NONCOOPERATIVE ENTRY DETERRENCE, UNCERTAINTY,
AND THE FREE RIDER PROBLEM

by

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Abstract

Previous authors who have considered the issue of noncooperative entry deterrence have concluded that the free rider problem is not a significant issue. In reaching this conclusion, however, these authors have only considered models in which the exact investment needed to deter entry is known with certainty. In this paper I add uncertainty to their models, and demonstrate that the free rider problem can be significant, but is not so in all cases. That is, for certain types of entry deterring investments the introduction of uncertainty causes the oligopoly to underinvest in entry deterrence, however, for other types no underinvestment result arises.
I. Introduction

Much recent theoretical work has been devoted to the role played by entry deterrence in the behavior of pre-established sellers. In looking at markets which are initially inhabited by an oligopoly, these studies have generally taken one of two approaches. First, some studies assume that the oligopoly behaves like a shared monopoly (e.g., Spence 1977). Second, other studies assume that the firms which comprise the oligopoly act in a purely noncooperative manner (e.g., Nti and Shubik 1981, Bernheim 1984, and Gilbert and Vives 1985). This paper falls into the latter category, and in particular is concerned with the role played by the free rider problem when oligopolists are unable to collude on an investment in entry deterrence.

Consider an oligopoly which cannot collude on an investment in entry deterrence. In such an environment entry deterrence has the properties of a public good. A public good in this context means that, because of the effect on the probability of entry, increasing the investment in entry deterrence yields a return to the oligopoly as a whole, some of which is not reflected as a return to the actual investor. Given such a situation, the free rider problem suggests there should be an underinvestment in entry deterrence. That is, the investment in entry deterrence should be less than that which would maximize the expected joint profits of the oligopoly.

Two previous papers which have addressed the issue of noncooperative entry deterrence and the free rider problem are the papers of Bernheim, and Gilbert and Vives. In each of these papers the formal analysis uncovered no evidence of the free rider problem, and in each paper it was subsequently suggested that the free rider problem may not be a significant factor whenever entry deterrence is an issue. Given the straightforward nature of the above argument, it seems difficult to believe that the free rider problem is never
an important factor in noncooperative entry deterrence. Hence, in the present paper I reconsider the models of Bernheim, and Gilbert and Vives in an attempt to get a clearer understanding of when the free rider problem is important.

One property shared both by the Bernheim model and the Gilbert and Vives model is a lack of uncertainty. That is, in each model there is a critical investment in entry deterrence such that, if the actual investment is less than this critical level, then the probability of entry is one. If, on the other hand, the actual investment is greater than the critical level, then the probability of entry is zero. This is significant in that a simple rationale suggests the free rider problem is more likely to be important when uncertainty is present. The argument follows. As indicated above, without uncertainty there is a critical investment in entry deterrence such that an \( \epsilon \) increase in the investment causes the probability of entry to change from one to zero. At this margin the oligopoly faces an infinite return to investing in entry deterrence, while everywhere else the oligopoly faces a zero return. Consider what happens if the oligopoly cannot collude on the investment. In that case the relevant return is the return to the oligopoly divided by the number of pre-established sellers. Dividing by the number of sellers does not change the return, however, since the initial return was either infinite or zero. Hence, one should not be surprised if the free rider problem is shown to be unimportant in a model which lacks uncertainty. Note, further, the addition of uncertainty eliminates the critical investment level, and the straightforward intuition is then that the free rider problem should be important.

In this paper I add uncertainty to both the Bernheim model and the Gilbert and Vives model, and investigate whether the conclusions found in the original papers still hold. The answer is that once uncertainty is introduced the free rider problem can be an important factor, but it will not be so in
all cases. In particular, adding uncertainty to Bernheim's model does cause the free rider problem to become important, with the result being that there is a strong tendency for the oligopoly to underinvest in entry deterrence. However, adding uncertainty to the Gilbert and Vives model only reinforces the conclusion contained in their original analysis that the free rider problem is not an important factor.

