived by individual $i$ from the consumption of good $B$. $\nu_i$ represents the utility directly derived by individual $i$ from the consumption of $M$. $L_i z_i(1+H)$ represents the disutility individual $i$ incurs when he obtains $M$ through the secondary market. Note, if individual $i$ does not consume a unit of $M$, $Q_i = 0$.

Each consumer $i$ also faces the following budget constraint.

\[(2) \quad b_i + e_i < Y\]

In specifying (2) we have assumed that each consumer has the same income, this
income being denoted simply as $Y$. Additionally, $e_i$ denotes the money individual $i$ spends in acquiring good $M$. Now, it is easily demonstrated that (2) must hold as an equality, and therefore the budget constraint can be substituted directly into the utility function, i.e.,

$$U_i = Y - e_i + vQ_i - L_i z_i (1 + H).$$

A number of aspects of the model remain to be specified. First, we assume that the $z_i$'s are not constant across individuals. Rather, $z_i$'s are distributed between the extreme values 0 and $Z$, where this distribution is described by a density function $g(.)$ which is continuously differentiable and nonzero in the specified interval. Second, equation (4) describes the primary source monopolist's total costs, denoted $TC(x,Q)$, of producing $x$ units of $M$ of quality $Q$.

$$TC(x,Q) = F(Q) + cx,$$

where $F(Q)$ denotes the fixed costs of production, and where $cx$ denotes the variable costs of production. Finally, the function $F(Q)$ is assumed to satisfy the following restrictions: $F(0) = 0$, $F'(.) > 0$, $F'(\infty) = \infty$, and there exists a $Q$ such that $\int_{vQ}^{Z} (vQ - c)g(z_i)dz_i > F(Q)$. This last restriction guarantees that the monopolist always has an incentive to produce.

In ending this section, a word is in order concerning how we modeled the costs of going through the secondary market. Since we did not want consumers to ever have access to a copying technology superior to the copying technology available to the monopolist, one of the copying costs faced by each consumer is a money cost equal to $c$. The additional costs of going through the
secondary market could have been modeled either as a money cost, a disutility, or some combination of the two. The results of the model do not depend on which of these three specifications is employed. There were two reasons, however, for why we chose to model these costs as a disutility. First, since entering these costs as a disutility separates these costs from the money cost c, this specification more clearly identifies these costs as costs which are in excess of the monopolist's marginal cost of production. Second, entering these costs as a disutility more closely matches what we think will be the effects of a recent out of court settlement of a suit related to this issue. According to the terms of a recent settlement between New York University and the American Association of Publishers, NYU is now relatively more restricted in terms of the amount of photocopied materials it can make available to students. We do not feel this will substantially change the money costs students face in obtaining a reproduction. Rather, students who now obtain reproductions will be forced to use self-service machines, which indicates to us that the extra costs will enter as a disutility or inconvenience.

II. Analysis

Before proceeding with the analysis, a number of points need to be made. First, the monopolist will only offer a single quality of M for sale. This is a consequence of each consumer having the same utility function (except for the z_i's which are unobservable to the monopolist). That is, if the monopolist were to attempt to price discriminate by offering a variety of qualities, it would always be the case that all consumers preferring to purchase from the monopolist would desire the same quality. Second, because secondary sources incur no costs in the lending process, in equilibrium they
will lend the same quality good the monopolist is selling, but at no charge to the borrower. This last point tells us which consumers will prefer to borrow rather than purchase. A consumer will prefer to borrow rather than purchase when his costs of acquiring the good through the secondary market, both monetary and nonmonetary, are less than the price being charged by the monopolist, denoted \( P \). Given that secondary sources lend the good at no charge, this implies that consumer 1 will prefer to borrow rather than purchase when \( c + z_1(1+H) < P \), or \( z_1 < (P-c)/(1+H) \).

The outline for the remainder of this section is as follows. First, we derive a pair of equations which characterize the socially optimal quality of \( M \). Second, we demonstrate that at any value for \( H \), the quality of \( M \) produced by the monopolist is less than the socially optimal quality. Third, we demonstrate that, given a restriction on the density function \( g(\cdot) \), the social welfare loss due to underproduction decreases as barriers limiting nonexcludability are raised -- a result consistent with the claims of previous authors. Fourth, we show through the analysis of a specific example that, when \( g(\cdot) \) does not satisfy this restriction, an increase in these barriers can actually increase the social welfare loss due to underproduction. Finally, we derive a Proposition which states that, in a global sense, the social welfare loss due to underutilization is a decreasing function of the height of government barriers limiting nonexcludability -- a result directly contrary to the claims of these authors.