We can understand these differing results by considering the properties of the entry deterring investment in each model. In Bernheim the entry deterring investment serves no role other than the entry deterring one, and in such an environment the free rider problem will be important as long as uncertainty is present. On the other hand, in the Gilbert and Vives setting the entry deterring investment does have another function since whether or not entry occurs is determined by the combined output of the pre-established sellers. This means that in their setting a second important factor is present. We know that in the absence of a potential entrant there is a tendency for a non-colluding oligopoly to "over-produce." Hence, in the Gilbert and Vives setting there are competing forces. The free rider problem suggests that there will be underproduction, while this second factor suggests overproduction. The analysis demonstrates that even with uncertainty the second factor will always be dominant, and hence, in their setting the free rider problem is not a significant factor.

The outline for the paper is as follows. Section II constructs a variant of the model analyzed by Bernheim. Section III demonstrates that in this setting the free rider problem becomes important once uncertainty is introduced. Section IV constructs a variant of the model analyzed by Gilbert and Vives. Section V demonstrates that in the Gilbert and Vives setting the free rider problem is not important even after uncertainty is introduced. Section
VI contains some concluding remarks, including a discussion of what the results tell us concerning other models where entry deterrence is an issue.

II. Model 1: A Variant of Bernheim's Model

There are two major differences between the model presented in this section and the one analyzed by Bernheim. First, as indicated earlier, here there will be uncertainty concerning the exact investment in entry deterrence needed to deter entry. Second, in this model there will be only a single potential entrant, rather than many potential entrants and the resultant sequential entry problem.

The structure of the model is as follows. There are $N$ risk neutral pre-established sellers who face a single potential entrant. Each pre-established seller can invest in entry deterrence prior to the entry decision, but the pre-established sellers are unable to collude on this investment. Denote as $x_i$ the investment in entry deterrence of pre-established seller $i$, and let $X = \sum_{i=1}^{N} x_i$. If the potential entrant decides not to enter, then the profits of pre-established seller $i$ equal $\pi(N) - x_i$. If, however, the potential entrant does enter, then the profits of pre-established seller $i$ equal $\pi(N+1) - x_i$, where $\pi(N+1) < \pi(N)$. There are two reasons for assuming $\pi(N+1) < \pi(N)$. First, when entry takes place the total profits of the industry must be divided among more firms. Second, entry may cause the degree of cooperation on price and output decisions to decrease.

The next aspect of the model to be described is the entry decision. The potential entrant is assumed to enter if and only if he anticipates positive profits. Further, if entry occurs, then the potential entrant earns profits equal to $\pi(N+1) - \theta D(X)$, where $D(0) = 0$ and $D' > 0$. $\theta D(X)$ is simply the extra costs imposed on the entrant which are due to the entry deterring activity of the pre-established sellers. The uncertainty in the model is
captured by the fact that 0 is a draw from a random variable which has a cumulative distribution function G( ). It is additionally assumed that the realization of 0 is only observed by the potential entrant. Thus, the pre-established sellers base their investments in entry deterrence solely on the distribution function G( ), while the potential entrant bases his entry decision on the actual realization of 0. G( ) is assumed to satisfy the following restrictions: G(0) = 0, G(\tilde{\theta}) = 1, G'(z) > 0 and G''(z) exists for z \in (0, \tilde{\theta}). That is, 0 falls somewhere between the extreme values 0 and \tilde{\theta}, and the density function for 0 is both strictly positive and differentiable in this interval.

Finally, before proceeding to the analysis, it is convenient to define some additional notation. Let \hat{\theta}(X,N) be such that \pi(N+1) - \hat{\theta}(X,N)D(X) = 0 for all X,N pairs, X > 0. Given this definition, the potential entrant will enter if and only if 0 < \hat{\theta}(X,N), or equivalently, the probability of entry equals G(\hat{\theta}(X,N)).

III. Analysis of Model 1

Bernheim's claim concerning his model was that the free rider problem is not a significant factor. His logic was that, even without the ability of collude, there always exists an equilibrium where the pre-established sellers behave in a joint profit maximizing manner. Since this model is a stochastic version of Bernheim's, the first step of the analysis is to demonstrate that in this model Bernheim's logic is correct when the stochastic element is removed. By doing so, I clearly demonstrate that later results concerning the importance of the free rider problem must be due to the presence of uncertainty.
Proposition 1: If $\theta$ can only take on a single value, then there necessarily exists an equilibrium where the total investment in entry deterrence is exactly as if the pre-established sellers could collude.

Proof: There are two cases. The collusive result could be that the oligopoly invests zero, or it could be that the oligopoly invests just enough to deter entry. Consider the latter case and call this critical investment level $\hat{X}$. I can prove the proposition for this case by demonstrating that if $N-1$ of the pre-established sellers set $x_1 = \frac{\hat{X}}{N}$, then the remaining seller will also have an incentive to invest $\frac{\hat{X}}{N}$. It is obvious that this seller cannot have an incentive to invest more than $\frac{\hat{X}}{N}$, because $\frac{\hat{X}}{N}$ deters entry and the entry deterring investment serves no other role. On the other hand, if he invests less than $\frac{\hat{X}}{N}$, then entry necessarily occurs. Thus, the only other possibility is that he invests zero. However, because we are dealing with the case where deterring entry is jointly profit maximizing, this investment level is also dominated by the investment level $\frac{\hat{X}}{N}$.

The proof for the other case follows similarly. Q.E.D.

The next step of the analysis is to return to the initial specification, i.e., where there is uncertainty concerning $\theta$, and establish a benchmark with which later results can be compared. Let $X^*(N)$ denote the total investment in entry deterrence which the oligopoly would make if it could collude on this investment, given that $N$ is the number of pre-established sellers. $X^*(N)$ is defined by equation (1).

$$X^*(N) = \arg \max_X N[1-G(\theta(X,N))][\pi(N)-\pi(N+1)] - X$$

Further, if $X^*(N)$ is such that $0 < G(\theta(X^*(N),N)) < 1$, then (1) yields the following first order condition.
We can now proceed to the main part of the analysis. That is, when the oligopoly cannot collude in the face of uncertainty, how does the total investment in entry deterrence compare with that which maximizes the expected joint profits of the oligopoly.\(^4,5\)

**Proposition 2:** If \(0 < G(\hat{\theta}(X^*(N), N)) < 1\), then any equilibrium must be characterized by \(X < X^*(N)\).

**Proof:** Let \(X_{-i} = \sum_{j \neq i} x_j\). The maximization problem faced by pre-established seller \(i\) is given by (3).

\[
(3) \quad \max_{x_1} \left[1-G(\hat{\theta}(X_{-i}+x_1, N)) \right] [\pi(N)-\pi(N+1)] - x_1
\]

If \(X = X^*(N)\), then (3) yields the following first order condition.

\[
(4) \quad -dG(\hat{\theta}(X^*(N), N)) \frac{d\hat{\theta}(X^*(N), N)}{dx} [\pi(N)-\pi(N+1)] - 1 = 0
\]

Comparing (2) and (4) immediately yields \(X \neq X^*(N)\).

I now need only demonstrate that \(X > X^*(N)\) also yields a contradiction. Suppose \(X = X'\), where \(X' > X^*(N)\). This means there is a pre-established seller who is investing at least \(\frac{X'}{N}\). Let this be seller \(i\). There are two cases to consider. Case 1 is \(X_{-i} < X^*(N)\). We know it is not profitable under collusion for the oligopoly to increase the investment from \(X^*(N)\) to \(X'\). Further, the return to firm \(i\) from this increase is strictly less than the return faced by the oligopoly under collusion, but the cost is the same. Thus, it must also not be profitable for the firm to increase the investment from \(X^*(N)\) to \(X'\), which implies a contradiction. The other case is \(X_{-i} \geq X^*(N)\). Given \(x_1 > \frac{X'}{N}\), the expected profits for firm \(i\) are less than what
it could get if the pre-established sellers were to collude, i.e., given that all the firms are treated symmetrically in the collusive agreement. However, because $X_{-1} > X^*(N)$, firm 1 could invest zero and do better than it would if the pre-established sellers were to collude, i.e., a contradiction.