To characterize the socially optimal quality of \( M \) we assume for the moment that the technology for producing \( M \) is in the hands of a social welfare maximizing government, which produces the good and makes units of the good available to consumers at marginal cost (note: this government is assumed to finance any deficits through lump sum taxes). We also assume that
the government's social welfare function is simply the sum of the utilities of all the individuals in the economy. Thus, the socially optimal quality of \( M \), denoted \( Q^* \), is defined by equation (5).\(^4\),\(^5\)

\[
Q^* = \arg \max_Q \int_0^Z (Y+\nu Q-c)g(z_1)dz_1 - F(Q)
\]

The maximization problem in (5), in turn, yields the following first order condition.

\[
\int_0^Z vg(z_1)dz_1 - F'(Q) = 0
\]

We now have a pair of equations which characterize the socially optimal quality of \( M \). Next, we compare this quality with that actually produced by the monopolist. This is done in Proposition 1.

Proposition 1: \( Q^M(H) < Q^* \) for all \( H \), where \( Q^M(H) \) denotes the quality of \( M \) produced by the monopolist.

Proof: In choosing a \((P,Q)\) pair, the monopolist has to take into account the following two facts. First, no consumer would be willing to pay for \( M \) an amount greater than his valuation of \( M \), i.e., \( vQ \) must exceed or equal \( P \). Second, each consumer for whom \( z_1 < (P-C)/(1+H) \) will prefer to go through the secondary market rather than purchase from the monopolist. Together these two facts imply that the monopolist's maximization problem is the following.
\begin{equation}
\max_{P,Q} \int_{\mathbb{R}} (P-c)g(z_1)dz_1 - F(Q) \\
\text{s.t. } vQ > P
\end{equation}

It is easy to demonstrate that the constraint, \( vQ > P \), must hold as an equality. Therefore, \( Q^M(H) \) is defined by the following equation.

\begin{equation}
Q^M(H) = \arg \max_Q \int_{\mathbb{R}} (vQ-c)g(z_1)dz_1 - F(Q), \\
S(Q,H)
\end{equation}

where \( S(Q,H) = (vQ-c)/(1+H) \). The maximization problem in (8), in turn, yields the following first order condition.

\begin{equation}
\int_{\mathbb{R}} v g(z_1)dz_1 - v(1+H)^{-1}(vQ-c)g(S(Q,H)) - F'(Q) = 0 \\
S(Q,H)
\end{equation}

A comparison of (6) and (9) yields \( Q^M(H) \neq Q^* \) for all \( H \). Furthermore, combining this with equations (5) and (8) yields that both (10) and (11) hold for all \( H \).

\begin{equation}
\int_{\mathbb{R}} (Y+vQ^M(H)-c)g(z_1)dz_1 - F(Q^M(H)) < \int_{\mathbb{R}} (Y+vQ^* - c)g(z_1)dz_1 - F(Q^*) \\
0
\end{equation}

\begin{equation}
\int_{\mathbb{R}} (vQ^M(H)-c)g(z_1)dz_1 - F(Q^M(H)) > \int_{\mathbb{R}} (vQ^* - c)g(z_1)dz_1 - F(Q^*) \\
S(Q(H),H)
\end{equation}

(10) and (11), in turn, imply that (12) holds for all \( H \).

\begin{equation}
\int_{\mathbb{R}} S(Q^M(H),H)(vQ^M(H)-c)g(z_1)dz_1 < \int_{\mathbb{R}} S(Q^*,H)(vQ^* - c)g(z_1)dz_1 \\
0
\end{equation}

Finally, (12) yields \( Q^M(H) < Q^* \) for all \( H \). Q.E.D.
Proposition 1 demonstrates that there is always a social welfare loss due to underproduction. Our next step is to investigate how a change in $H$ affects this social welfare loss.

Proposition 2: If $g'(z_i) > 0$ for all $0 < z_i < Z$, then $Q^M(H_2) > Q^M(H_1)$ for any $H_1, H_2$ pair, where $H_2 > H_1$.

Proof: Consider an $H_1, H_2$ pair, where $H_2 > H_1$. Equation (8) yields both (13) and (14).

\[
\int_{Q^M(H_2),H_2}^{Z} (\nu Q^M(H_1) - c)g(z_i)dz_i - F(Q^M(H_1)) > \int_{Q^M(H_2),H_1}^{Z} (\nu Q^M(H_2) - c)g(z_i)dz_i - F(Q^M(H_2))
\]

\[
\int_{Q^M(H_2),H_2}^{Z} (\nu Q^M(H_1) - c)g(z_i)dz_i - F(Q^M(H_2)) > \int_{Q^M(H_1),H_1}^{Z} (\nu Q^M(H_2) - c)g(z_i)dz_i - F(Q^M(H_1))
\]

(13) and (14), in turn, yield equation (15).

\[
\frac{S(Q^M(H_1),H_1)}{S(Q^M(H_1),H_2)} (\nu Q^M(H_1) - c)g(z_i)dz_i < \frac{S(Q^M(H_2),H_1)}{S(Q^M(H_2),H_2)} (\nu Q^M(H_2) - c)g(z_i)dz_i
\]

Furthermore, given our restriction on the density function $g(\cdot)$, (15) implies $Q^M(H_2) > Q^M(H_1)$.

Now, given the above result, $Q^M(H_2) = Q^M(H_1)$ can only occur if at $H_1$ the partial derivative of the lefthand side of (9) with respect to $H$ is equal to zero. Taking this derivative, setting it equal to zero, and rearranging yields,

\[
2g(S(Q^M(H_1),H_1) = -S(Q^M(H_1),H_1)g'(S(Q^M(H_1),H_1)).
\]
(16) violates our restriction on \( g(.) \). \[ \text{Q.E.D.} \]

Proposition 2 demonstrates that, given a restriction on the density function \( g(.) \), our model displays a property consistent with the first claim of economists who have previously considered this type of market. That is, given this restriction, an increase in the height of government barriers limiting nonexcludability necessarily results in a decrease in the social welfare loss due to underproduction. When this restriction is not satisfied, however, it is possible for an increase in the height of these barriers to actually lead to an increase in the social welfare loss due to underproduction. This can be seen through the following example. Let \( Z = 1 \), \( v = 1 \), \( c = 0 \), \( P(Q) = Q^2 \), \( g(z_1) = 6 \) for all \( 0 < z_1 < \frac{1}{4} \) and \( g(z_1) = 3 \) for all \( \frac{1}{4} < z_1 < 1 \). For a local maximum in this example where \( S(Q,H) \) lies between 0 and \( \frac{1}{4} \), (9) reduces to (17a); while for a local maximum where \( S(Q,H) \) lies between \( \frac{1}{4} \) and 1, (9) reduces to (17b).

\[
(17a) \quad \int_{\frac{1}{4}}^{1} 6dz_1 + \int_{\frac{1}{4}}^{1} 3dz_1 - \frac{6Q}{1+H} - 2Q = 0
\]

\[
(17b) \quad \int_{\frac{1}{4}}^{1} 3dz_1 - \frac{3Q}{1+H} - 2Q = 0
\]

(17a) simplifies to \( Q = \frac{15(1+H)}{56+8H} \), while (17b) simplifies to \( Q = \frac{3(1+H)}{8+2H} \). Substituting these expressions for \( Q \) back into (7) yields that \( Q^M(1) = 3/5 \) and \( Q^M(3/2) \approx .55 \).

One might ask why, in contrast to the claims of previous authors, the social welfare loss due to underproduction is not always a decreasing function of the height of government barriers limiting nonexcludability. The logic behind the claims of previous authors is as follows. As barriers limiting
nonexcludability are raised more individuals will be forced to purchase from the monopolist. Thus, the additional revenue which will result from a marginal increase in quality will be higher, the higher are these barriers. Furthermore, this in turn implies that the higher are these barriers, the higher will be the quality of good produced by the monopolist and the lower will be the social welfare loss due to underproduction.

Now consider equation (9). This equation tells us that an increase in the height of government barriers limiting nonexcludability actually affects the monopolist's choice of quality through two channels. The first is the one recognized by previous authors. That is, at any given quality level, the number of individuals who purchase from the monopolist will be increasing with the height of government barriers limiting nonexcludability. As argued by previous authors, this channel tends to make $Q^M(.)$ a positive function of $H$. As $Q$ is increased, however, some consumers shift from the primary market to the secondary market. This gives rise to the second channel referred to above. The shifting of consumers from the primary to the secondary market exerts a negative influence on $Q$, the magnitude of this influence being positively related to the number of people who shift. When $g(.)$ is a nondecreasing function, the number of people who shift at any given value for $Q$ will be decreasing with $H$. This in turn implies that as $H$ increases the severity of this negative influence will decrease. This then explains Proposition 2. That is, when $g(.)$ is a nondecreasing function, both channels work in the same direction and cause $Q^M(.)$ to be a positive function of $H$. When $g(.)$ is a decreasing function, however, this second channel may work in the direction of making $Q^M(.)$ a negative function of $H$. Thus, when this is the case, the intuition of previous authors may be incorrect because a factor they did not recognize can push $Q^M(.)$ in the
direction opposite from what they expected.  