Q.E.D.

Proposition 2 demonstrates the first main point of the paper. If in an uncertain environment an oligopoly cannot collude on an investment in entry deterrence, then it is quite possible the oligopoly will underinvest. The intuition for the result is straightforward. Investing in entry deterrence yields a return to the oligopoly as a whole, some of which is not reflected as a return to the actual investor. Thus, the free rider problem suggests there should be underinvestment.

One question concerning Proposition 2 is why are there restrictions? The answer for the restriction $G(\hat{\theta}(X^*(N), N)) < 1$ is straightforward. $G(\hat{\theta}(X^*(N), N)) = 1$ says that the probability of entry is one, which in turn means that $X^*(N) = 0$. Further, it is clear the oligopoly cannot underinvest if the optimal investment equals zero. The rationale for the restriction $G(\hat{\theta}(X^*(N), N)) > 0$ is as follows. $G(\hat{\theta}(X^*(N), N)) = 0$ says that the probability of entry is zero. This means that under collusion the oligopoly is choosing a corner solution. Further, it is not surprising that, given a corner solution, changing the incentive to invest will not necessarily change the investment.

IV. Model 2: A Variant of the Gilbert and Vives Model

The only difference between the model presented in this section and the one analyzed by Gilbert and Vives is that here there will be uncertainty concerning the exact investment in entry deterrence needed to deter entry.
The structure of the model follows. There are $N$ risk neutral pre-established sellers who produce a homogeneous product and who face a single potential entrant. Each pre-established seller chooses an output level prior to the entry decision, and the pre-established sellers are unable to collude on this choice. Let $x_i$ now denote the output level of pre-established seller $i$, and let $X = \sum_{i=1}^{N} x_i$. The potential entrant is assumed to enter if and only if he anticipates positive profits. That is, the firm will enter if, given the pre-established sellers' choice of $X$, the residual demand curve the firm faces is such that positive profits are possible.

The next aspect of the model to be described is the specification of costs. Each pre-established seller $i$ incurs a cost $C(x)$ when it produces $x$ units of output, where

$$C(x) = S + vx.$$ \hspace{1cm} (5)

In (5) $S$ represents a sunk cost while $v$ is the constant marginal cost of production. On the other hand, the potential entrant incurs a cost $\hat{C}(x)$ when it produces $x$ units of output, where

$$\hat{C}(x) = \begin{cases} F + vx & \text{if } x > 0 \\ 0 & \text{if } x = 0. \end{cases}$$ \hspace{1cm} (6)

The uncertainty in the model is captured by the fact that $F$ is a draw from a random variable which has a cumulative distribution function $H(\ )$. Similar to the previous model, it is assumed that the realization of $F$ is only observed by the potential entrant. Thus, the pre-established sellers base their investments in entry deterrence solely on the distribution function $H(\ )$, while the potential entrant bases his entry decision on the actual realization of $F$. $H(\ )$ is assumed to satisfy the following restrictions: $H(F) = 0$, $H(\bar{F}) = 1$, $H'(z) > 0$ and $H''(z)$ exists for $z \in (F, \bar{F})$. 
Let \( P(X^I) \) be the industry inverse demand function, where \( X^I \) is the industry output. It is assumed that \( P(\cdot) \) is twice continuously differentiable, downward sloping (\( P' < 0 \)), and concave (\( P'' < 0 \)).

Before proceeding it is again convenient to define some additional notation. For all \( X \), let \( \hat{F}(X) \) equal the realization of \( F \) such that the potential entrant just breaks even if he enters. Given this, the potential entrant will enter if and only if \( F < \hat{F}(X) \), or equivalently, the probability of entry equals \( H(\hat{F}(X)) \).

V. Analysis of Model 2

Section III demonstrated that adding uncertainty to Bernheim's model causes the free rider problem to become a significant factor. In this section I demonstrate that in the Gilbert and Vives setting the addition of uncertainty does not have this effect. The analysis in this section follows along the same lines as the analysis of Section III. I begin by presenting a result which is derived by Gilbert and Vives.