We will now investigate how a change in $H$ affects the social welfare loss due to underutilization.

Proposition 3: For every $H_1$, there exists an $H_2$, $H_2 > H_1$, such that $W(H_3) < W(H_1)$ for every $H_3 > H_2$. Note, $W(H)$ denotes the social welfare loss due to underutilization.

Proof: Note first that in our model all consumers obtain $M$ in equilibrium. Thus, the social welfare loss due to underutilization is simply the costs incurred in the secondary market, which are in excess of what would be incurred if the good was purchased from the monopolist. Equation (18) captures this formally.  

\[
W(H) = \int_0^{S(Q^*(H),H)} z_1 (1+H)g(z_1)dz_1
\]

(18)

Now consider some random value for $H$, denoted $H_1$. We will denote as $H_2$ the value for $H$ at which (19) holds.

\[
\int_0^{S(Q^*,H_2)} (vQ^* - c)g(z_1)dz_1 = W(H_1)
\]

(19)

Proposition 1 together with the definition of $W(H)$ yields (20).

\[
W(H_3) < \int_0^{S(Q^*,H_2)} (vQ^* - c)g(z_1)dz_1 \text{ for all } H_3 > H_2.
\]

(20)

(19) and (20) together, in turn, yield $W(H_3) < W(H_1)$ for all $H_3 > H_2$.

Q.E.D.
Proposition 3 demonstrates that the prevailing orthodoxy concerning how the social welfare loss due to underutilization behaves as barriers limiting nonexcludability are raised, is not supported by our model. That is, there seems to be no tendency for an increase in the height of government barriers limiting nonexcludability to increase the social welfare loss due to underutilization.

As with our previous result, one might ask why we reach a different conclusion than do previous authors. One difference between our analysis and the analysis employed by previous authors is that these previous authors did not take into account the costs involved in obtaining a partially nonexcludable good through a secondary market. The reason such costs are important is the following. Since the monopolist is pricing above marginal cost, consumers at the margin are expending more resources in acquiring the good through the secondary market than would be incurred if the good was purchased from the monopolist. Thus, when individuals shift from the secondary market to the primary market as a result of the government raising barriers limiting nonexcludability, the social welfare loss due to underutilization tends to decrease.

The reason our results on this aspect of the issue so strongly contradict the claims of previous authors is that our consumers did not vary as regards their valuations on the quality of the partially nonexcludable good. In a model where this is not the case, as barriers limiting nonexcludability are raised some individuals may stop consuming the partially nonexcludable good entirely. We conjecture, however, that even in a world where consumers vary in terms of their valuations on the quality of the partially nonexcludable good, the basic point we wanted to demonstrate would be valid. That is, it would still be the case that an increase in the height of government barriers
limiting nonexcludability could lead to a decrease in the social welfare loss due to underutilization.

Before ending this section, one final point needs to be addressed. It is widely recognized that there exists a social welfare loss when economic agents expend resources surmounting or evading barriers. For example, the social welfare loss attributable to the resources expended in tariff evasion has been analyzed by, among others, Bhagwati and Hansen (1973), Johnson (1972), Bhagwati and Srinivasan (1973), Kemp (1976), Falvey (1978), and Ray (1978); while Tullock (1967) argues that a major portion of the social welfare loss associated with theft derives from the resources agents invest in the activity of theft. All of these previous papers share one common thread. The activity analyzed in each paper is an example of a directly unproductive profit-seeking activity. That is, the activities "yield pecuniary returns but do not produce goods or services that enter a utility function directly or indirectly via increased production or availability to the economy of goods that enter a utility function." (Bhagwati 1982, p. 989) In this paper we have shown that this type of social welfare loss can even arise in a case where the activity under consideration is associated with positive production. That is, making a reproduction of a partially nonexcludable good does entail positive production. However, because the original producer's price exceeds his marginal cost, some of the resources agents expend in this activity will constitute a social welfare loss.