**Proposition 3:** If \( F \) can only take on a single value, then in any equilibrium \( X \) will be at least as large as the collusive value of \( X \).

Proposition 3 simply states that, similar to the Bernheim model, in the absence of uncertainty the free rider problem is not a significant factor. We can now go back to the uncertainty case and derive a benchmark with which later results are compared. Let \( X^* \) denote the aggregate output level of the pre-established sellers under the assumption they can collude on output. \( X^* \) is defined by (7).

\[
X^* = \arg \max \left( 1 - H(\hat{F}(X)) \right) P(X)X + H(\hat{F}(X)) \left( P(X + x_E(X))X - vX - NS \right),
\]

where \( x_E(X) \) is the output of the potential entrant given entry, and given
that the pre-established sellers are producing $X$. Further, if $X^*$ is such that $0 < H(\hat{F}(X^*)) < 1$, then (7) yields the following first order condition.\(^9\)

\begin{equation}
(8) \\
A X^* + B = 0,
\end{equation}

where

\begin{equation}
(9a) \\
A = \left[ -\frac{dH(\hat{F}(X^*))}{d\hat{F}(X^*)} \frac{d\hat{F}(X^*)}{dX} (P(X^*) - P(X^* + x_E(X^*))) \\
+ (1-H(\hat{F}(X^*))) P'(X^*) + H(\hat{F}(X^*)) [P'(X^* + x_E(X^*)) \left(1 + \frac{dx_E(X^*)}{dX}\right)] \right],
\end{equation}

and

\begin{equation}
(9b) \\
B = (1-H(\hat{F}(X^*))) P(X^*) + H(\hat{F}(X^*)) P(X^* + x_E(X^*)) - v.
\end{equation}

The next step is to again consider the oligopoly when it cannot collude in the face of uncertainty, and ask how the total investment in entry deterrence compares with that which maximizes the expected joint profits of the oligopoly.\(^10,11\)

**Proposition 4:** If $H(\hat{F}(X^*)) \geq 0$, then any equilibrium must be characterized by $X \geq X^*$.

**Proof:** Let $X_{-i} = \Sigma_{j \neq i} x_j$. The maximization problem faced by pre-established seller $i$ is given by (10).

\begin{equation}
(10) \\
\max_{x_i} \left(1-H(\hat{F}(X_{-i}+x_i))\right) P(X_{-i}+x_i) x_i \\
+ H(\hat{F}(X_{-i}+x_i)) P(X_{-i}+x_i+x_E(X_{-i}+x_i)) x_i - vx_i - S
\end{equation}

If $X = X^*$ and $H(\hat{F}(X^*)) > 0$, then (10) yields the following first order condition.

\begin{equation}
(11) \\
A x_i + B = 0
\end{equation}

Since $B > 0$, a comparison of (8) and (11) yields $X \neq X^*$. 

I now need only demonstrate that $X < X^*$ also yields a contradiction. Suppose $X < X^*$, and let $\delta$ represent the change in the profits of the oligopoly as output is increased from $X$ to $X^*$. $\delta$ is given by (12).

\begin{equation}
\delta = B(X^*-X) + (B-B)X,
\end{equation}

where

\begin{equation}
\hat{B} = (1-H(F(X))) P(X) + H(F(X))P(X + x_E(X)) - v.
\end{equation}

Now hold fixed the output of all pre-established sellers except $i$. Further, let $\delta_i$ represent the change in the profits of seller $i$ as its output is increased from $x_i$ to $x_i + (X^*-X)$. $\delta_i$ is given by (14).

\begin{equation}
\delta_i = B(X^*-X) + (B-B)x_i
\end{equation}

Since $B > 0$ and $\delta > 0$, a comparison of (12) and (14) yields $\delta_i > 0$. Hence, $X < X^*$.