III. Conclusion

Consider commodities characterized by what we have termed partial nonexcludability. A market containing such a good can be described as having two types of social welfare losses. First, there is a social welfare loss
which stems from the fact that some individuals who have access to the good will not have paid the producing agent, i.e., there is a social welfare loss due to underproduction. Second, there is a social welfare loss which stems from the nonequivalence of the original producer's price and marginal cost, i.e., there is a social welfare loss due to underutilization. Economists who have previously considered this type of market have claimed that an increase in the height of government barriers limiting nonexcludability will lead first, to a decrease in the social welfare loss due to underproduction, and second, to an increase in the social welfare loss due to underutilization. In order to investigate the validity of these claims in a formal setting, we here analyzed a model in which consumers only differed in terms of their costs of making a reproduction. We first demonstrated that, given a restriction on the distribution of reproduction costs in the population, an increase in the height of government barriers limiting nonexcludability will indeed decrease the social welfare loss due to underproduction. However, we then showed through the analysis of a specific example that, when this restriction is not satisfied, an increase in the height of these barriers can actually lead to an increase in the social welfare loss due to underproduction. Finally, as opposed to the partial support given to the first claim of economists who have previously considered this type of market, our analysis did not give any support to the second claim. That is, we derived a Proposition more consistent with the social welfare loss due to underutilization decreasing, rather than increasing, when barriers limiting nonexcludability are raised.

There are a number of different ways that the analysis contained in this paper could be extended. Two particular examples come to mind. First, as was discussed at the end of Section II, consumers could be allowed to vary in terms of their valuations on the quality of the partially nonexcludable
good. With this type of variation the monopolist would need to be concerned with self-selection constraints, an issue central to a number of recent papers (e.g., Mussa and Rosen 1978, Spence 1978, Chiang and Spatt 1982, and Maskin and Riley 1982). Second, in the real world it is not only the case that consumers expend resources when they obtain a partially nonexcludable good from someone other than the original producer, but also the case that producers expend resources in an attempt to make this type of acquisition more difficult. Thus, a second worthwhile extension might be to incorporate these types of expenditures into our model.
Footnotes

1. We can provide a precise definition for our term partial nonexcludability by contrasting it with the concept of pure nonexcludability. Pure nonexcludability occurs when one individual cannot get access to a commodity without every other individual in the economy getting free access to the commodity. Partial nonexcludability occurs when the situation deviates from the preceding in one or more of the following ways.

   (1) When one individual gets access to a commodity only a segment of the rest of the population gets access to that commodity.

   (2) When one individual gets access to a commodity other individuals are only afforded access to the commodity at some cost.

   (3) When one individual gets access to a commodity other individuals are only afforded access to a related commodity.

2. An alternative way to incorporate the height of government barriers would be to have the disutility from making a reproduction equal $z_1 + H$. We have analyzed the model under this alternative specification and found that the results do not change significantly.

3. A seemingly more general specification would be to have the utility derived from the consumption of $M$ be a general function $v(Q)$. In actuality, however, this yields little additional generality since it would always be possible to rescale the quality variable to yield the linear form used in this paper.

4. This is a correct method of finding the socially optimal quality of $M$ because in our equilibrium all individuals, either through the primary market or the secondary market, acquire a unit of $M$. 
5It is implicitly being assumed that \( Y \) is sufficiently large so that the constraint, \( b_1 > 0 \), is not binding. Note also, we can treat the fixed costs of production as a separate term because our model displays no wealth effects.

6Proposition 1 holds for all finite values of \( H \). When \( H = \infty \), (6) and (9) are equivalent. This simply means that, when the good is fully excludable, there is no social welfare loss due to underproduction.

7One density function which satisfies this restriction is the uniform density function.

8One might wonder whether our results concerning \( Q^M(.) \) in our example derive from the discontinuous nature of \( g(.) \), as opposed to the decreasing nature of \( g(.) \). Given that the following can easily be demonstrated, our results concerning \( Q^M(.) \) in this example must derive from the decreasing nature of \( g(.) \). Corollary to Proposition 2: Assuming nothing concerning the continuity of \( g(.) \), if \( g(z_2) > g(z_1) \) for any \( z_1, z_2 \) pair, where \( 0 < z_1 < z_2 < Z \), then \( Q^M(H_2) > Q^M(H_1) \) for any \( H_1, H_2 \) pair, where \( H_2 > H_1 \).

9This is not the correct method of finding the social welfare loss due to underutilization if the government limits nonexcludability by utilizing taxes. An example of this is the proposed legislation which would place a tax on videocassette recorders. We have, however, analyzed a model wherein the government limits nonexcludability by utilizing taxes, and derived results very similar to those contained in Proposition 3.
References