Proposition 4 demonstrates that the free rider problem is not a significant factor in the Gilbert and Vives setting even after uncertainty is introduced. The intuition is as given in the introduction. In the absence of a potential entrant, the pre-established sellers in this setting have an incentive to "over-produce". Hence, there are competing forces. The free rider problem suggests underproduction, while this second factor suggests overproduction. Further, the analysis demonstrates that the second factor is dominant, with the result being that the free rider problem is not important in this setting.

VI. Conclusion

Previous authors who have considered the issue of noncooperative entry deterrence have concluded that the free rider problem is not an important factor. In reaching this conclusion, however, these authors have only
considered models in which the exact investment needed to deter entry is known with certainty. In this paper I have added uncertainty to the models investigated by these previous authors, and shown that once uncertainty is introduced the free rider problem may be important. In particular, adding uncertainty to the model previously considered by Bernheim causes the free rider problem to become important, with the result being that the pre-established sellers have a strong tendency to underinvest in entry deterrence. The logic here is that in the Bernheim setting the entry deterring investment serves no role other than the entry deterring one, and in such an environment the free rider problem will be important as long as uncertainty is present. On the other hand, adding uncertainty to the model previously investigated by Gilbert and Vives does not cause the free rider problem to become important. The reason is that in their setting the entry deterring investment serves another role, such that in the absence of a potential entrant there is a tendency toward overinvestment. Further, as the analysis demonstrates this second factor is always dominant.

One final point concerns what the above conclusions tells us about entry deterring investments other than the specific ones considered in this paper. For example, one might be interested in knowing what would occur in a world where excess capacity deters entry (see e.g., Spence 1977), but where pre-established sellers are unable to collude on investments in capacity. In the model investigated by Spence there is no reason why pre-established sellers would overinvest in capacity in the absence of a potential entrant. Hence, the above results suggest that in a Spence type world the free rider problem will be important as long as uncertainty is present, with the result being an underinvestment in entry deterrence. On the other hand, one could imagine a world where entry is deterred either through expenditures on advertising or
expenditures on R&D. Here one would expect a natural tendency to overinvest in the absence of a potential entrant. Hence, the above results suggest that in this type of world the free rider problem may very well not serve an important role.
Footnotes


2Let \( \hat{\theta}(0,N) = \frac{\theta}{\theta} \) if \( \pi(N+1) > (\xi) 0 \).

3Equation (1) does not necessarily uniquely define \( X^*(N) \). The text will be written as if \( X^*(N) \) were uniquely defined, while in footnote 4 I make it clear how the main result changes if \( X^*(N) \) can take on more than one value.

4Suppose \( X^*(N) \) is not uniquely defined, and call \( X^*(N) \) the lowest of these multiple values for \( X^*(N) \). If \( 0 < G(\theta(X^*(N),N)) < 1 \), then any equilibrium must be characterized by \( X < X^*(N) \).

5In a proof available upon request I demonstrate that for the situation considered in Proposition 2 an equilibrium necessarily exists.

6Note that, as opposed to what was true in Model 1, here the probability of entry only directly depends on the investment in entry deterrence and not directly on the number of pre-established sellers.

7Note that, as opposed to what was true in Model 1, here the collusive entry deterrence level does not depend on the number of pre-established sellers.

8Similar to equation (1) (see footnote 3), equation (7) does not necessarily uniquely define \( X^* \). The text will be written as if \( X^* \) were uniquely defined, while in footnote 11 I make it clear how results change if
X* can take on more than one value.

9If $H(\hat{F}(X^*)) = 0$ the first order condition need not hold. This is because of a possible discontinuity in the marginal benefit to investing in entry deterrence.

10Note that when $H(\hat{F}(X^*)) = 0$, it can only be shown that $X > X^*$. This is because the first order conditions are not necessarily valid for this case (see footnote 9).

11Suppose $X^*$ is not uniquely defined, and call $\bar{X}^*$ the highest of these multiple values for $X^*$. If $H(\hat{F}(\bar{X}^*)) \geq 0$, then any equilibrium must be characterized by $X \geq (\geq) \bar{X}^*$.

12See Waldman (1982) for a formal demonstration of this point.
References


